Tutorial 1: Modeling

Finite dimensional MS: We consider the mass spring of Figure (1).
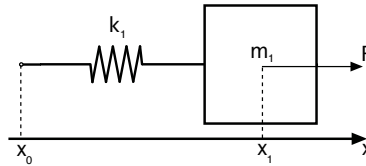

Figure 1: Mass spring system.

- What are the extensive variables characterizing this system ?

- Give the expression of the total energy as a function of the energy variables.

- Write the balance equations associated to this system.

- Give the port Hamiltonian representation of this system considering the external force at point 1 and the velocity at point 0 as inputs. What are the conjugated output ?

Chain of MS: We consider now the interconnexion of two MS systems as described in Figure 2.
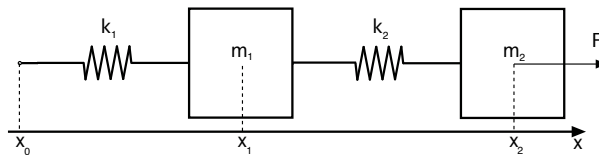

Figure 2: Interconnexion of MSD systems.

- Give the port Hamiltonian representation of each subsystem considering the same input/output than in the previous subsection.

- Write the interconnexion relations between the two subsystems.

- Give the port Hamiltonian representation of the overall system.

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- Extend this representation to a n-elements chain.

Longitudinal vibration of a beam: We consider now the longitudinal deformation of a beam of length L. $u(z,t)$ represents the deformation of the beam, with z the spatial variable, $z \in [0,L]$. The properties of the beam are defined by the distributed mass ρ and the Young modulus E (A is the cross section of the beam). The strain of the beam ε is defined by $\varepsilon = \frac{\partial u}{\partial z}(z,t)$. We consider the stress $\sigma(z,t)$ depends linearly to the strain $\sigma(z,t) = E\varepsilon(z,t)$.
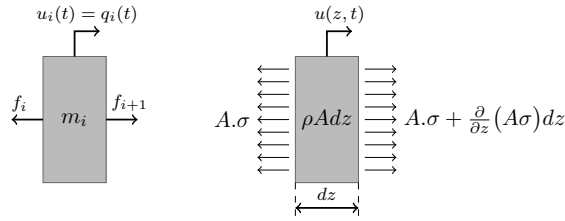

Figure 3: Local description of a beam.

- By analogy with the previous subsections, define the extensive variables associated with this system.

- Give the expression of the total energy of this system.

- Define the co-energy variables and deduce from the balance equations the port Hamiltonian representation of this system.

- Write the balance equation on the energy and the conditions associated with the skew symmetry of the differential operator.