

A model- and derivative-free arclength continuation method for tracing bifurcation diagrams in nonlinear systems

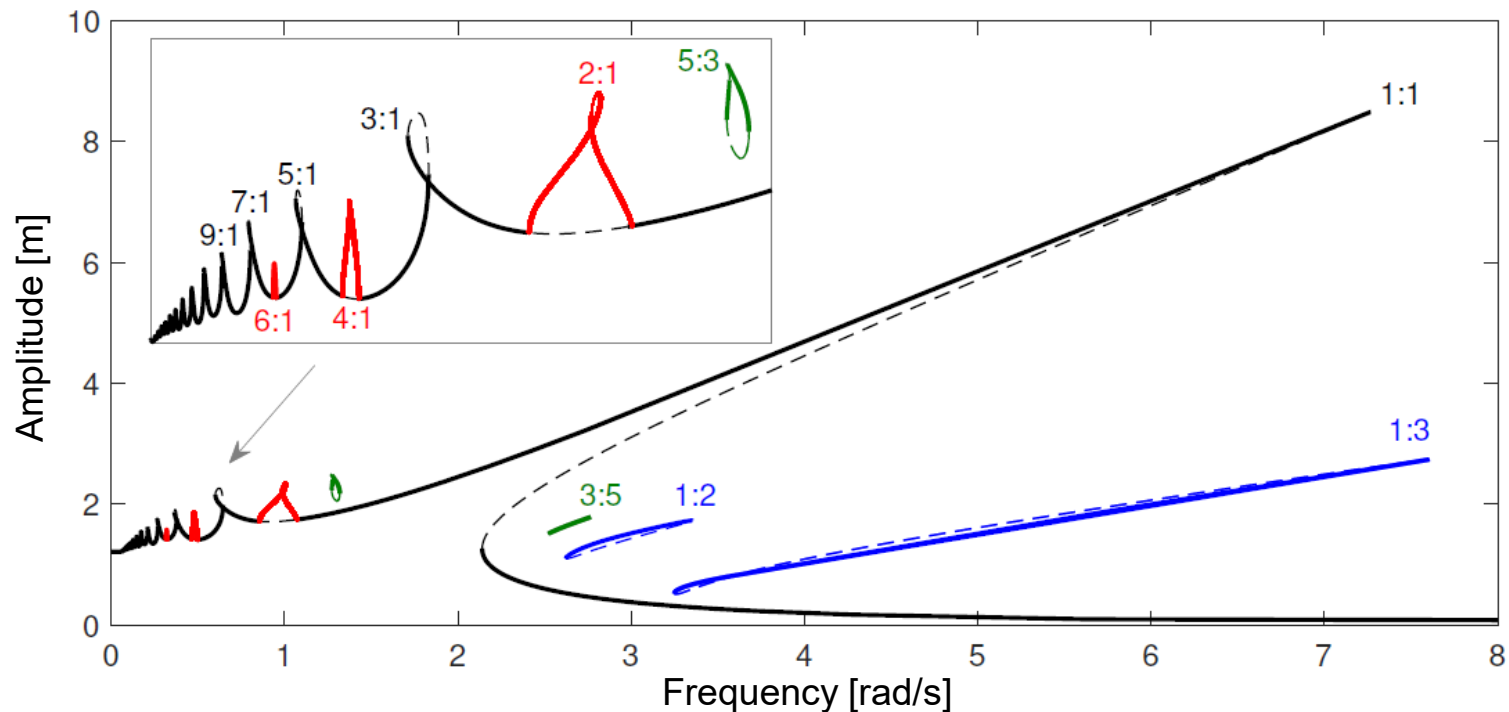
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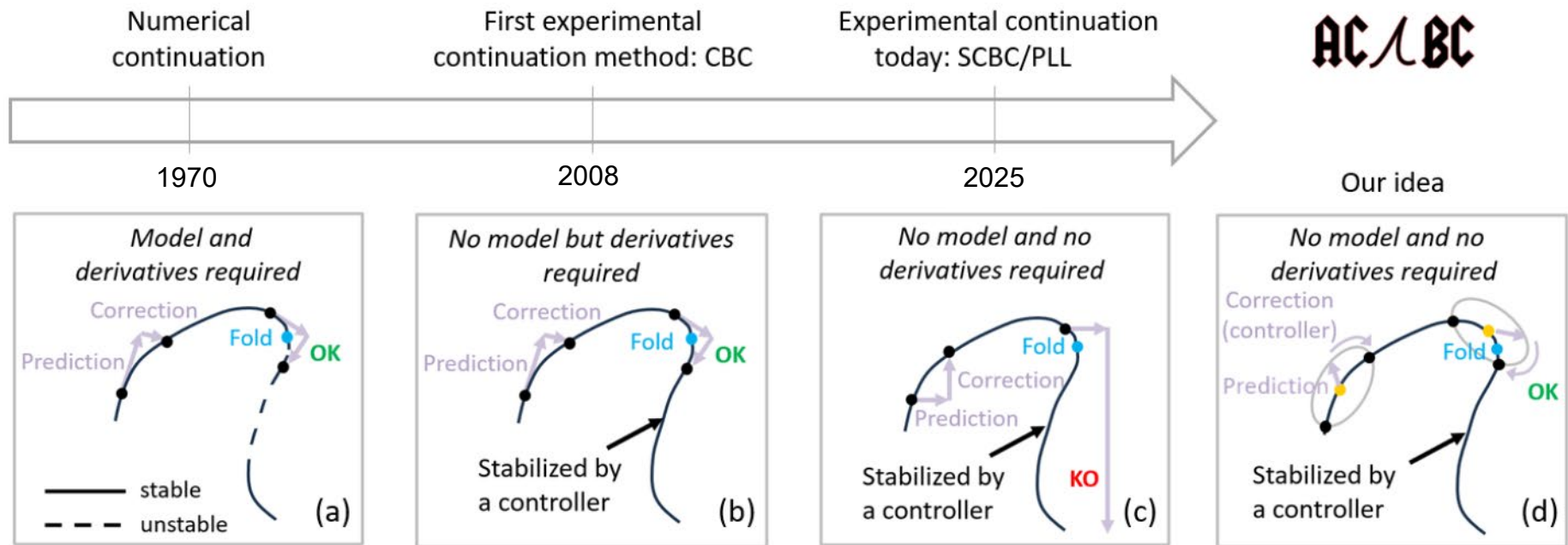
A Duffing oscillator exhibits complex resonant dynamics

$$\ddot{x} + 0.05 \dot{x} + x + x^3 = 3 \sin \omega t$$



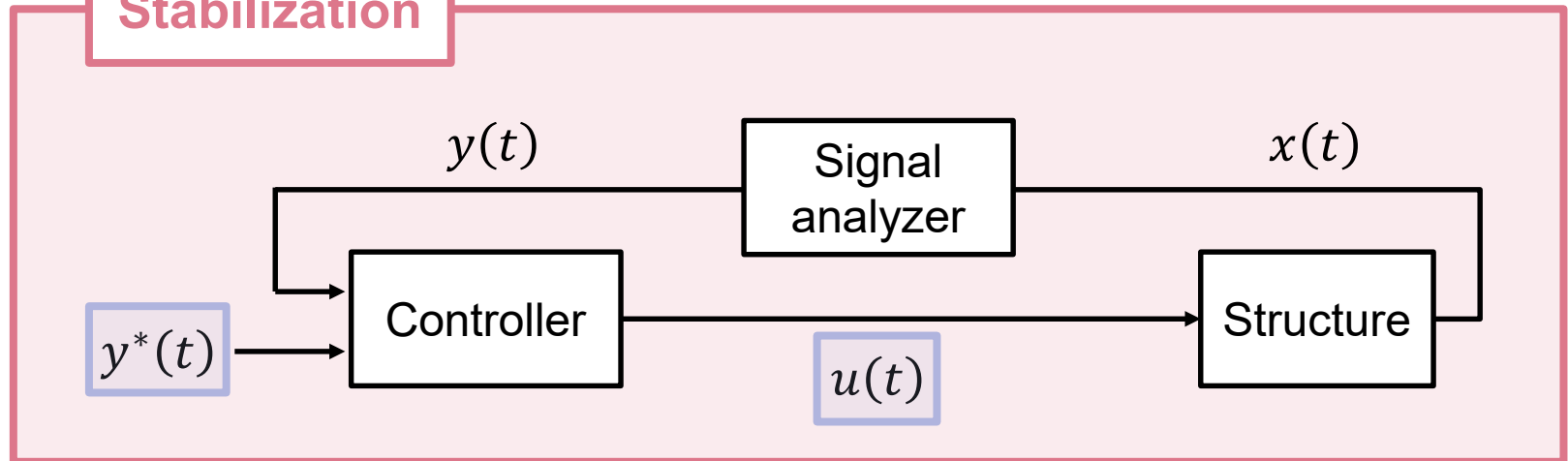
Can we experimentally identify this bifurcation diagram in a model-less manner?

Historical perspective



PLL and SCBC identify responses under harmonic forcing

Stabilization



Sequential continuation

$$u(t) = U \sin(\Omega t)$$

→ PLL and SCBC fix Ω or U and adjust the other

Response
amplitude

Folding

A softening
hardening oscillator

PLL

Forcing frequency

SCBC

constant forcing amplitude U

constant forcing frequency Ω

No folding

Controller:
forcing
Frequency Ω

Response phase

No folding

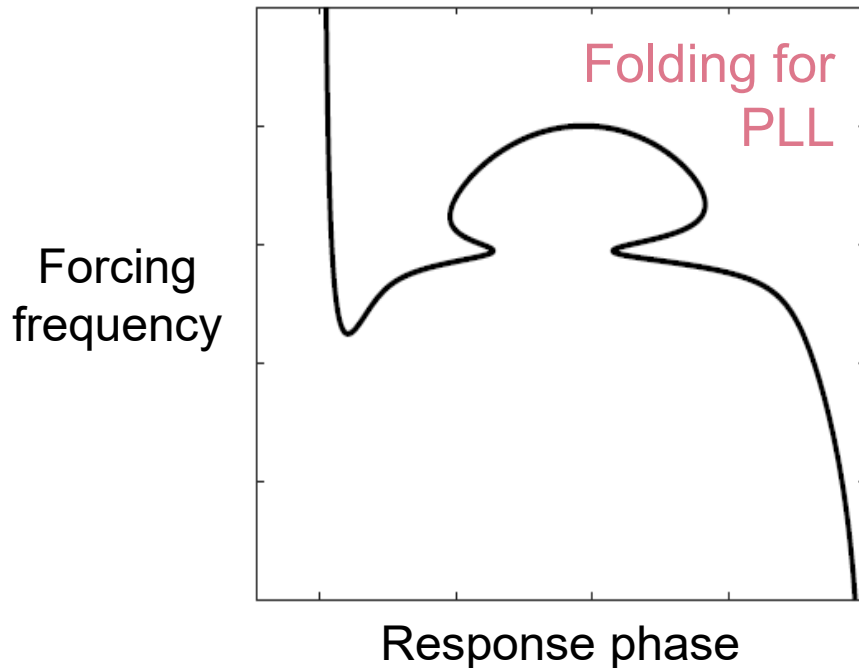
Controller:
forcing
Amplitude U

Response harmonic

Limitation of existing derivative-free approaches

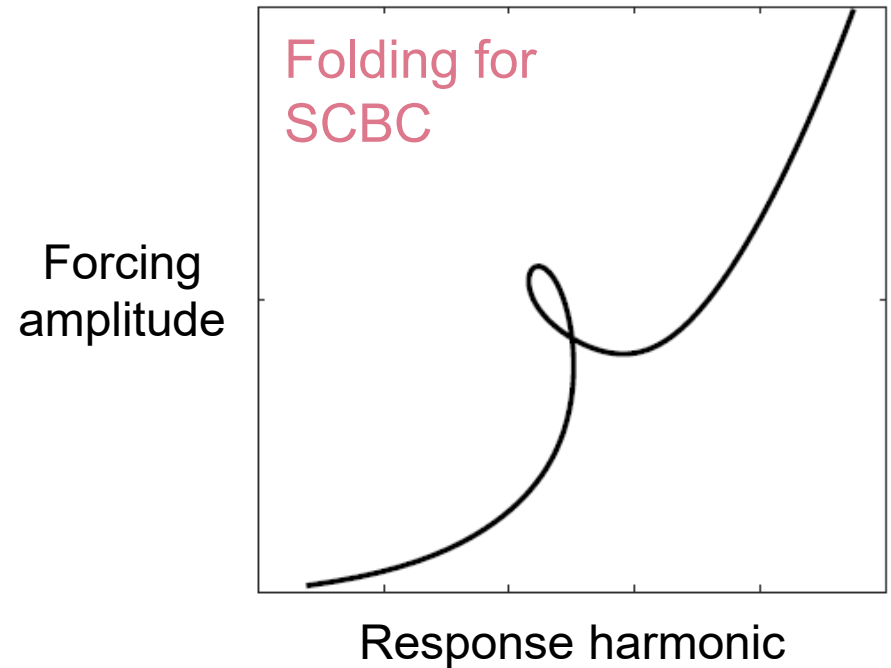
Merging of an isola with a resonance peak

constant forcing amplitude



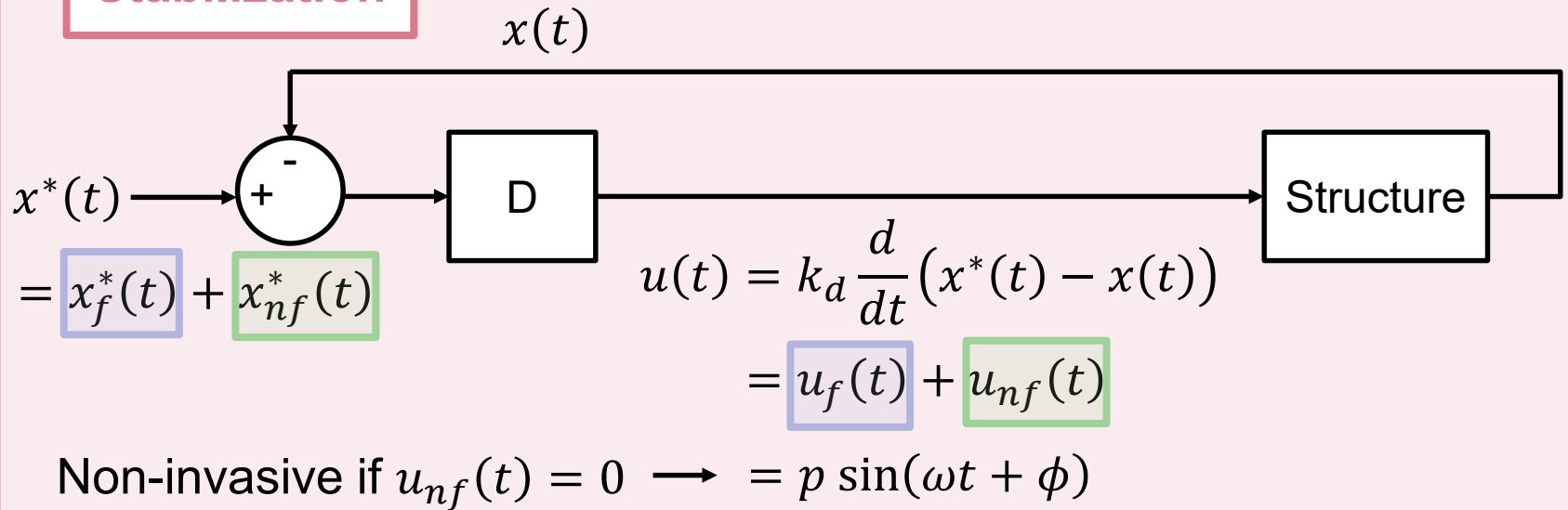
3:1 superharmonic resonance

constant forcing frequency



ACBC identifies responses under harmonic forcing

Stabilization



Arclength continuation

$$u_f(t) = p \sin(\omega t + \phi)$$

→ Modifies $x_f^*(t)$ until $p = p^*$

Adaptive filters

$$u_{nf}(t) = 0$$

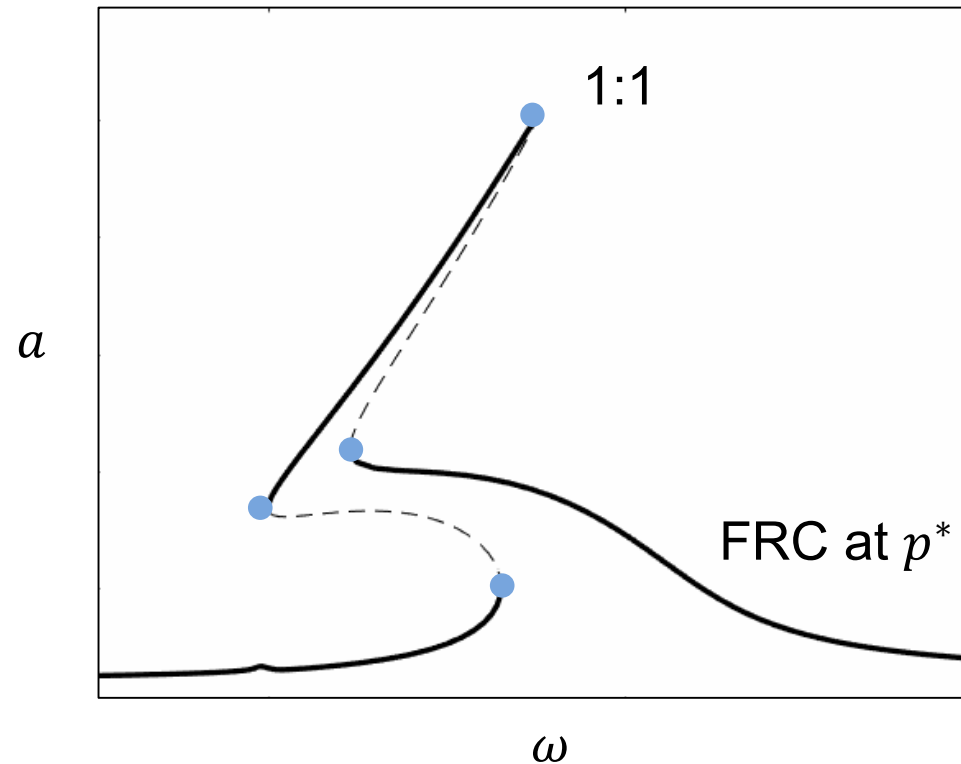
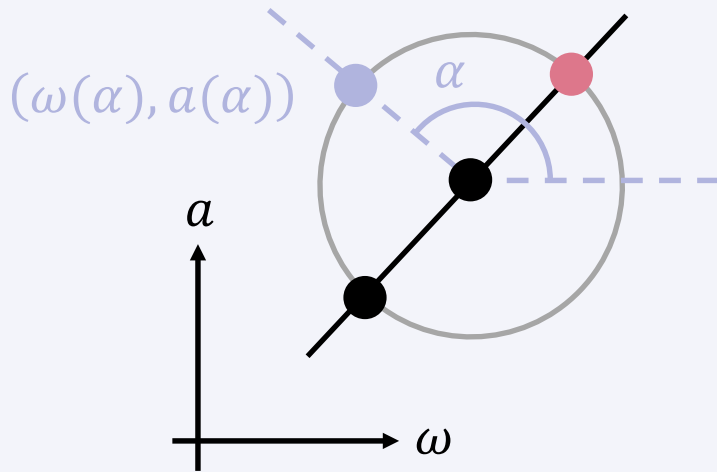
→ Enforce $x_{nf}^*(t) = x_{nf}(t)$

ACBC goes through bifurcations in a derivative-free manner

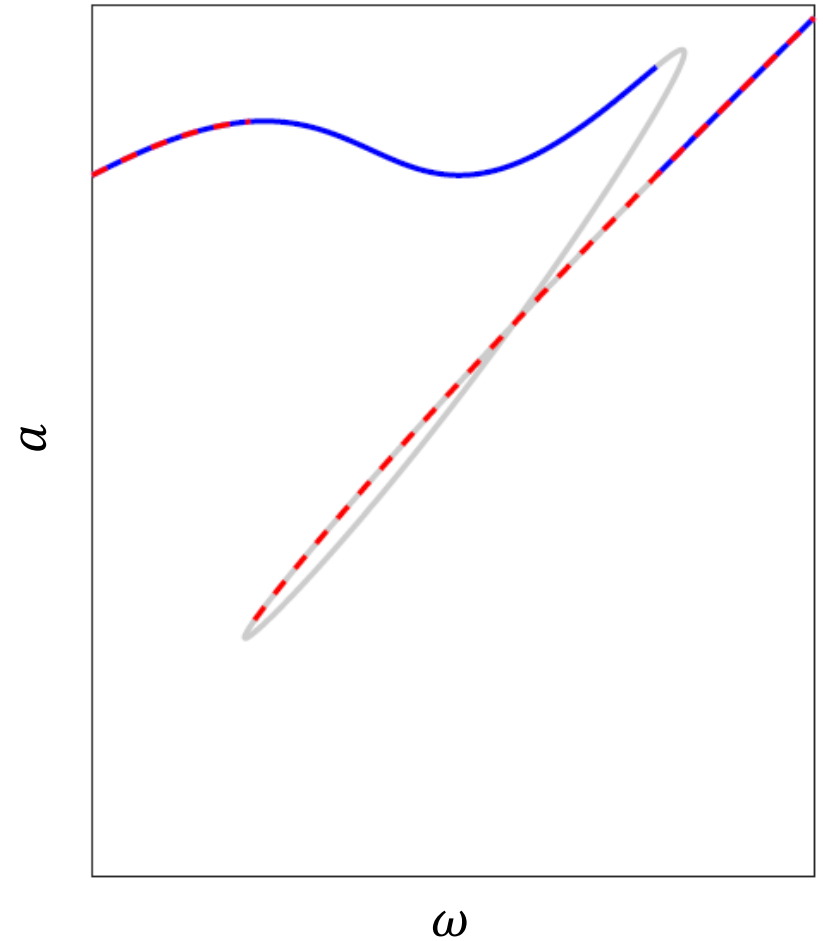
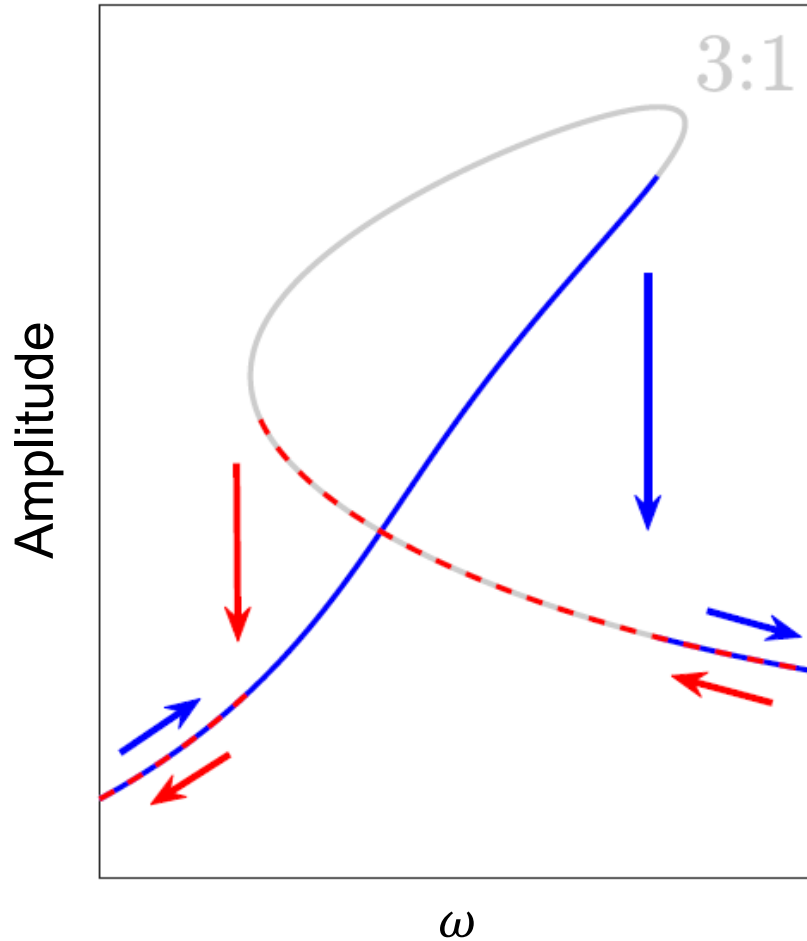
Arclength continuation

→ Modifies $x_f^*(t) = a \sin(\omega t)$

$$p(\omega, a) - p^* = 0$$



ACBC fails for secondary resonances



Unlike ACBC, x-ACBC uncovers secondary resonances

$$u(t) = u_f(t) + u_{nf,r}(t) + u_{nf,nr}(t) = p \sin(\omega t + \phi)$$

Control of the resonant harmonic component

$$u_{nf,r}(t) = 0$$

→ Enforces $x_{nf,r}^*(t) = x_{nf,r}(t)$ and reaches the desired branch

Arclength continuation

$$u_f(t) = p \sin(\omega t + \phi)$$

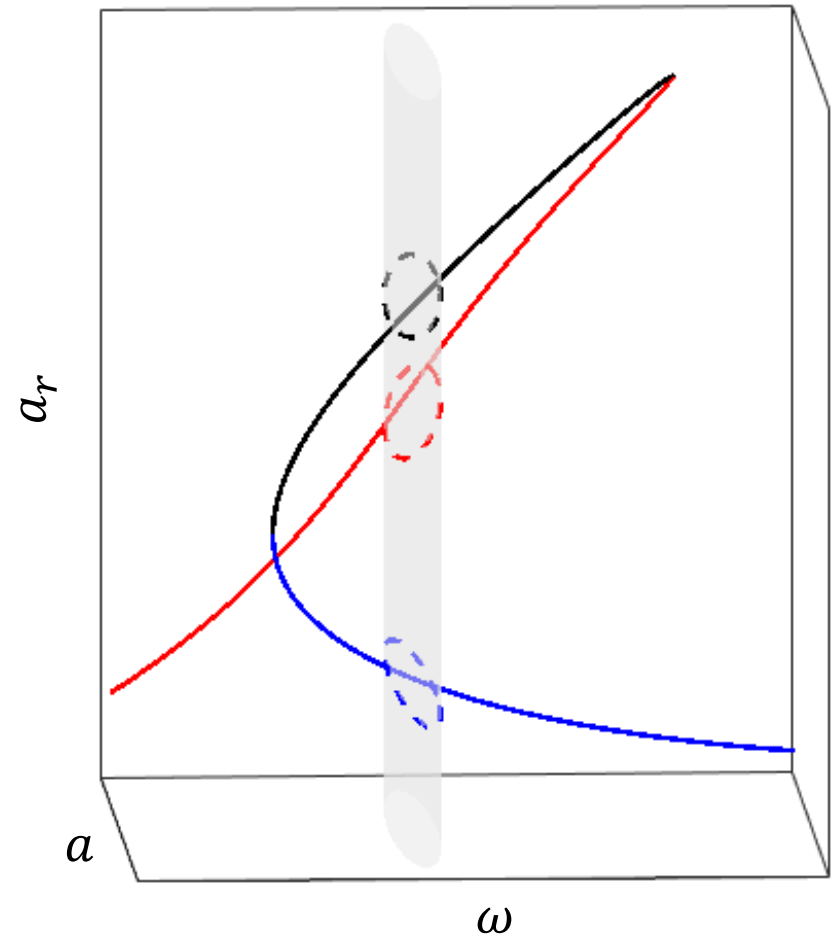
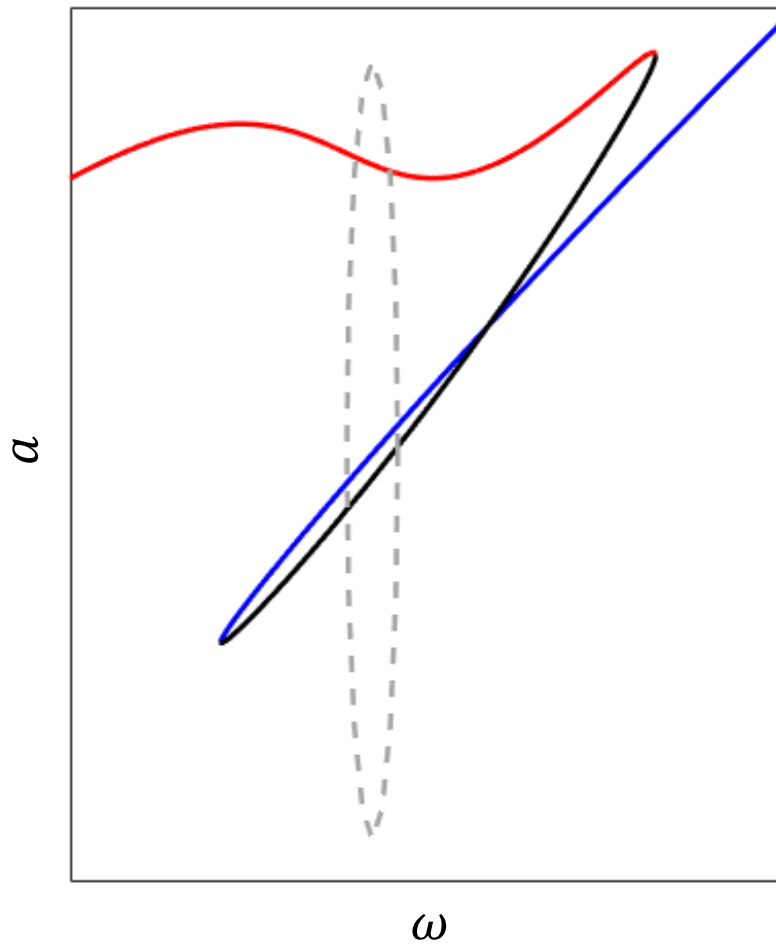
→ Modifies $x_f^*(t)$ until $p = p^*$

Adaptive filters

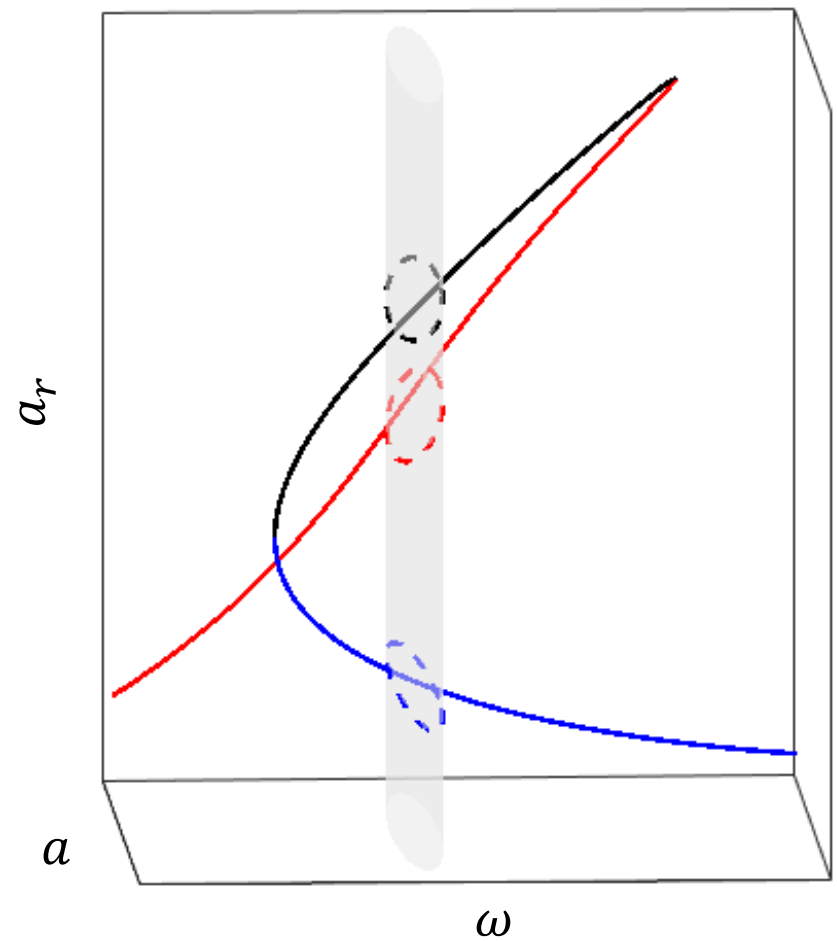
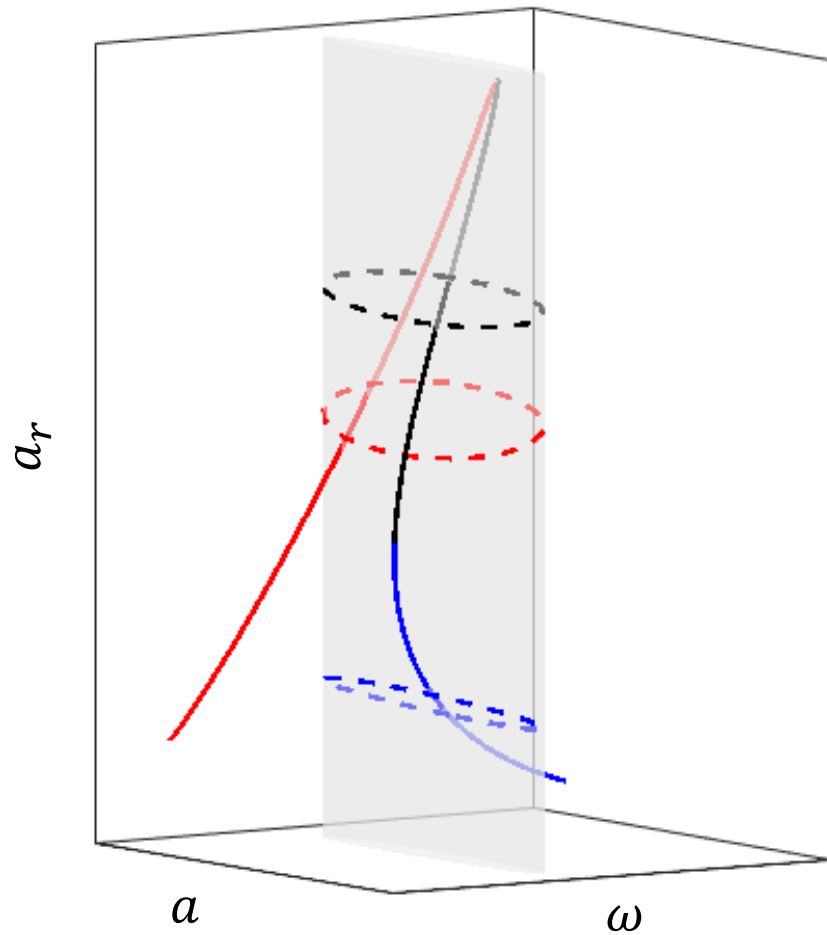
$$u_{nf,nr}(t) = 0$$

→ Enforce $x_{nf,nr}^*(t) = x_{nf,nr}(t)$

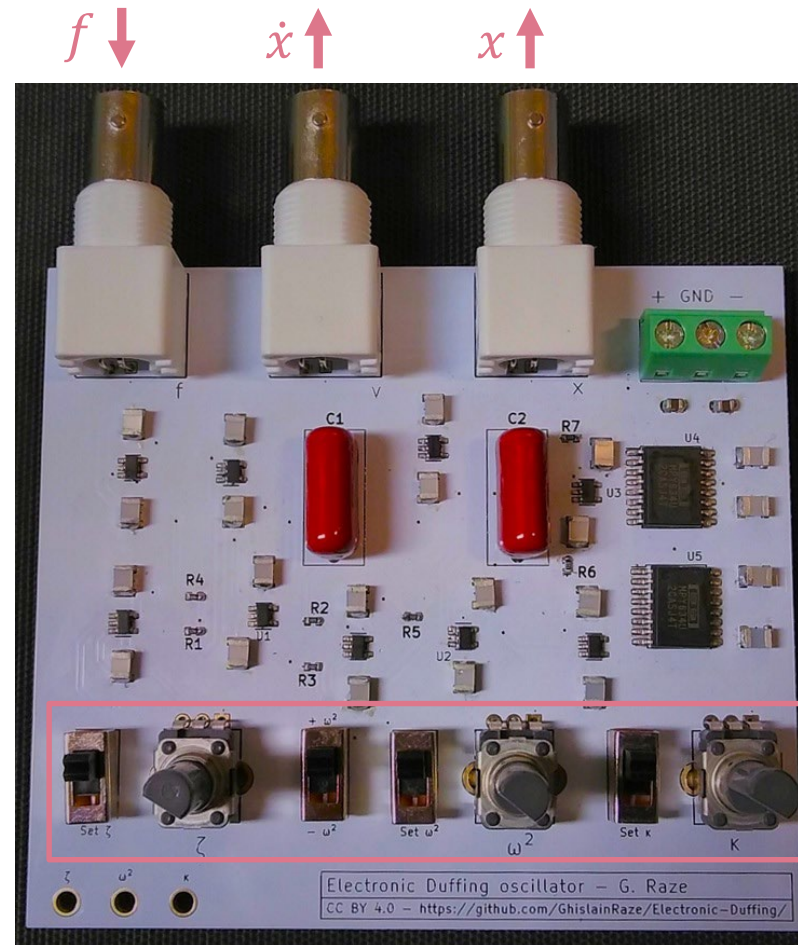
Influence of the resonant non-fundamental harmonic



Influence of the resonant non-fundamental harmonic

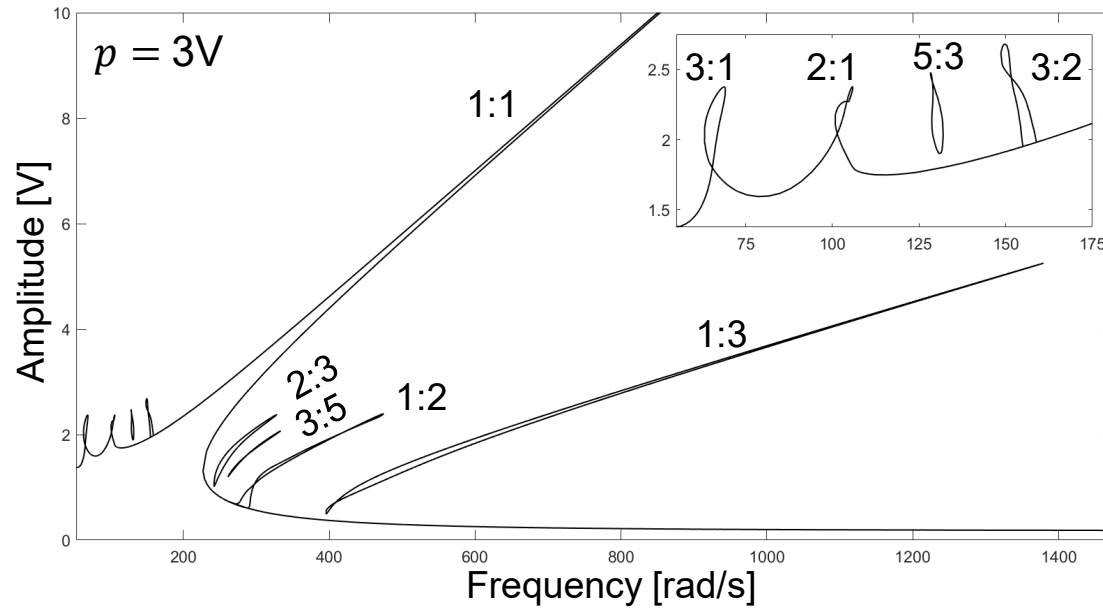


Experimental validation on an electronic Duffing oscillator

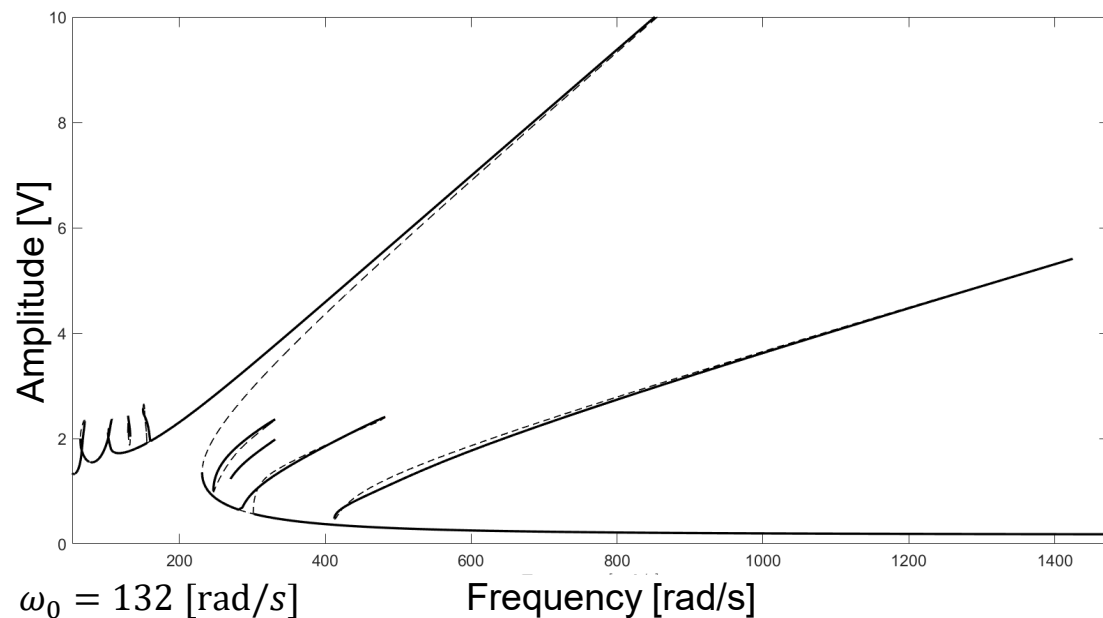


$$10^{-4} \ddot{x} + 1.49 \times 10^{-4} \dot{x} + 1.75 x + 0.99 x^3 = p \sin \omega t = f(t)$$

ACBC fully identifies the complex resonant behavior



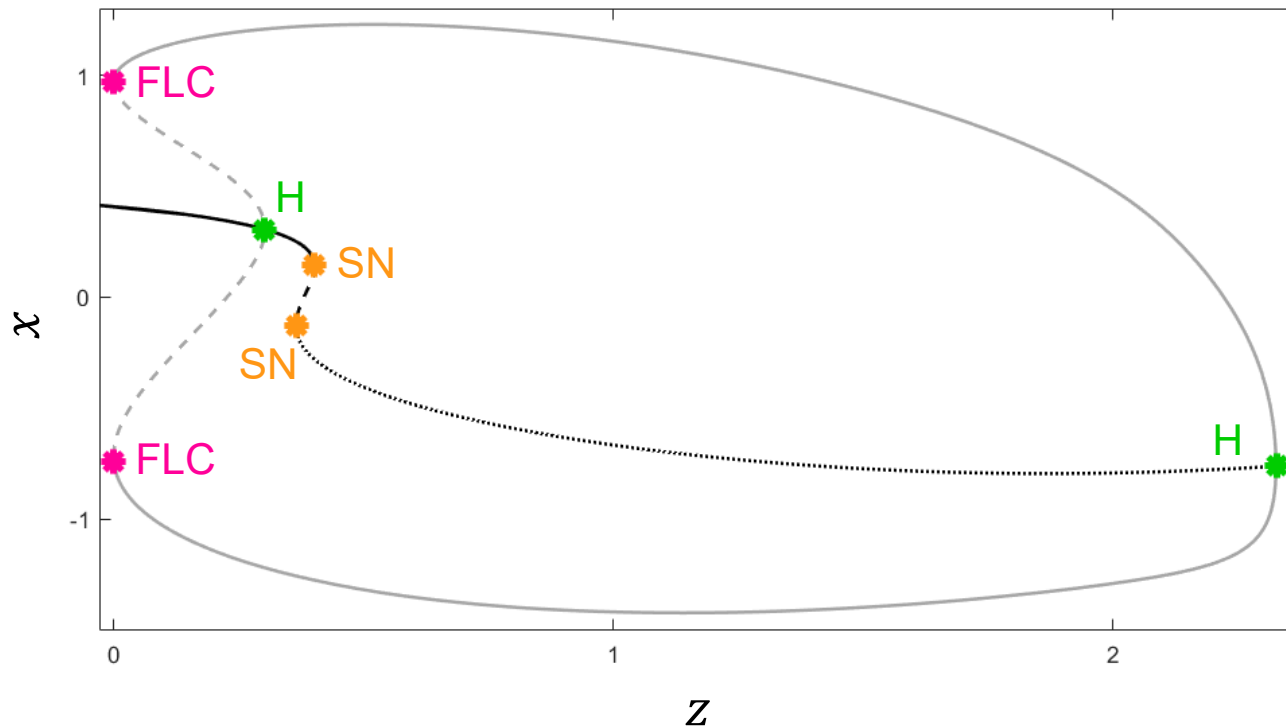
Experimental
(x-ACBC)



Numerical
(HB + pseudo-arclength
continuation)

Conclusions

- ▶ A first: x-ACBC can identify all resonances of a Duffing oscillator including its subharmonic and ultrasubharmonic resonances.
- ▶ ACBC can also be applied to autonomous systems to uncover fixed points and limit cycles.



Thank you for your
attention!

Stay tuned, an article is on the way!

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