Laboratoire de Mécanique et d'Acoustique

Vers la continuation expérimentale de points de bifurcation de Fold

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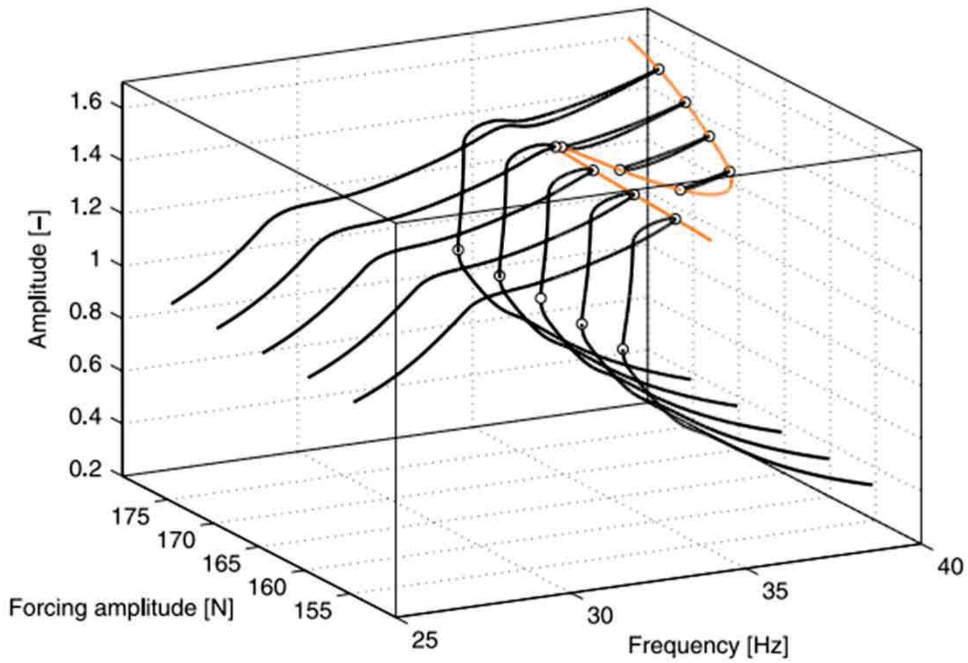
EX-MODELI
EXPLOITATION & MODÉLISATION des DYNAMIQUES NON LINÉAIRES
GdR CNRS 2134

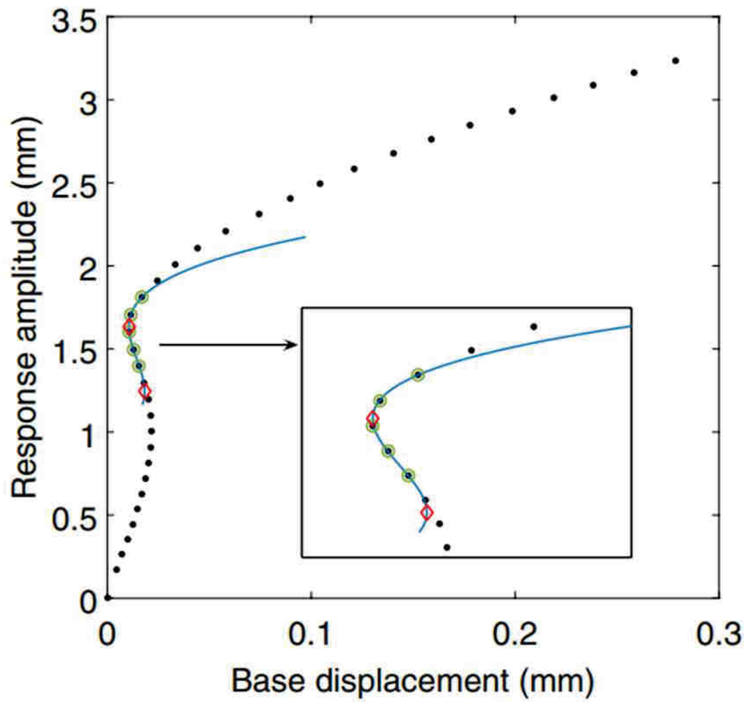
Journées du GdR
Octobre 2025

Tracking fold bifurcation

Numerical continuation of Fold bifurcations



Experimental detection of Folds



T. Detroux, L. Renson, L.Masset, G. Kerschen, *The harmonic balance method for bifurcation analysis of large-scale nonlinear mechanical systems* , CMAME, 2015
L. Renson, D. Barton, S. Nield, *Experimental Tracking of Limit-Point Bifurcations and Backbone Curves Using Control-Based Continuation*, IJBC, 2016

S-Curve Control Based Continuation

Derivative feedback

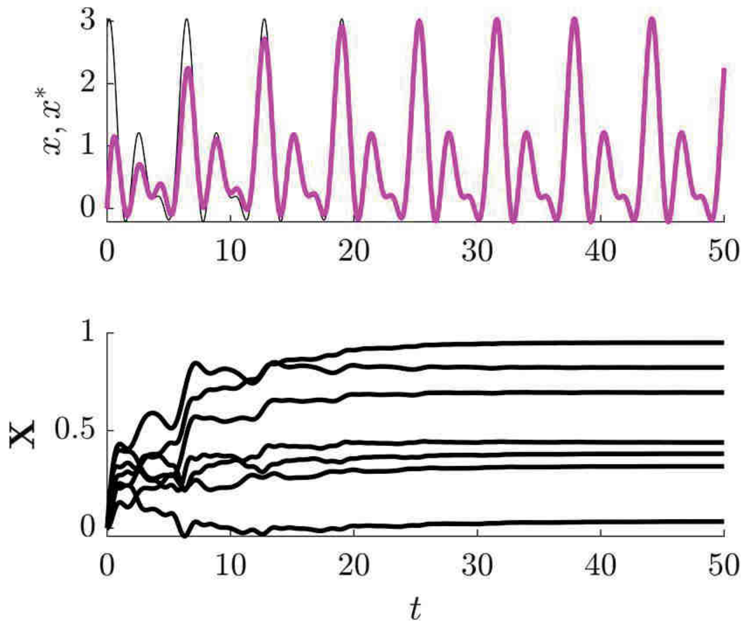
$$\ddot{x} + x + f_{nl}(x, \dot{x}) = K_d(\dot{u} - \dot{x})$$

Adaptive filter

Online estimation of Fourier coefficients

$$x^*(t) = X_0 + \sum_{n=1}^{N_h} X_{c,n} \cos(n\Omega t) + X_{s,n} \sin(n\Omega t) = \boldsymbol{Q}^T \boldsymbol{X}$$

$$\dot{\boldsymbol{X}} = \mu \boldsymbol{Q}(x - \boldsymbol{Q}^T \boldsymbol{X})$$



S-Curve Control Based Continuation

Synthesis of the control signal $u(t)$

Non-fundamental harmonics

$$u(t) = \boxed{X_0} + \boxed{U_{c,1}\cos(\Omega t)} + \boxed{\sum_{n=2}^{N_h} X_{c,n}\cos(n\Omega t) + X_{s,n}\sin(n\Omega t)} = \boldsymbol{Q}^T\boldsymbol{U}$$

User supplied fundamental harmonic

S-Curve Control Based Continuation

Synthesis of the control signal $u(t)$

Non-fundamental harmonics

$$u(t) = \boxed{X_0} + \boxed{U_{c,1}\cos(\Omega t)} + \boxed{\sum_{n=2}^{N_h} X_{c,n}\cos(n\Omega t) + X_{s,n}\sin(n\Omega t)} = \boldsymbol{Q}^T\boldsymbol{U}$$

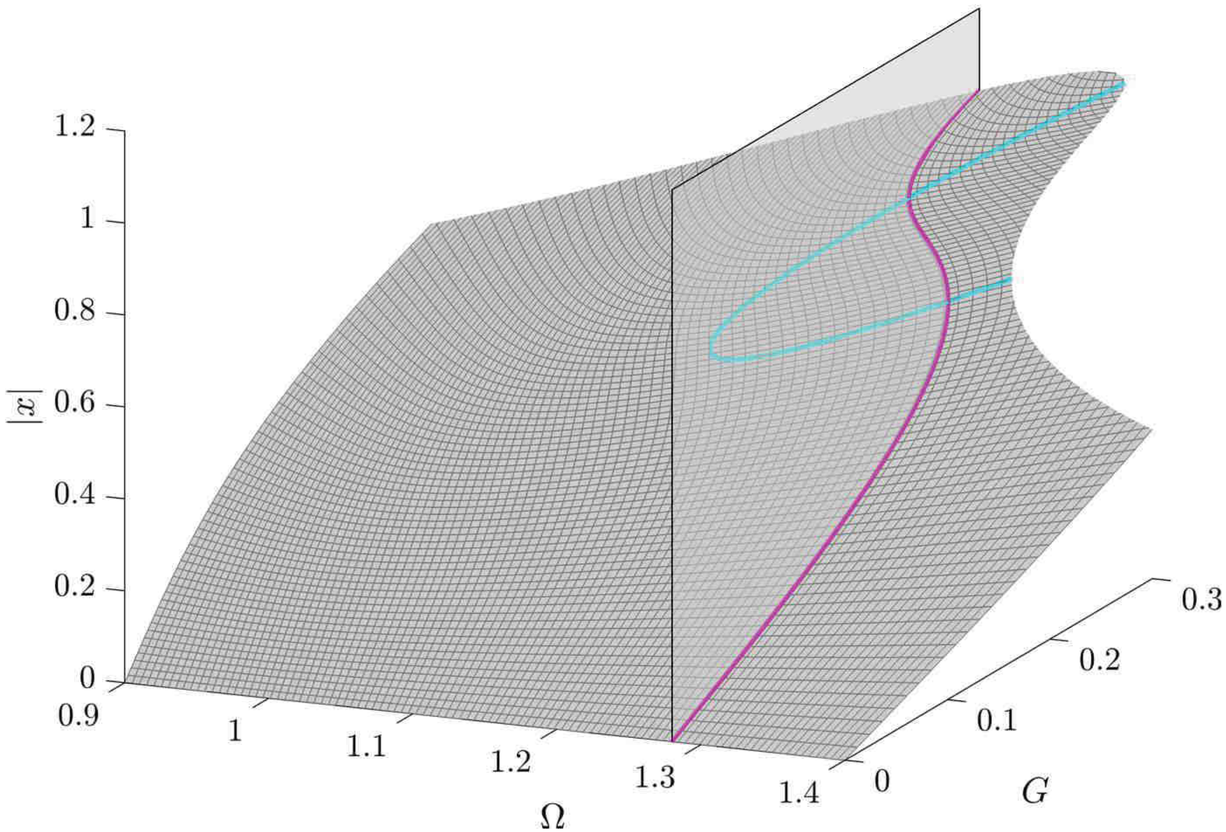
User supplied fundamental harmonic

Considering steady state (i.e. $\dot{\boldsymbol{X}} \doteq 0$) and perfect adaptive filter (i.e. $x^* = x$)

Equivalent forcing amplitude

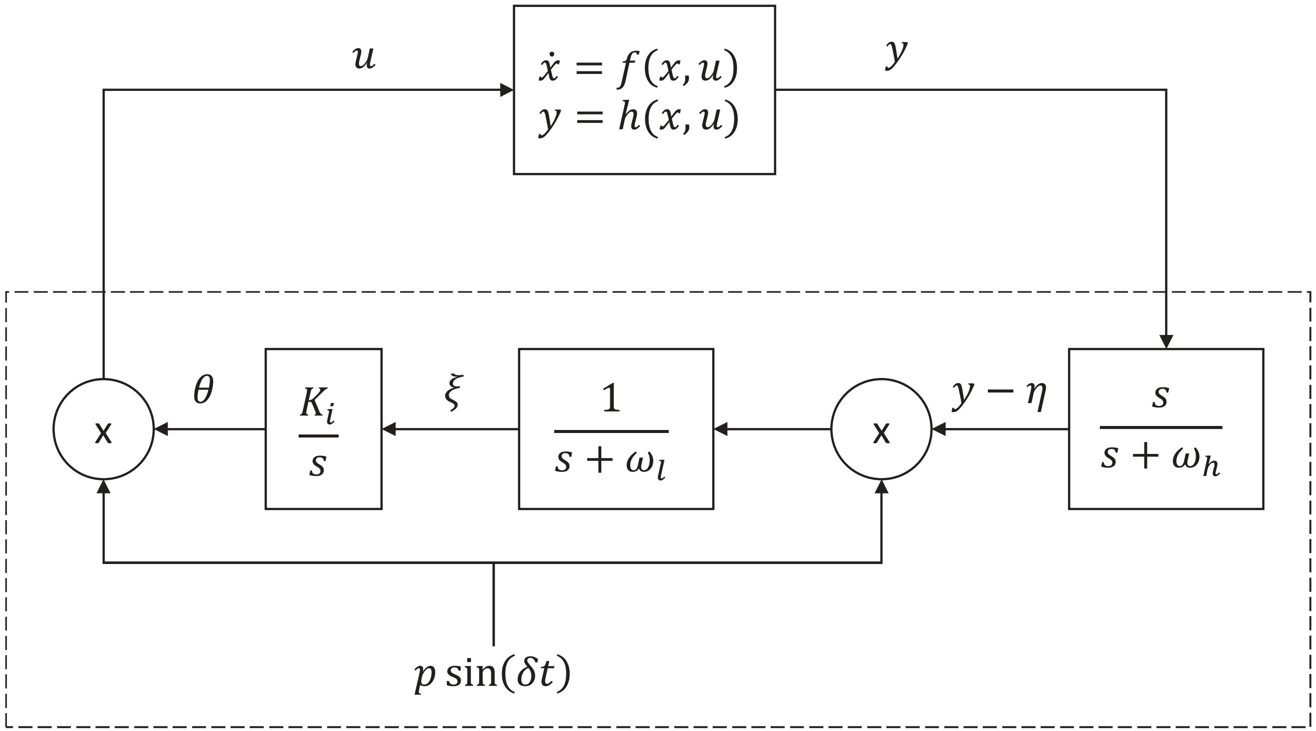
$$g(t) = K_d(\dot{u} - \dot{x}) = G\cos(\Omega t) \qquad \text{with} \qquad G = K_d\Omega\sqrt{\left(U_{c,1} - X_{c,1}\right)^2 + X_{s,1}^2}$$

S-Curve Control Based Continuation



Extremum seeking control: an adaptive controller

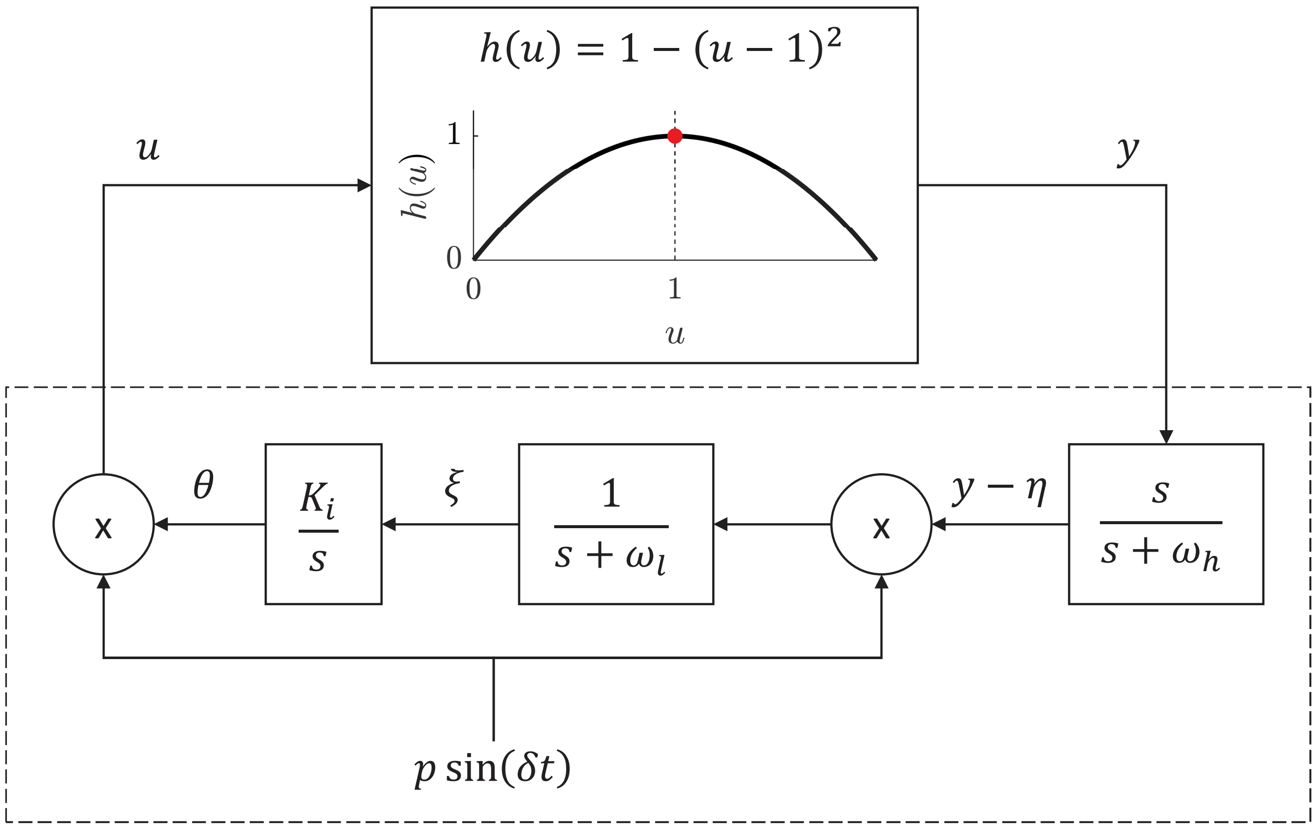
Objective maximize the output y of an unknown dynamical system



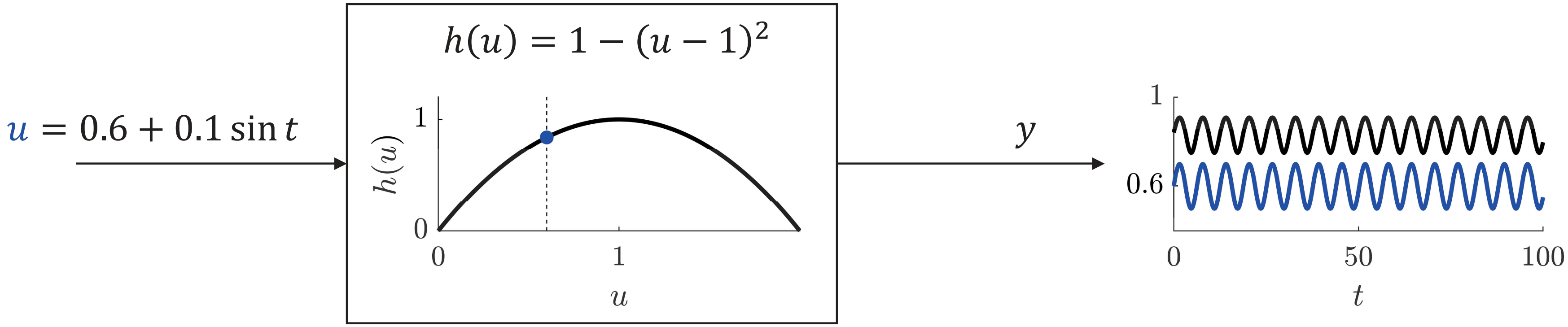
M. Leblanc, *Sur l'électrification des chemins de fer au moyen de courants alternatifs de fréquence élevée*, Revue générale de l'électricité, 1922

M. Krstic, H-H Wang, *Stability of extremum seeking feedback for general nonlinear dynamic system*, Automatica, 2000

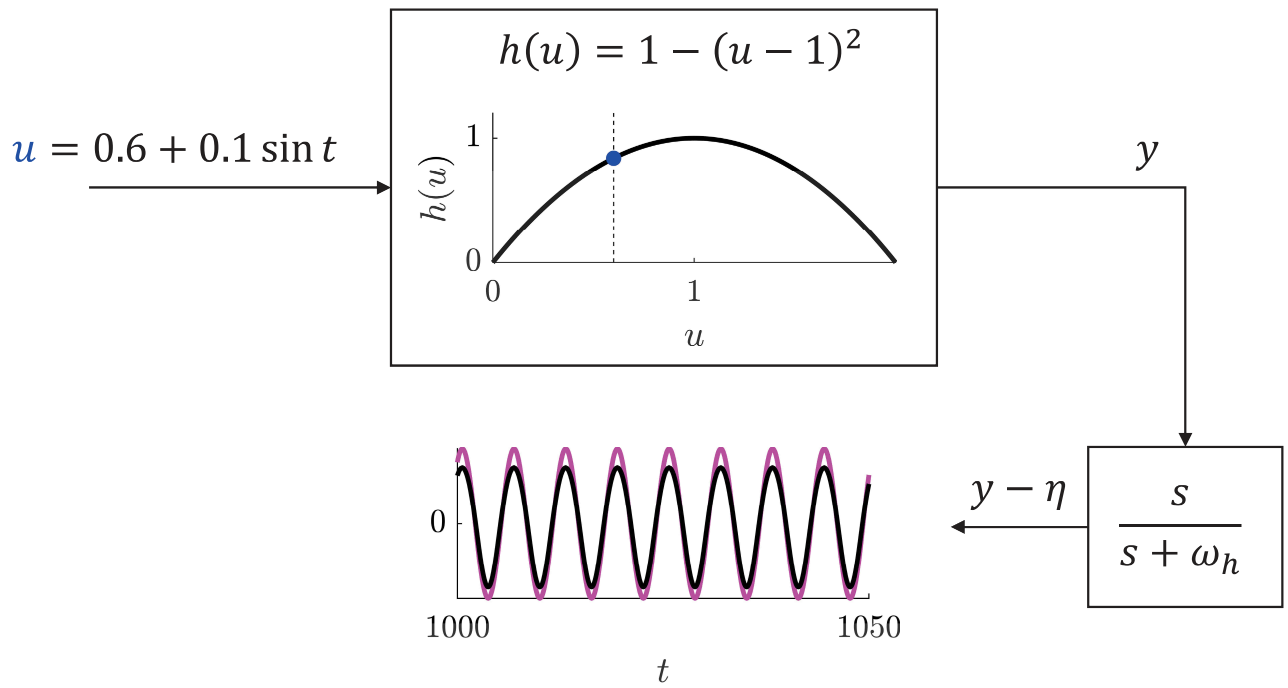
Extremum seeking control: how it works



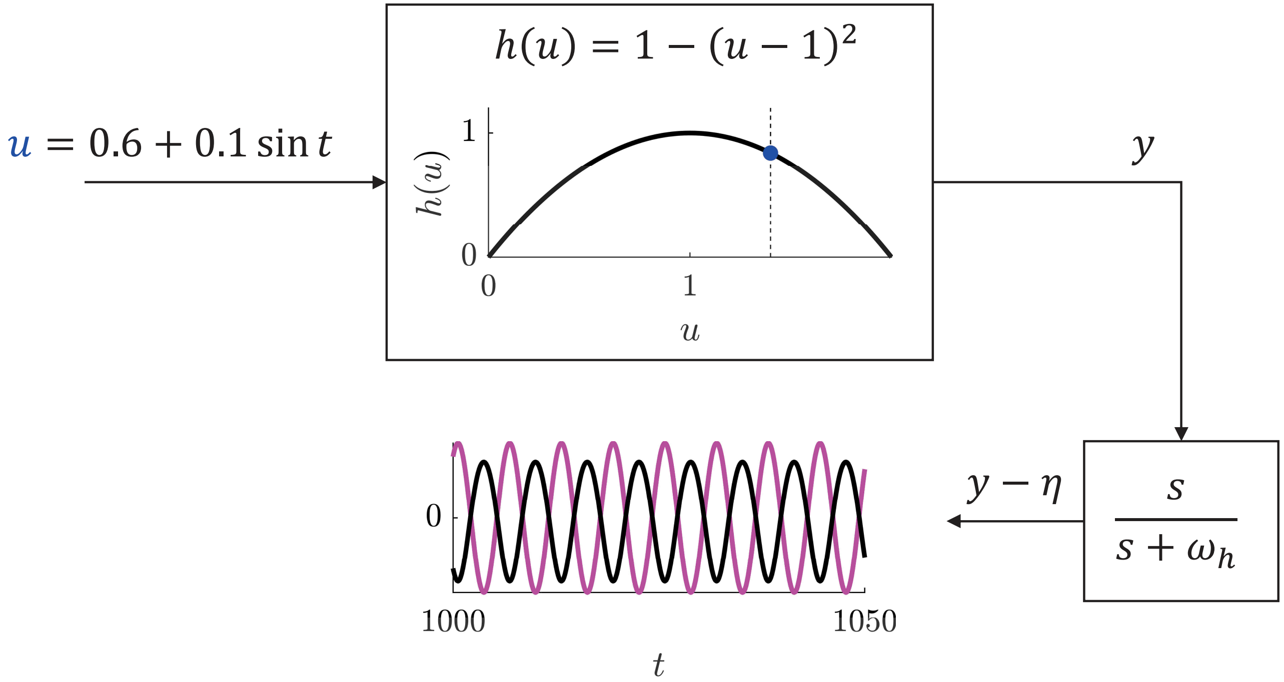
Extremum seeking control: how it works



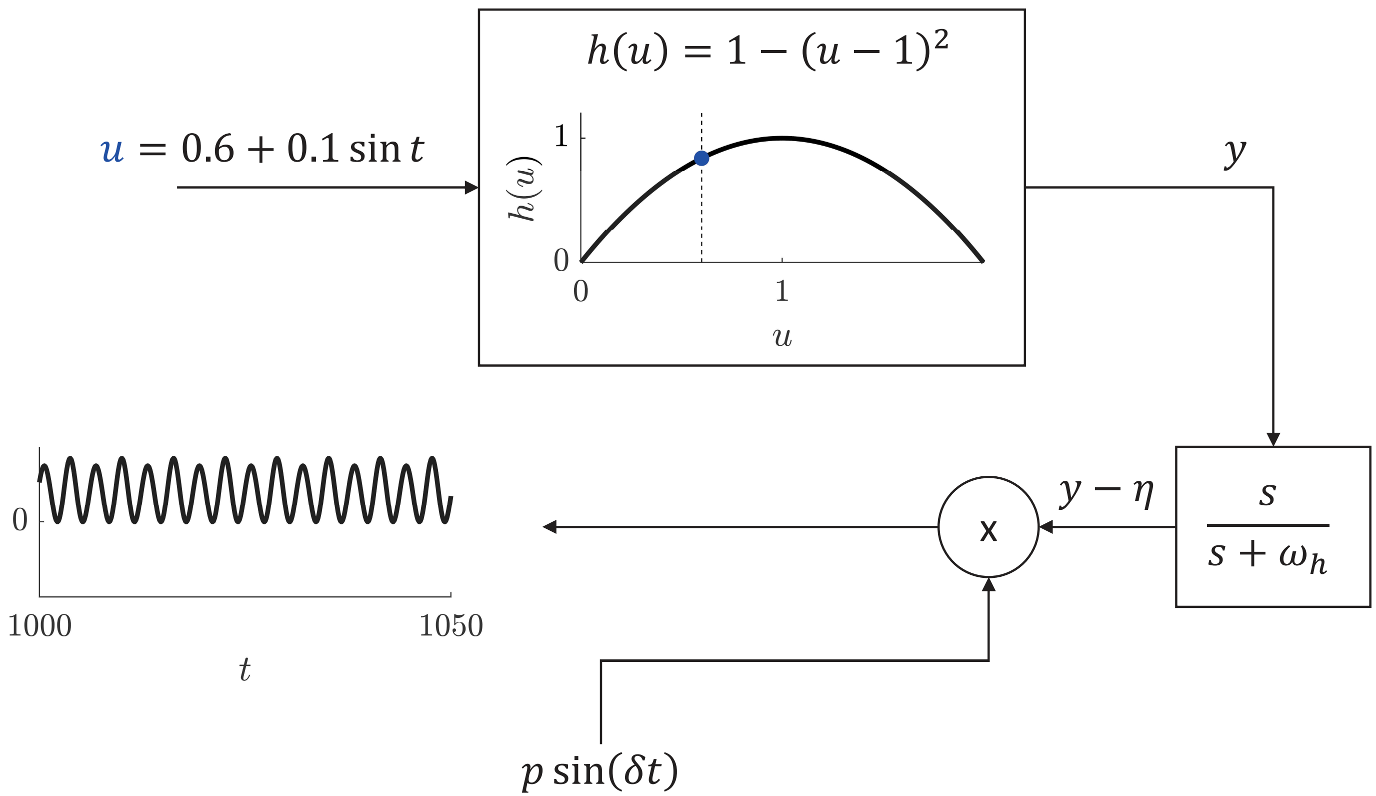
Extremum seeking control: how it works



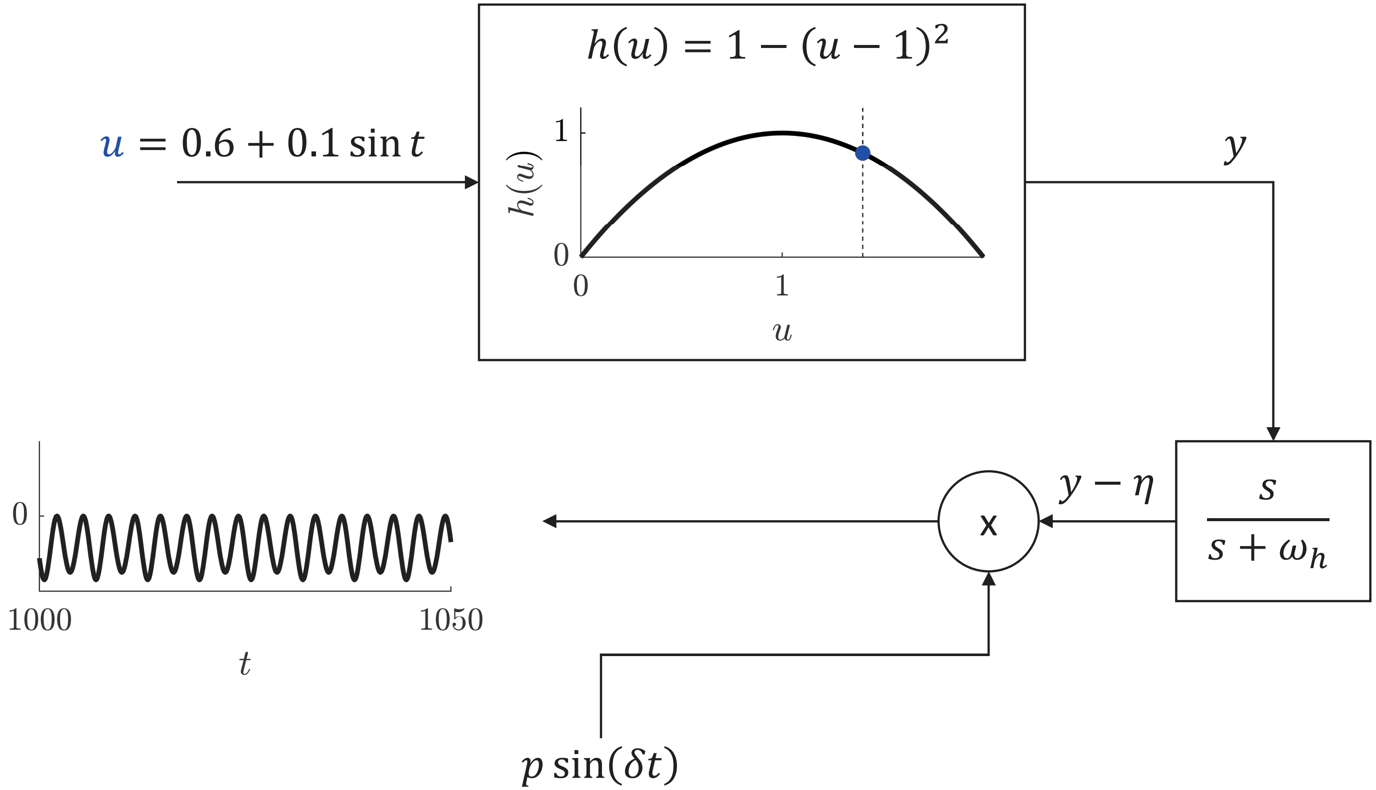
Extremum seeking control: how it works



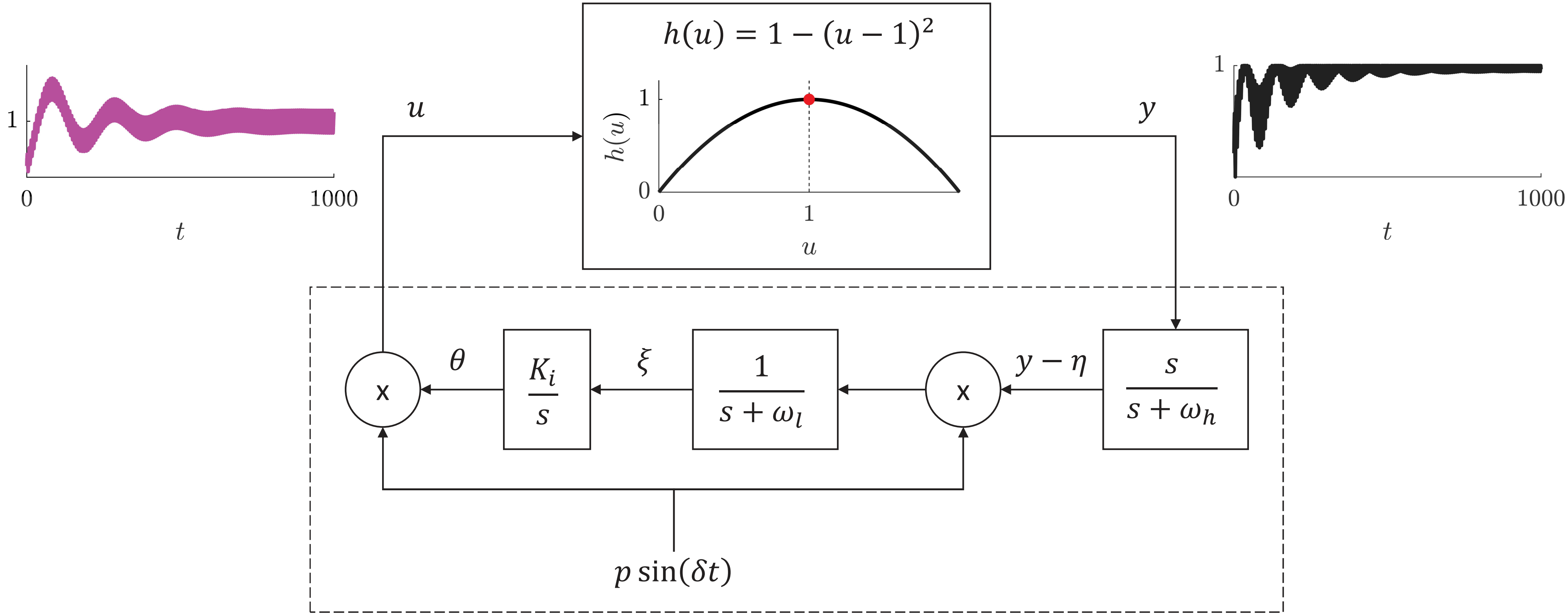
Extremum seeking control: how it works



Extremum seeking control: how it works



Extremum seeking control: how it works



Summary of the controller

$$\ddot{x} + x + f_{nl}(x, \dot{x}) = K_d(\boldsymbol{Q}^T\boldsymbol{U} - \dot{x})$$

Derivative feedback control

$$\dot{\boldsymbol{X}} = \mu\boldsymbol{Q}(x - \boldsymbol{Q}^T\boldsymbol{X})$$

Adaptive filter

$$\dot{\eta} = \omega_h(y - \eta)$$

$$\dot{\xi} = \omega_l\big((y - \eta)\sin(\delta t) - \xi\big)$$

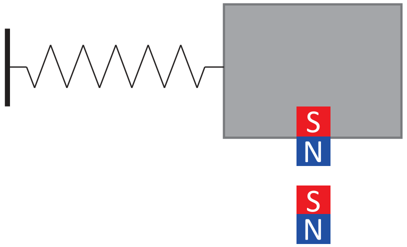
$$\dot{\theta} = K_i\xi$$

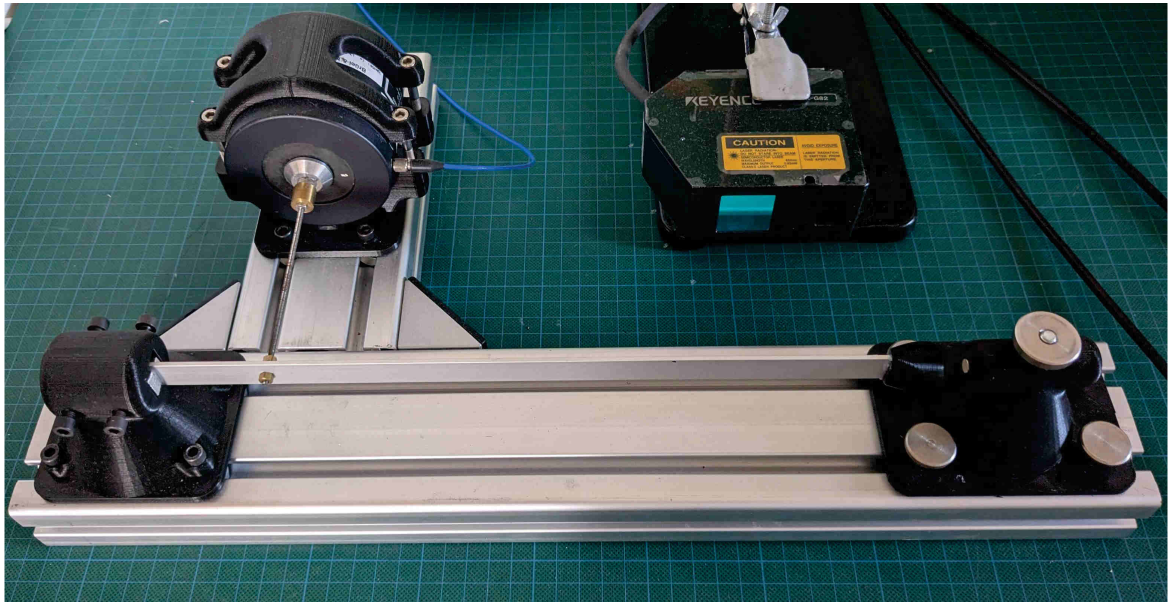
Extremum seeking control

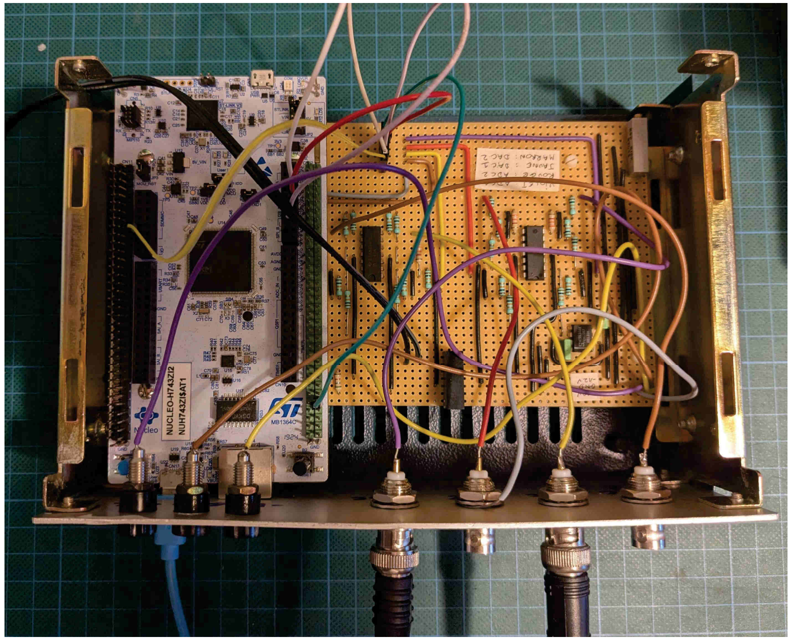
$$y \equiv G = K_d\Omega\sqrt{\left(\boxed{U_{c,1}} - X_{c,1}\right)^2 + X_{s,1}^2}$$

$$U_{c,1} = \theta + p\sin(\delta t)$$

Experimental setup



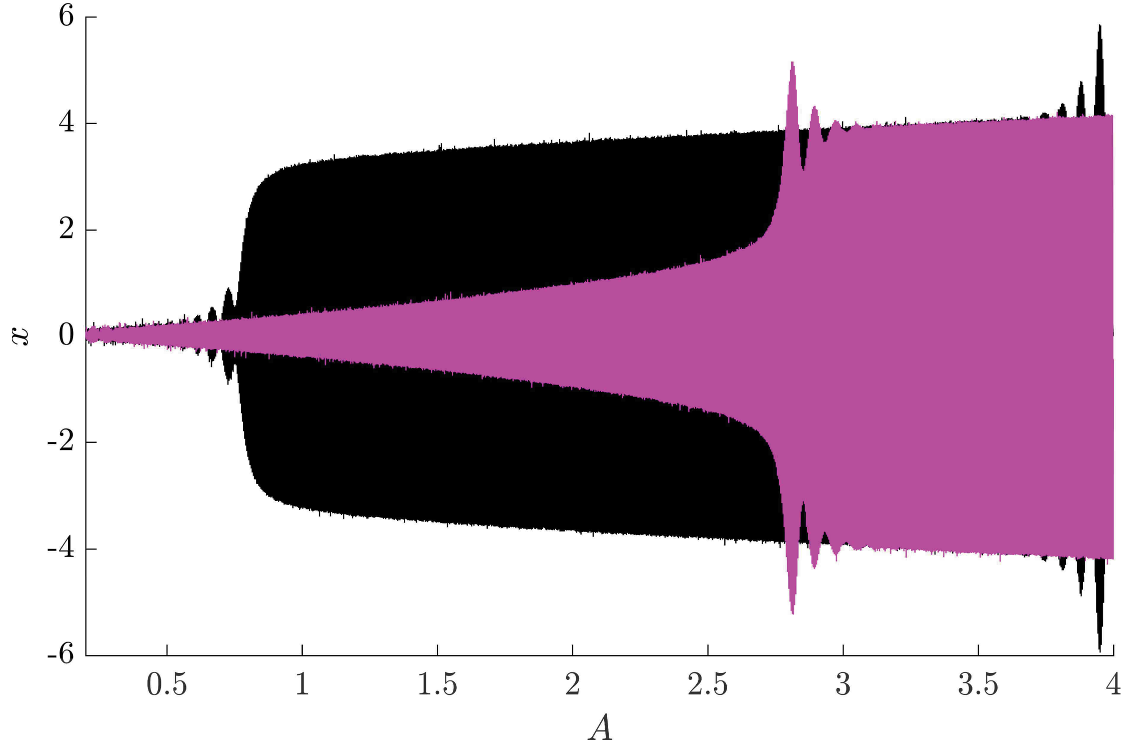




STM32 MCU

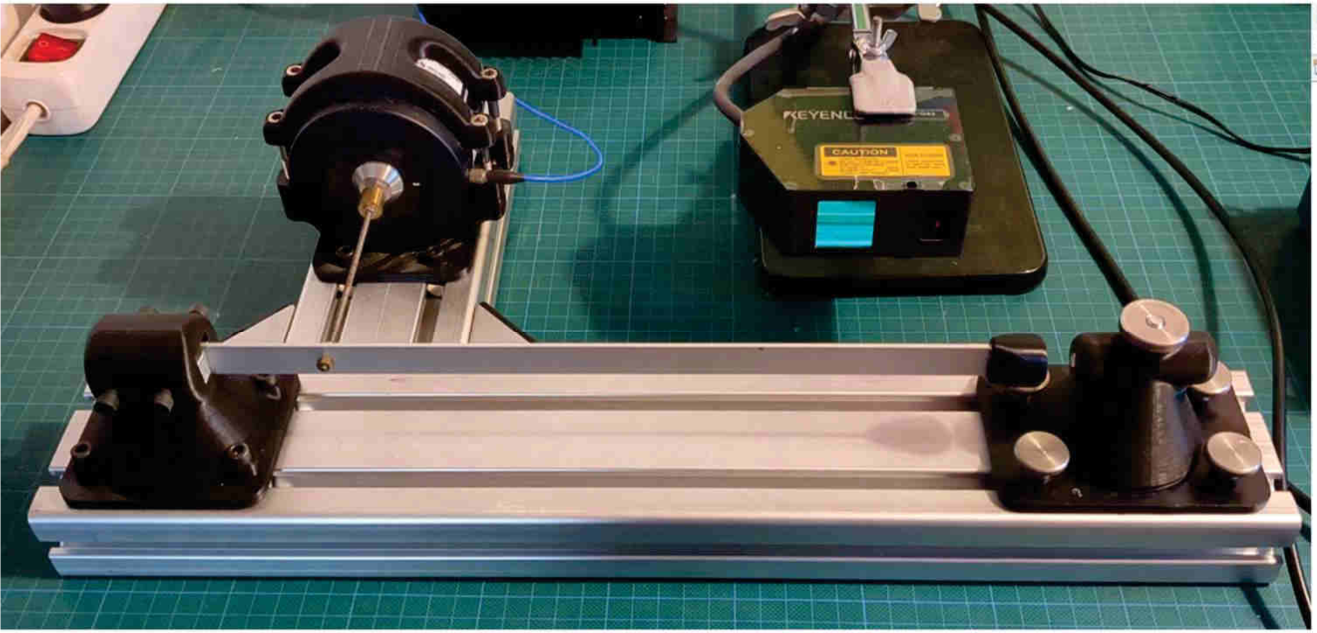
$f_s = 1\ kHz$

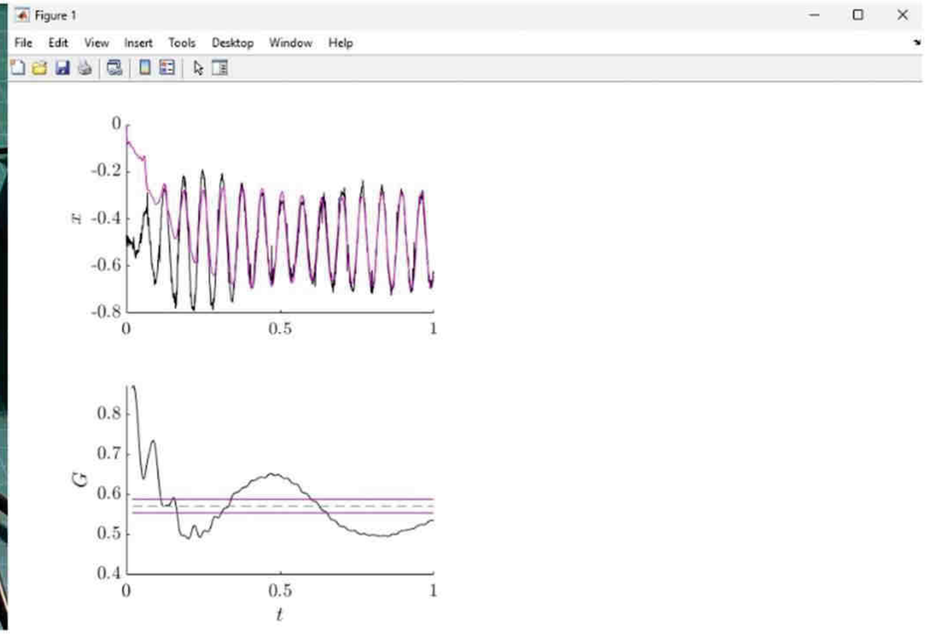
Amplitude sweep @15.5 Hz



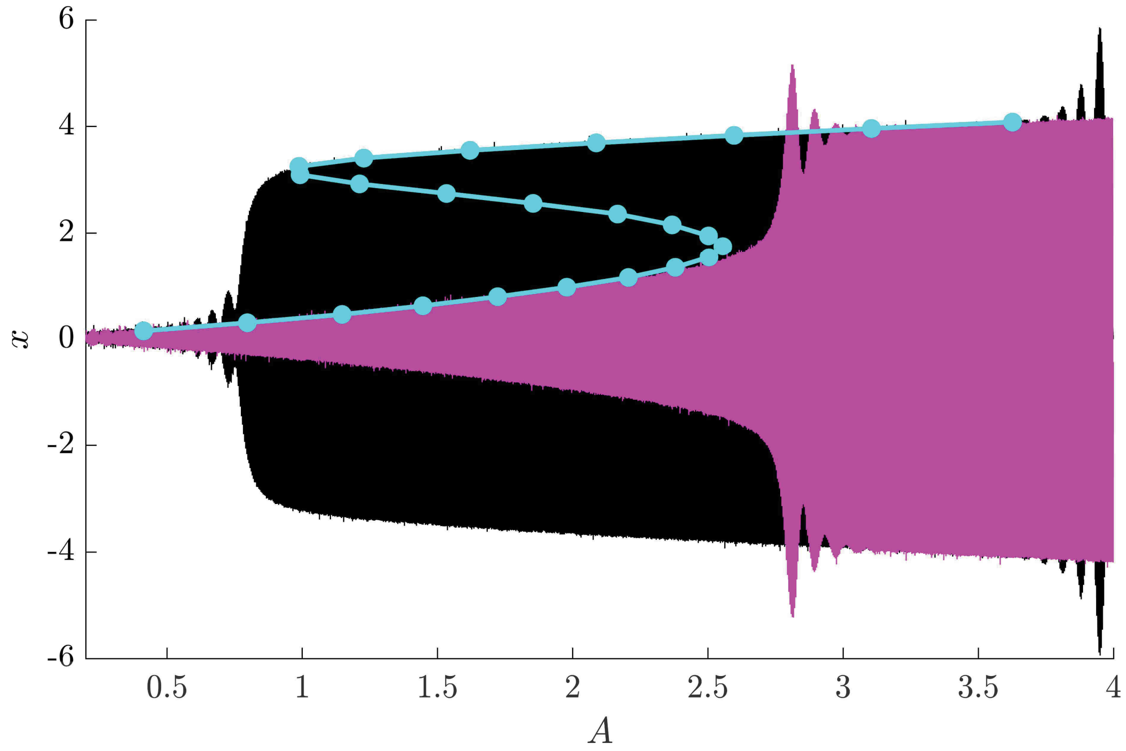
Amplitude sweep and CBC

```
%% Serial port config

port = "COM9";
baud = 115200;
s = serialport(port, baud, 'ByteOrder','little-endian');

%% Run CBC

Kd = 0.03;
mu = 10;
Omega = 15.5*2*pi;
Nh = 5;
Uc1 = 1/4;

out = controlBasedContinuation(s,Kd,mu,Omega,Nh,Uc1);
```

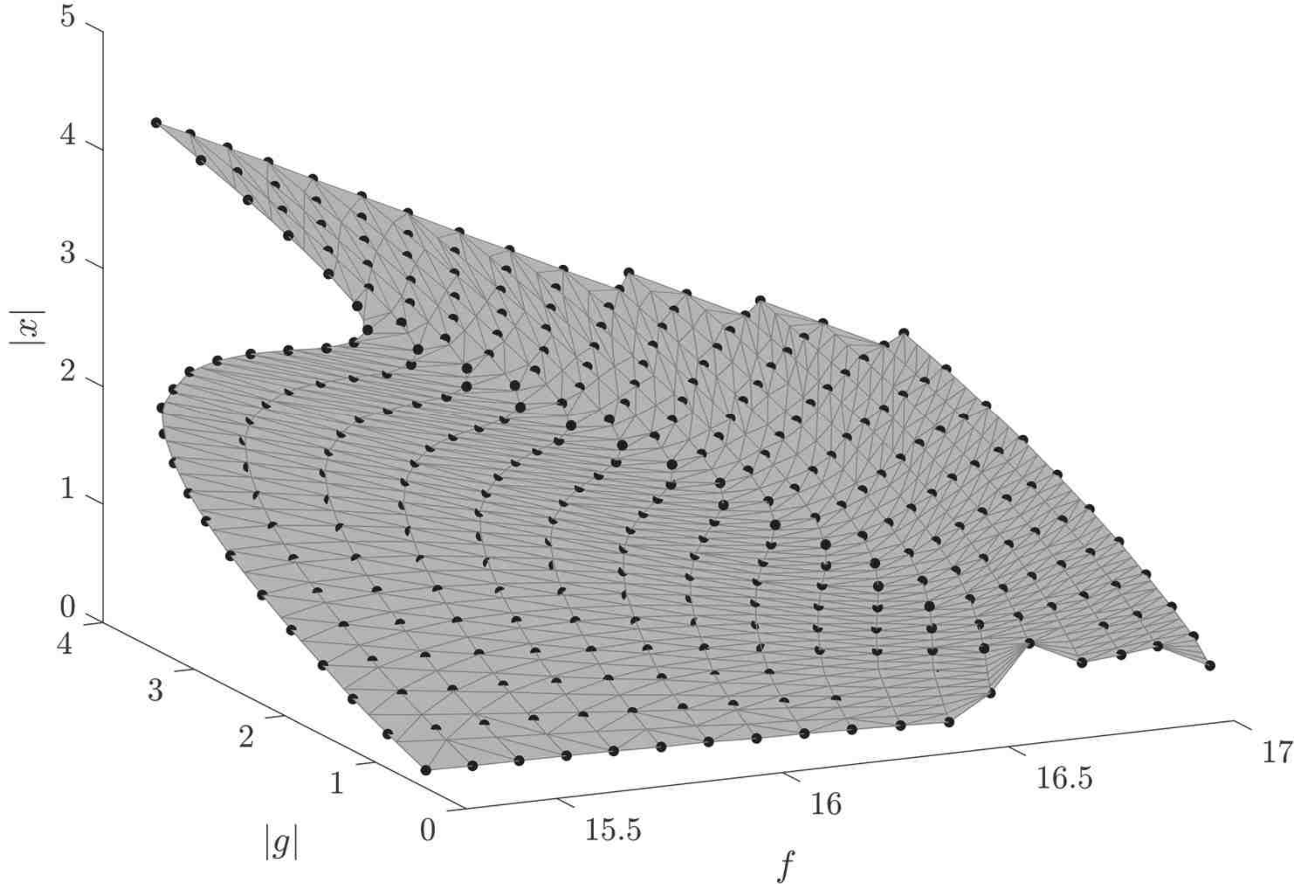




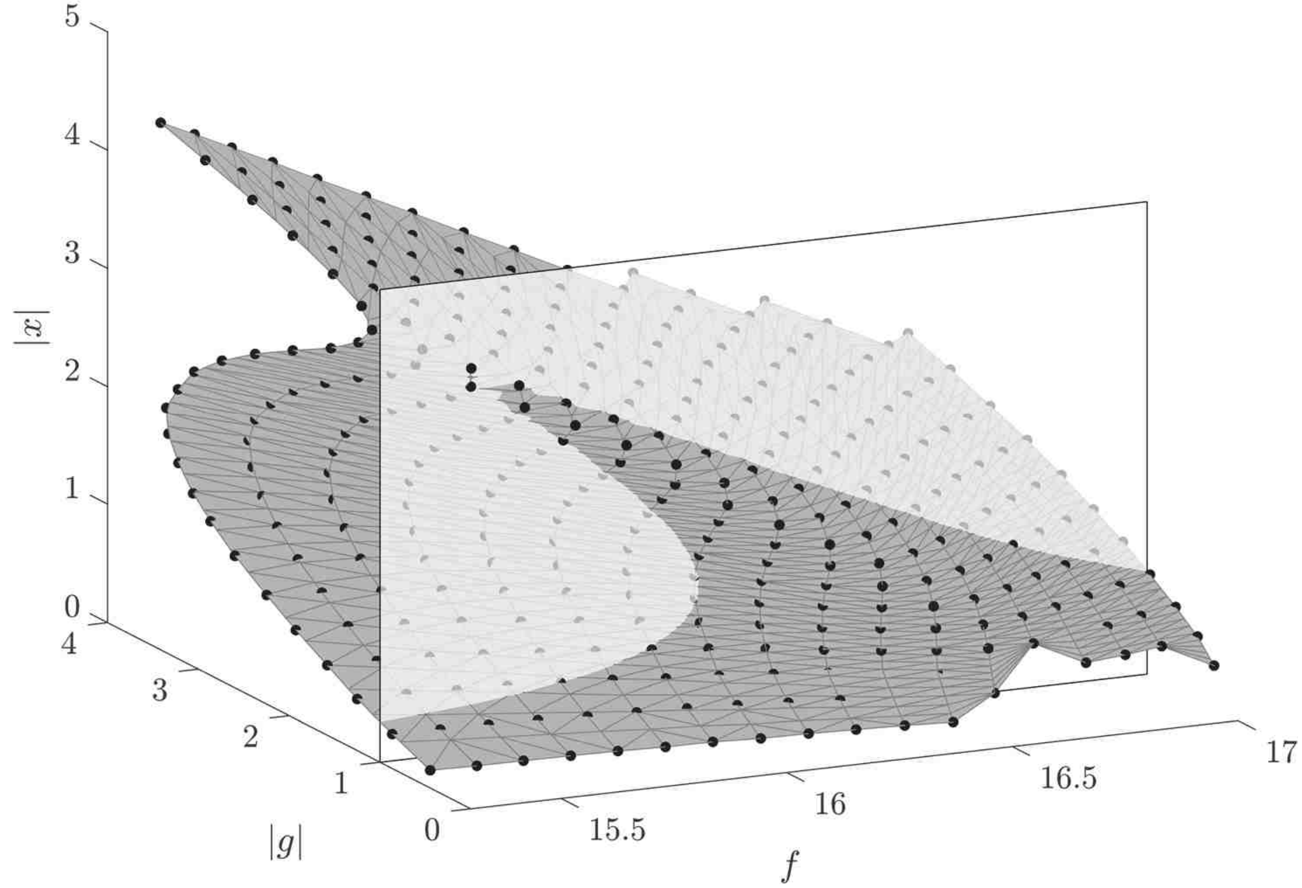
Amplitude sweep and CBC @15.5 Hz



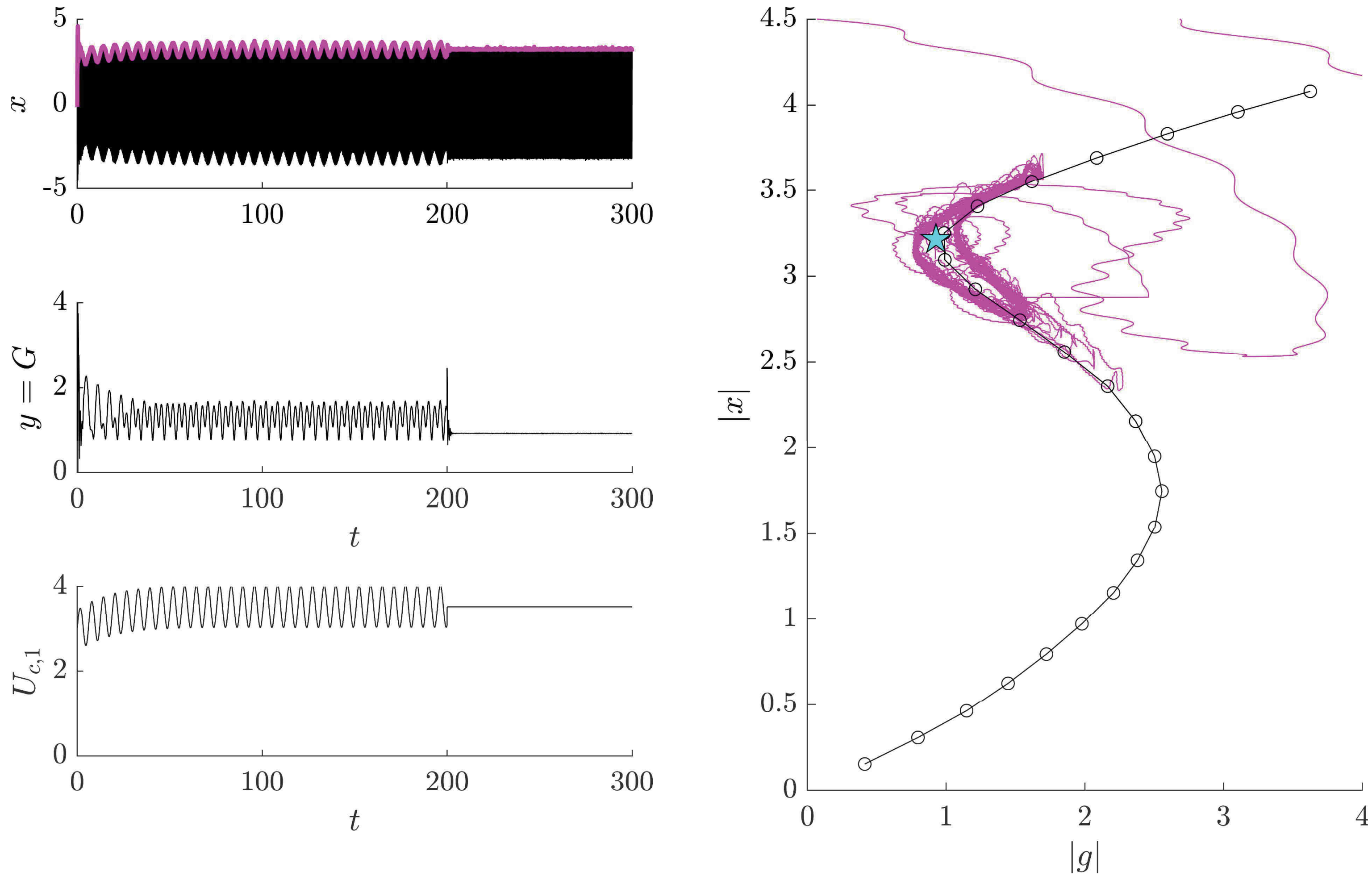
Amplitude sweep and CBC



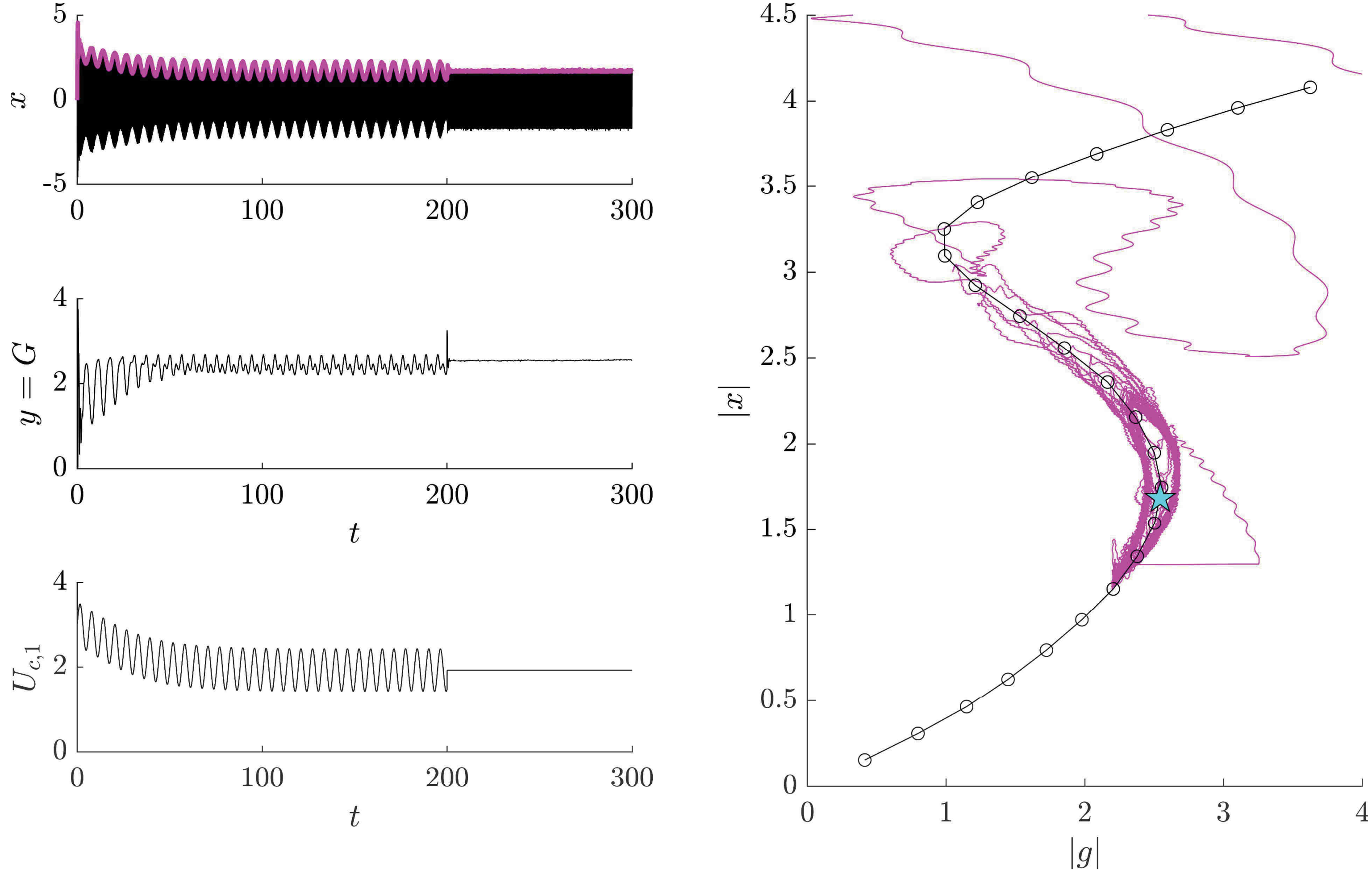
Amplitude sweep and CBC



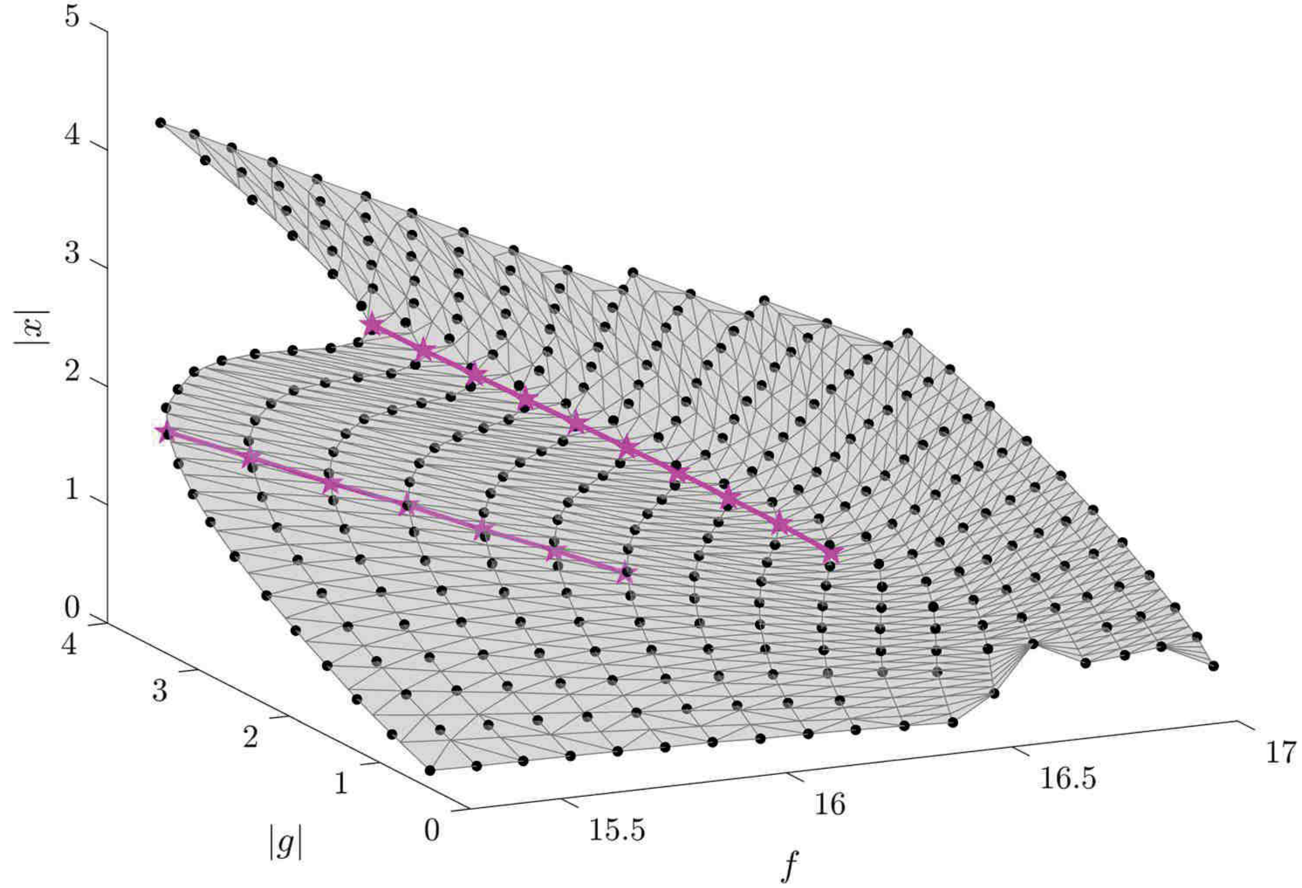
Experimental extremum seeking control



Experimental extremum seeking control

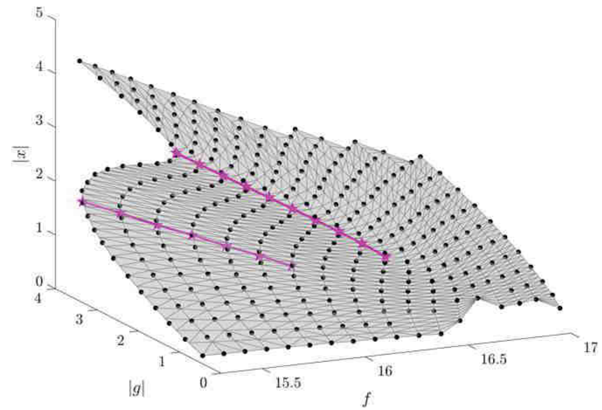


"Continuation" of Fold points



Conclusion

- A non-iterative control algorithm that automatically converge to a Fold bifurcation
    - Control based continuation + Extremum seeking
- Experimental validation on a nonlinear beam
    - Practical implementation on STM32 MCU



Perspectives

- It's slooooooooooowww!
    - Proportional-Integral ESC
- Application to the detection of isola

M. Guay, D. Dochain, *A proportional-integral extremum-seeking controller design technique*, Automatica, 2017

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