Tutorial 4: Input/output

In oder to describe the vibrational behaviour of a flexible structure we consider the undamped Timosheko beam model. This model incorporates shear and rotational inertia effects, which makes it a more precise model than Euler Bernoulli model or Rayleigh models. It means that the neutral fiber is no longer perpendicular to the section. In this model it is assumed that:

- All the related deformations and strains are small, which permits the validity of Hookes law.
- Plane cross sections remain plane after the deformation.

The transverse displacement of the beam $w(\zeta, t)$ and the angle of rotation of the neutral fiber $\phi(\zeta, t)$ are defined over the spatial domain $\zeta \in [0, L]$. The shear displacement is defined by $\frac{\partial w}{\partial \zeta}(\zeta, t) - \phi(\zeta, t)$. The motion of the beam is the result of the action of the shear force and bending moment defined by:

$$\begin{aligned} F(\zeta, t) &= K(\zeta) \left(\frac{\partial w}{\partial \zeta}(\zeta, t) - \phi(\zeta, t) \right) \\ M(\zeta, t) &= EI(\zeta) \frac{\partial \phi}{\partial \zeta}(\zeta, t) \end{aligned}$$

and defined from the balance equations on the momenta $\rho(\zeta) \frac{\partial w}{\partial t}(\zeta, t)$ and $I_\rho(\zeta) \frac{\partial \phi}{\partial t}(\zeta, t)$ *i.e.*

$$\begin{aligned} \rho(\zeta) \frac{\partial^2 w}{\partial t^2}(\zeta, t) &= \frac{\partial}{\partial \zeta} \left[K(\zeta) \left(\frac{\partial w}{\partial \zeta}(\zeta, t) - \phi(\zeta, t) \right) \right] \\ I_\rho(\zeta) \frac{\partial^2 \phi}{\partial t^2}(\zeta, t) &= \frac{\partial}{\partial \zeta} \left(EI(\zeta) \frac{\partial \phi}{\partial \zeta}(\zeta, t) \right) + K(\zeta) \left(\frac{\partial w}{\partial \zeta}(\zeta, t) - \phi(\zeta, t) \right) \end{aligned}$$

In what follows we consider the energy variables (extensive variables) as state variables *i.e.*

$$\begin{array}{lcll} x_1(\zeta, t) &=& \frac{\partial w}{\partial \zeta}(\zeta, t) - \phi(\zeta, t) & \text{Shear strain} \\ x_2(\zeta, t) &=& \rho(\zeta, t) \frac{\partial w}{\partial t}(\zeta, t) - \phi(\zeta, t) & \text{Momentum} \\ x_3(\zeta, t) &=& \frac{\partial \phi}{\partial \zeta}(\zeta, t) & \text{Angular strain} \\ x_4(\zeta, t) &=& I_\rho \frac{\partial \phi}{\partial t}(\zeta, t) & \text{Angular momentum} \end{array}$$

Modeling

1. Write the total energy of the system (sum of the potential and kinetic energies) as a quadratic form of the state variables.
2. Derive the co energy variables by differentiating the total energy with respect to the state variables.

3. From balance equations on state variables derive the port Hamiltonian model of this system.

4. Find the matrices P_1 and P_0 defining this PHS and derive the boundary port variables.

Input/output - BCS

1. From the previous parametrization, define the linear combination of boundary port variables that allows to consider the beam is clamped at the left side and controlled through the application of an external force and a torque at the right side.

2. Make explicit the domain such that the differential operator is the generator of C_0-semigroup. Does it define a contraction semigroup or a unitary semigroup.

3. Write the balance equation on the total energy.

4. Do the same work in the case the beam is connected at the both side with mass-spring-damper systems.