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A hybrid Nonlinear energy sink with adaptive nonlinear stiffness tuning for high-energy external excitations

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Context and motivations

How to dampen vibration when the resonant frequency changes?



Ariane V



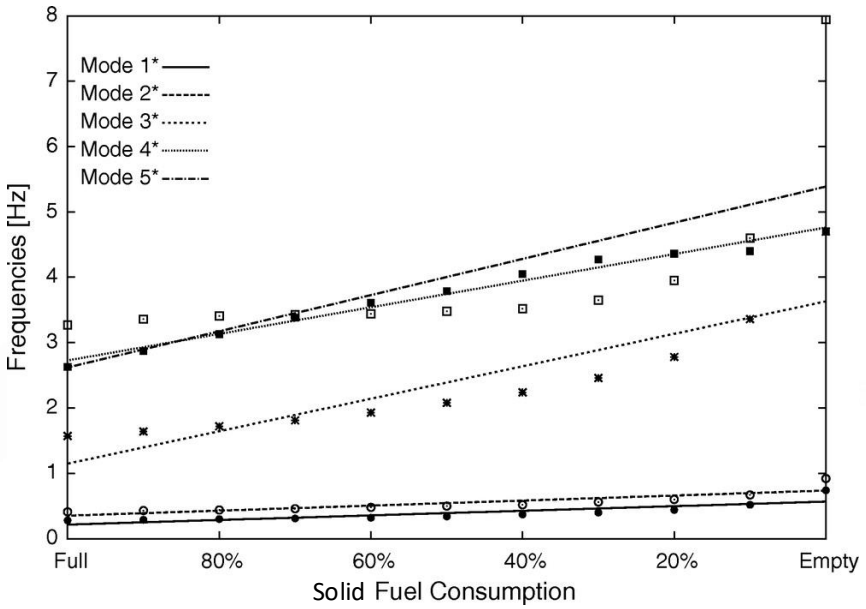
Ariane V rocket liftoff

Context and motivations

How to dampen vibration when the resonant frequency changes?



Ariane V



Shows the variations in the first five frequencies when the burned fuel is varied.

Mass variation can be very important as can the new energy level!

[1] Carrera, E., Cavallo, T., & Zappino, E. (2017). Effect of solid mass consumption on the free-vibration analysis of launchers. Journal of Spacecraft and Rockets, 54(3), 774-781.

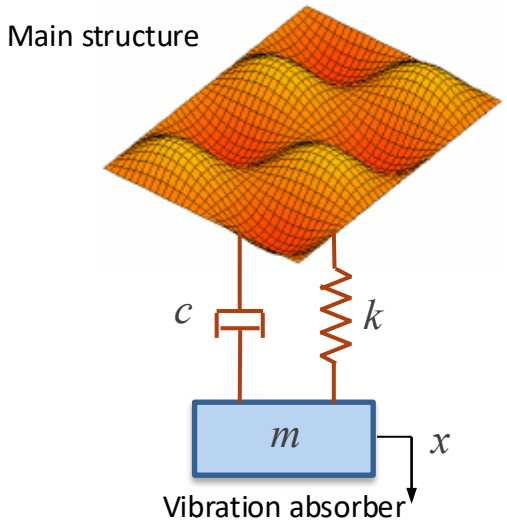
Context and motivations

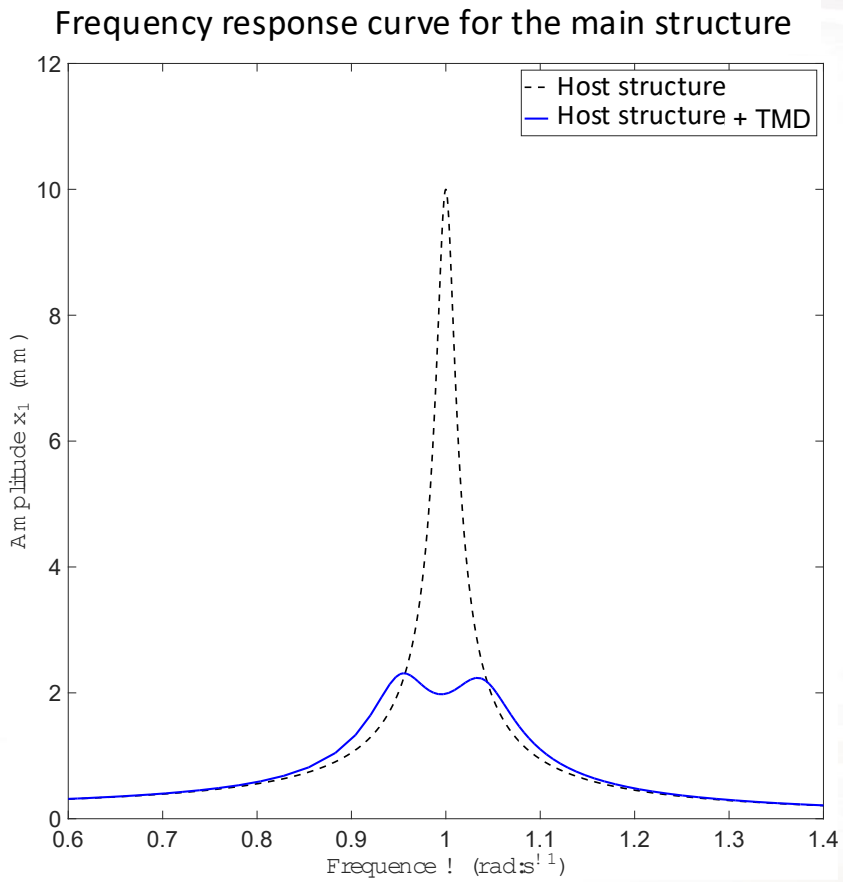
INSTITUT CLÉMENT ADER

How to reduce vibrations ?

Passive linear vibration absorber

- ⊕ Easy to implement
- ⊖ Tuned to a given frequency
- ⊖ Limited performance with an NL host structure







Taiwan : Taipei
tower 101

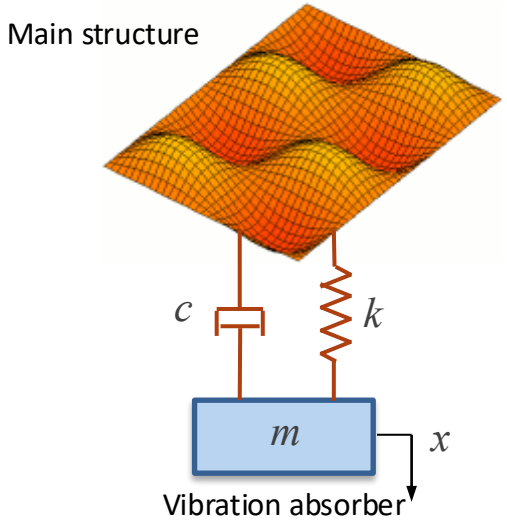


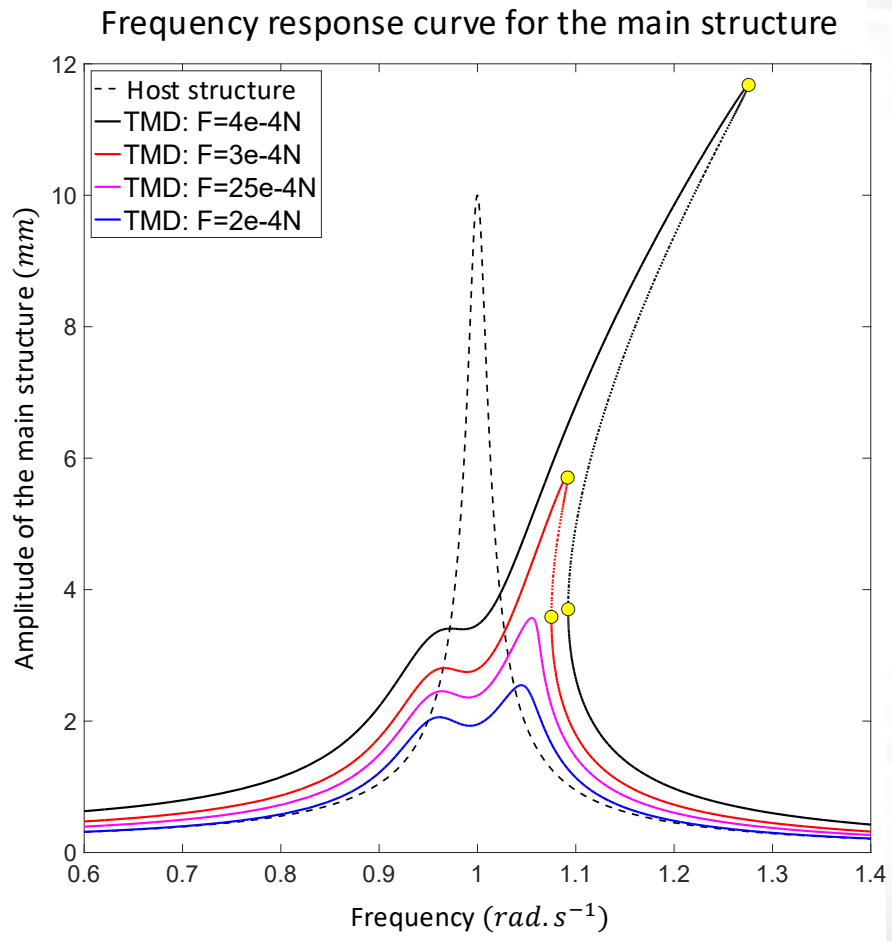
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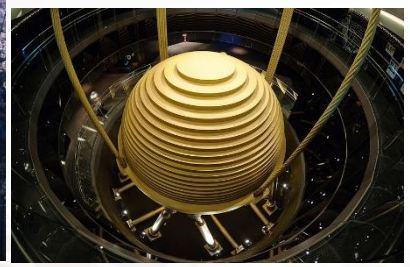
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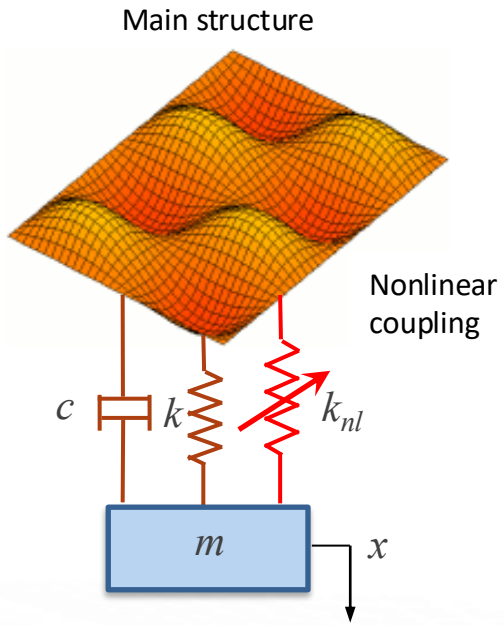
What if the frequency varies?

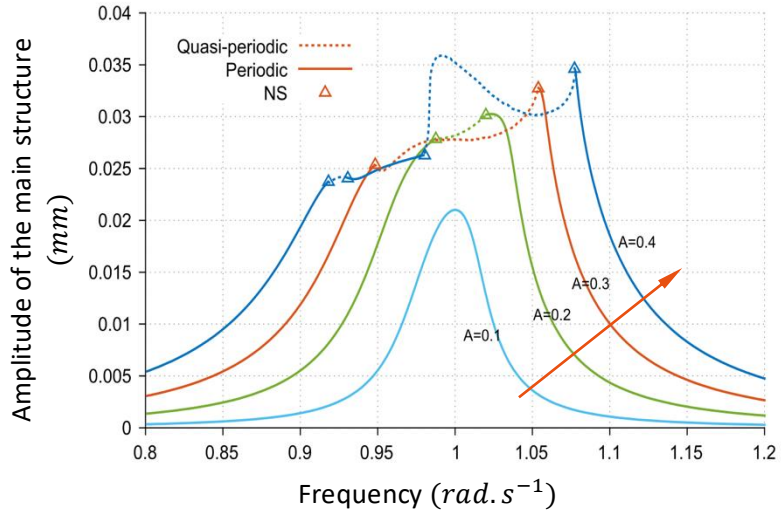
Nonlinear passive vibration absorber

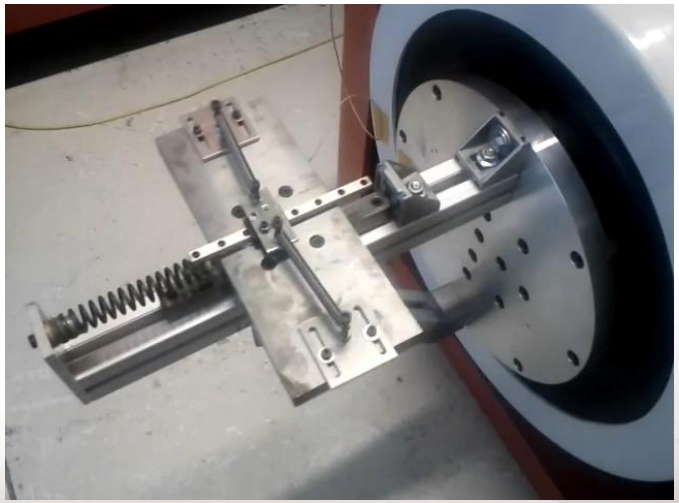
- ⊕ Effective on a bigger frequency range
- ⊕ Targeted energy transfer
- ⊕ Amplitude saturation when the force increase

Adverse nonlinear phenomena:

- ⊖ activation threshold
- ⊖ **Not very effective for high level of vibration**







Context and motivations

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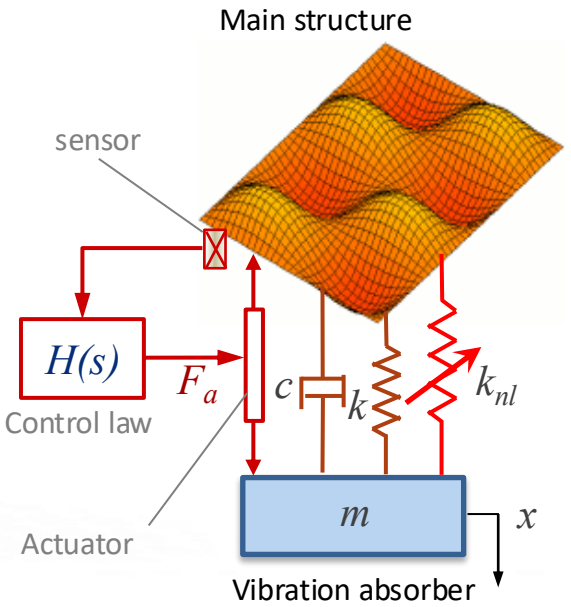
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What if the excitation or the environment fluctuates ?

Nonlinear hybrid vibration absorber

- ⊕ Real-time control of absorber parameters
- ⊕ Adaptation to environmental variations, fault correction, etc.
- ⊕ Improving performance compared with the passive part
- ⊕ Fail-safe aspect

Context and motivations

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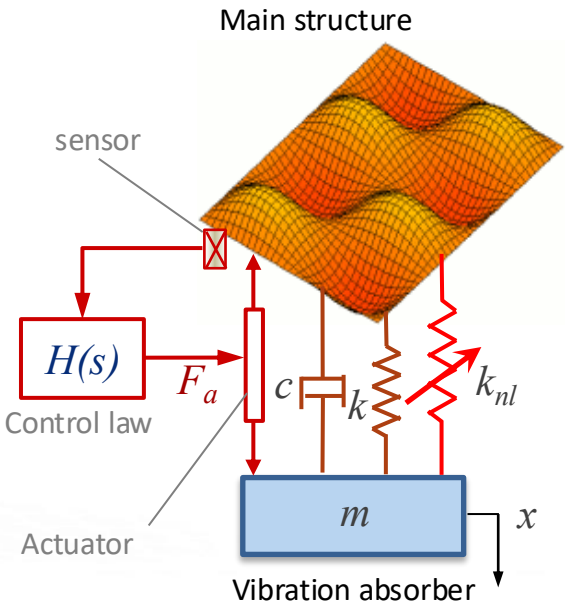
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Hybrid nonlinear energy sink

Context and motivations

Problem: How does active control help to reduce excessive vibration levels?

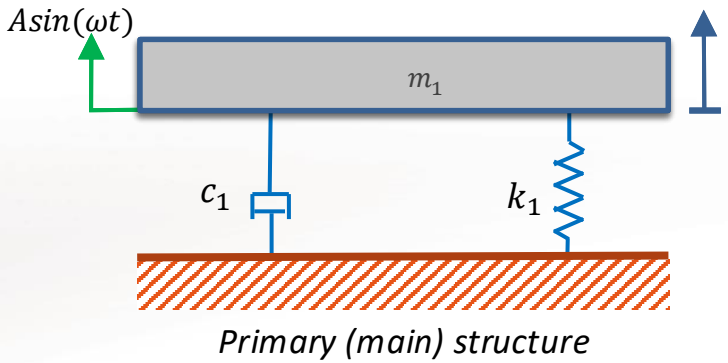
Contents

1. Context and motivations
2. Mathematical modeling
3. Numerical results
4. Experimental setup
5. Experimental results
6. Synthesis and improvements

Mathematical modeling

Equations of motion

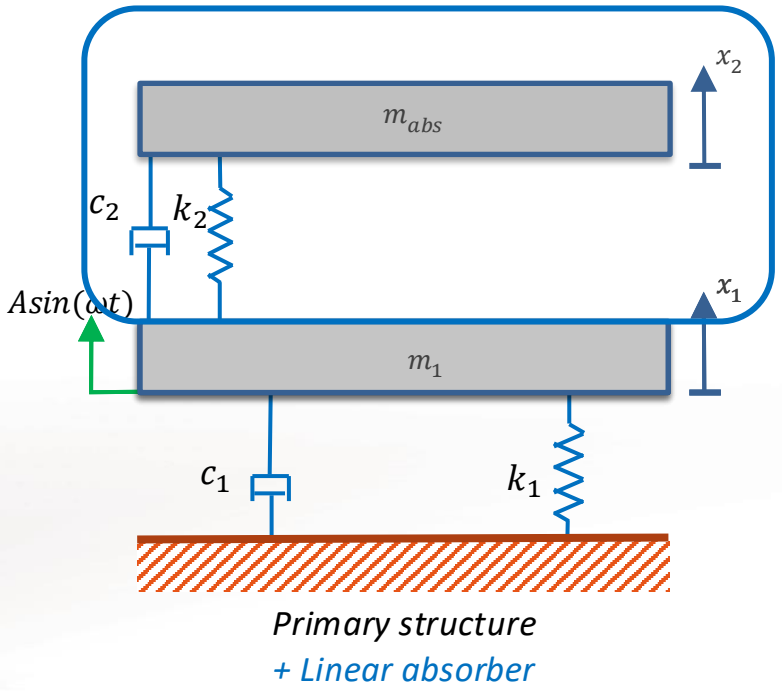
$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = Asin(\omega t)$$



Mechanical linear terms

Mathematical modeling

Linear absorber



Primary structure

+ Linear absorber

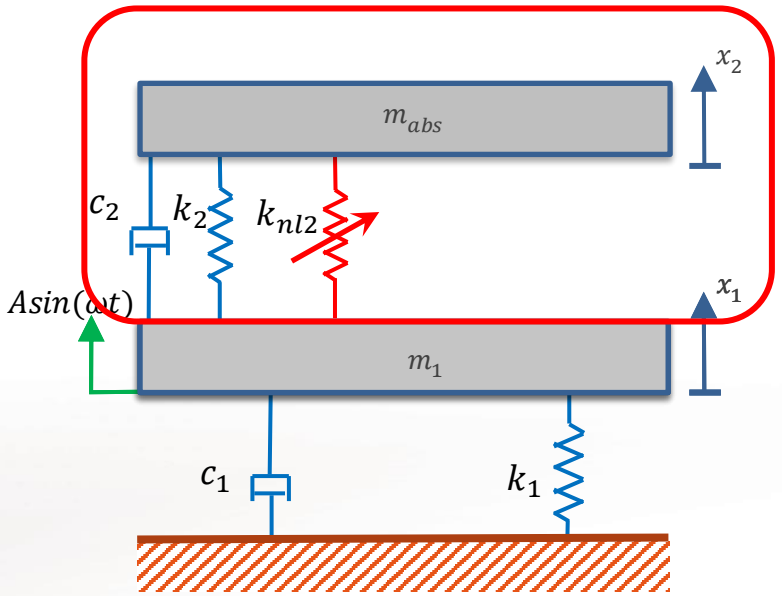
Equations of motion

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) = Asin(\omega t) \\ m_{abs}\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = 0 \end{cases}$$

Mechanical linear terms

Mathematical modeling

Nonlinear absorber



Linear absorber +
Nonlinearity

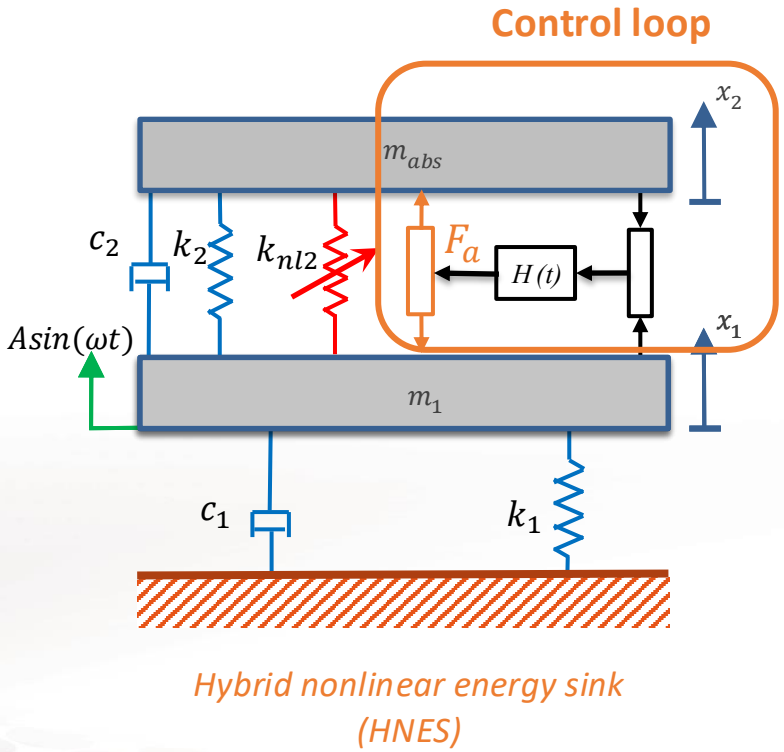
Equations of motion

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) + k_{nl}(x_1 - x_2)^3 \\ \qquad = Asin(\omega t) \\ m_{abs}\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) + k_{nl}(x_2 - x_1)^3 = 0 \end{cases}$$

Mechanical linear terms

Geometric non-linearities

Mathematical modeling



Hybrid nonlinear energy sink (HNES)

Equations of motion

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) + k_{nl}(x_1 - x_2)^3 \\ \qquad = Asin(\omega t) + F_a \\ m_{abs}\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) + k_{nl}(x_2 - x_1)^3 = -F_a \end{cases}$$

Mechanical linear terms

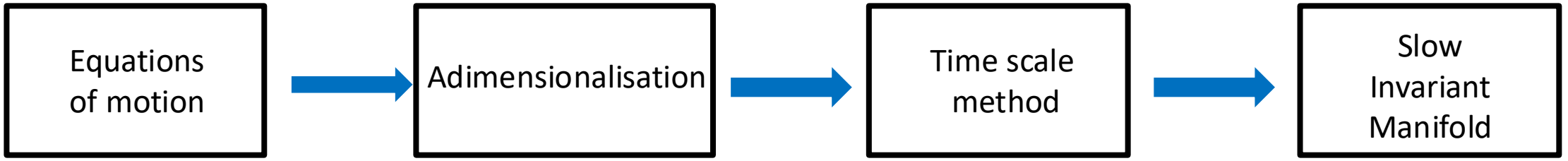
Geometric non-linearities

Active force to **modify the mechanical nonlinearities (reactive law)**

Change the activation threshold to return to quasi-periodic

Mathematical modeling

Slow Invariant Manifold (SIM)



Including the control terms

Introduction of dimensionless time $\tau = \omega_0 t$

Introduction of the complex variables of Manevitch [1]

Study of slow dynamics

The slow invariant manifold is calculated taking into account the active part

[1] Manevitch, L. I., Mikhlin, Y. V., Pilipchuk, V. N., & Zevin, A. A. (2001). Normal modes and localization in nonlinear systems.

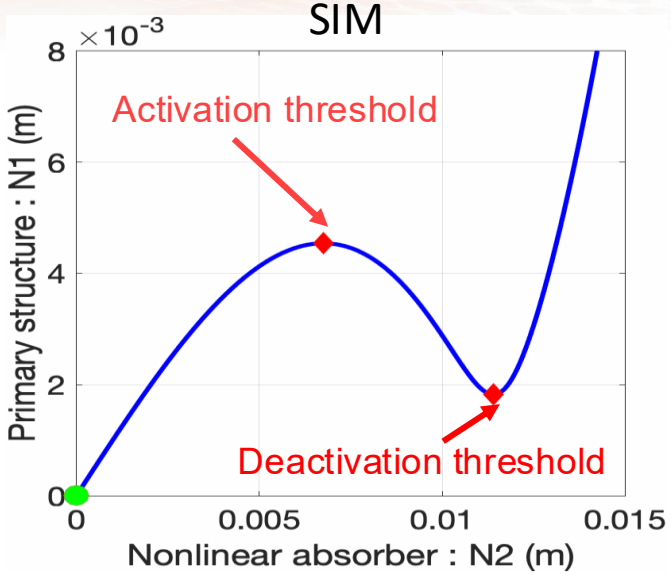
Mathematical modeling

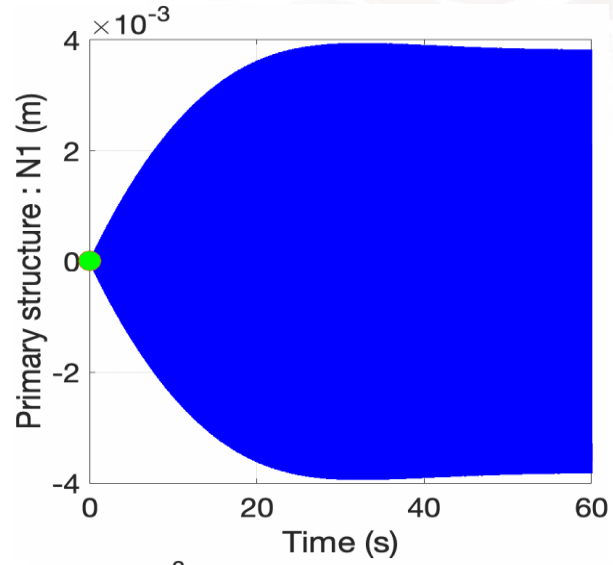
SIM

Temporal response

Case 1 : small level of external force

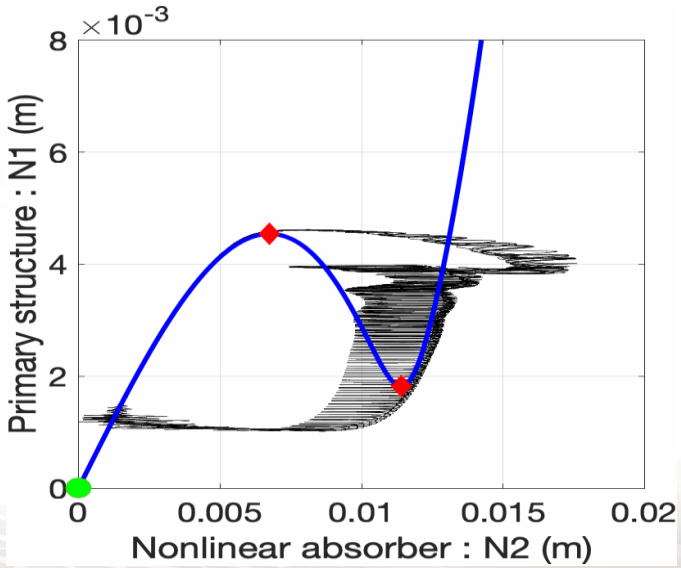
The amplitude does not exceed the threshold.
The absorber is not triggered.

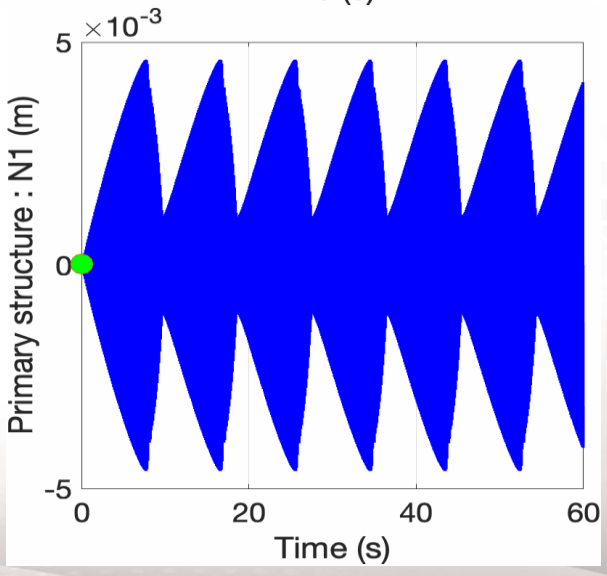




Case 2 : large level of external force

Strongly Modulated Response (SMR)
The amplitude exceeds the threshold. The absorber is triggered.

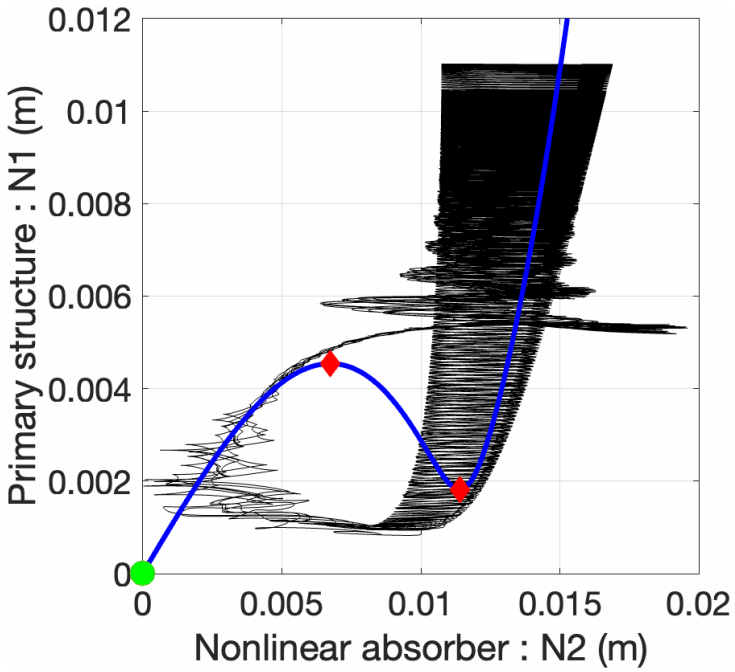




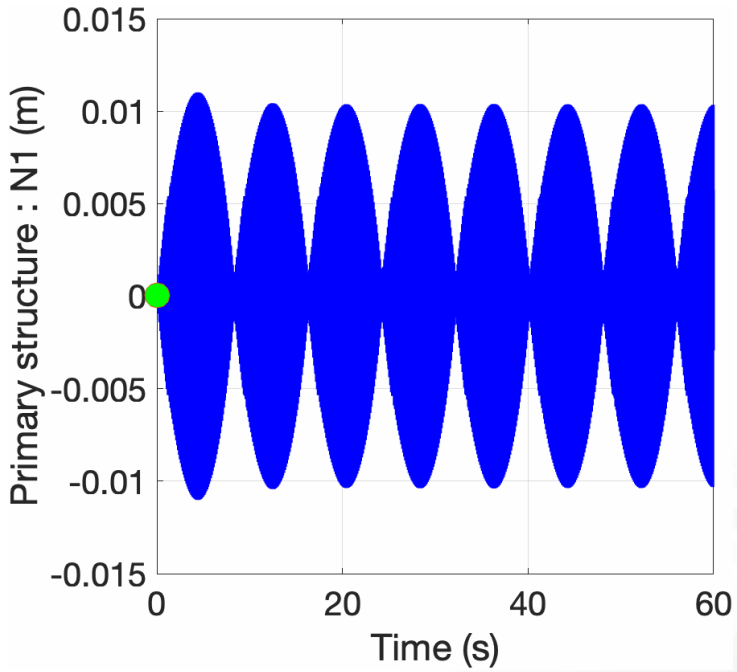
Mathematical modeling

Case 3 : Too large level of external force

The amplitude exceeds the threshold. The absorber has been triggered, but is not very effective.

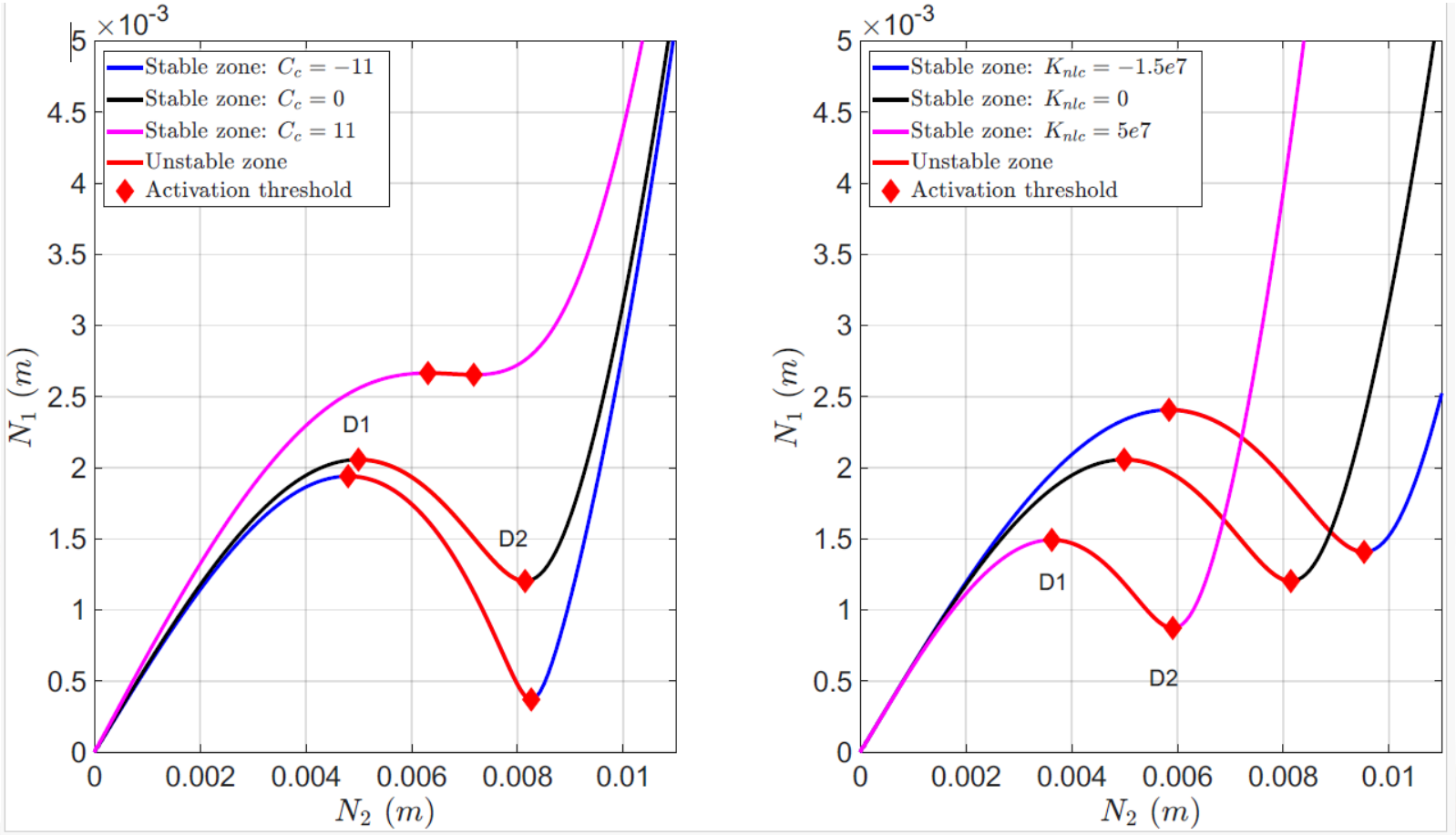


Temporal response



Numerical results

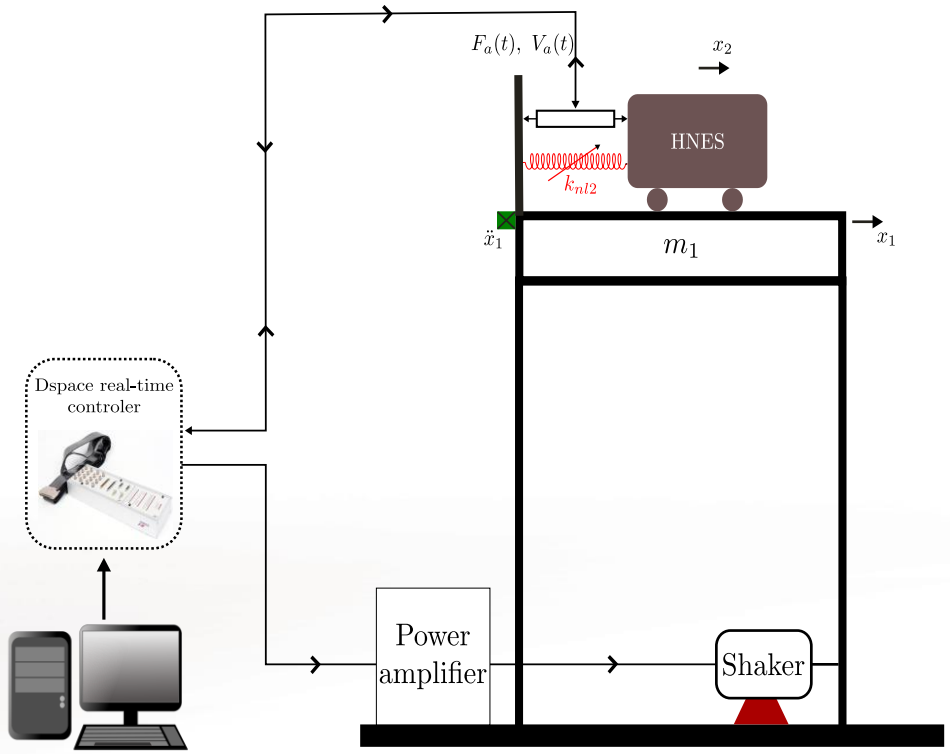
Here, active control is used to modulate the dynamics of the entire system.



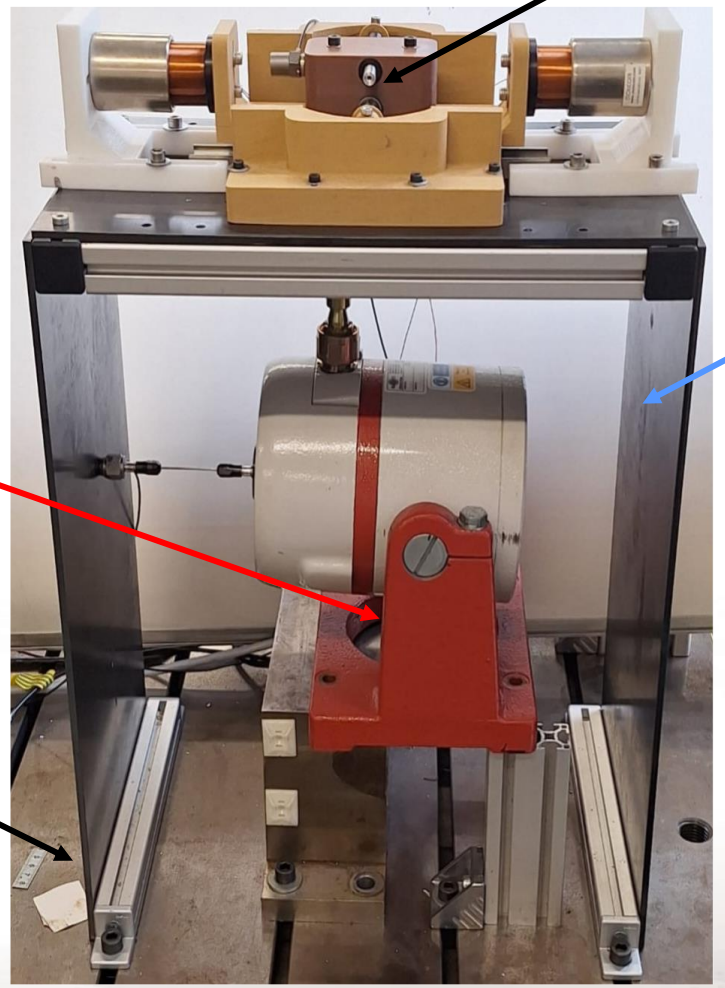
Slow invariant manifold: numerical damping (left) and analysis of non-linear stiffness (right) variation

Experimental setup

Case study: 1-storey building with absorber

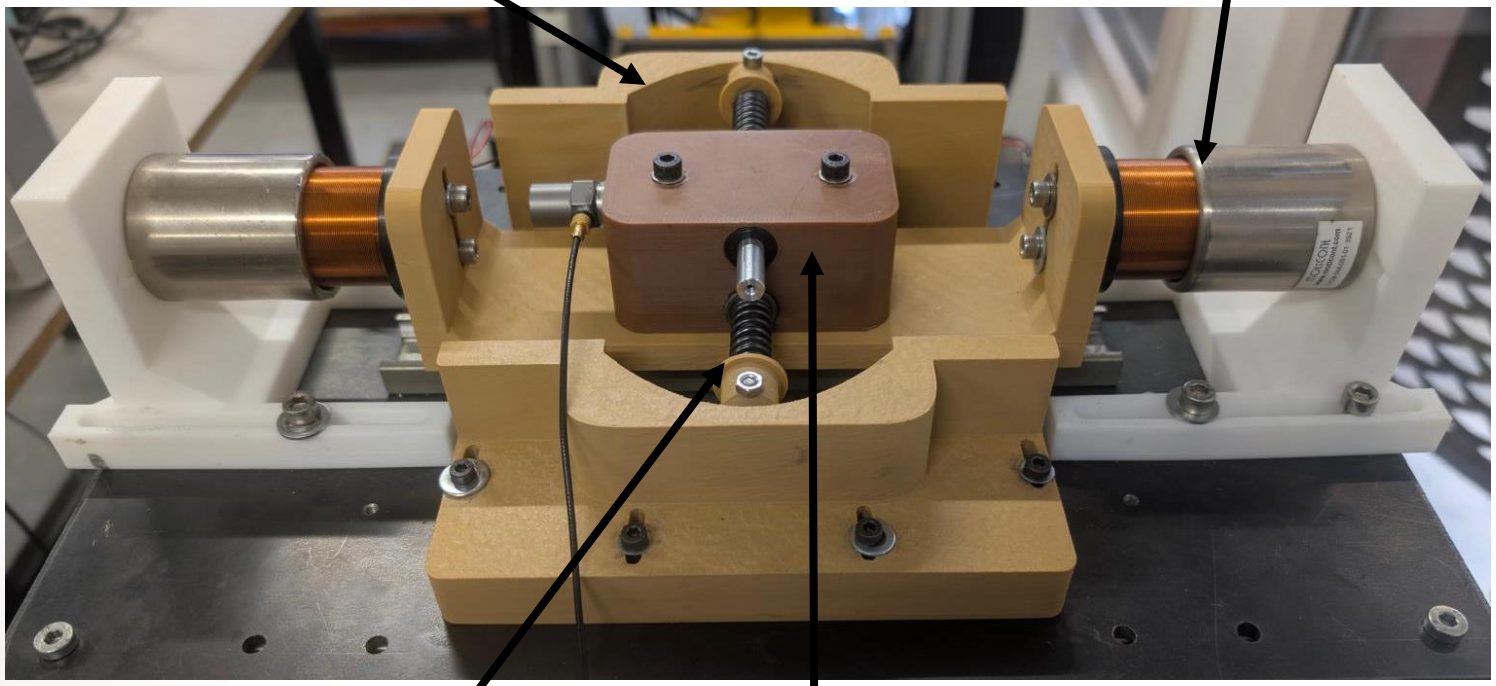


Schematic diagram

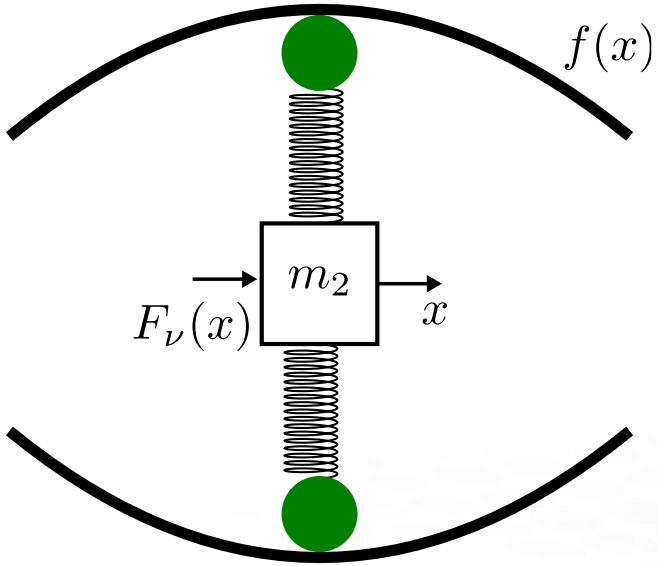


$f_0 \approx 13Hz, Height: 50cm$

Experimental setup

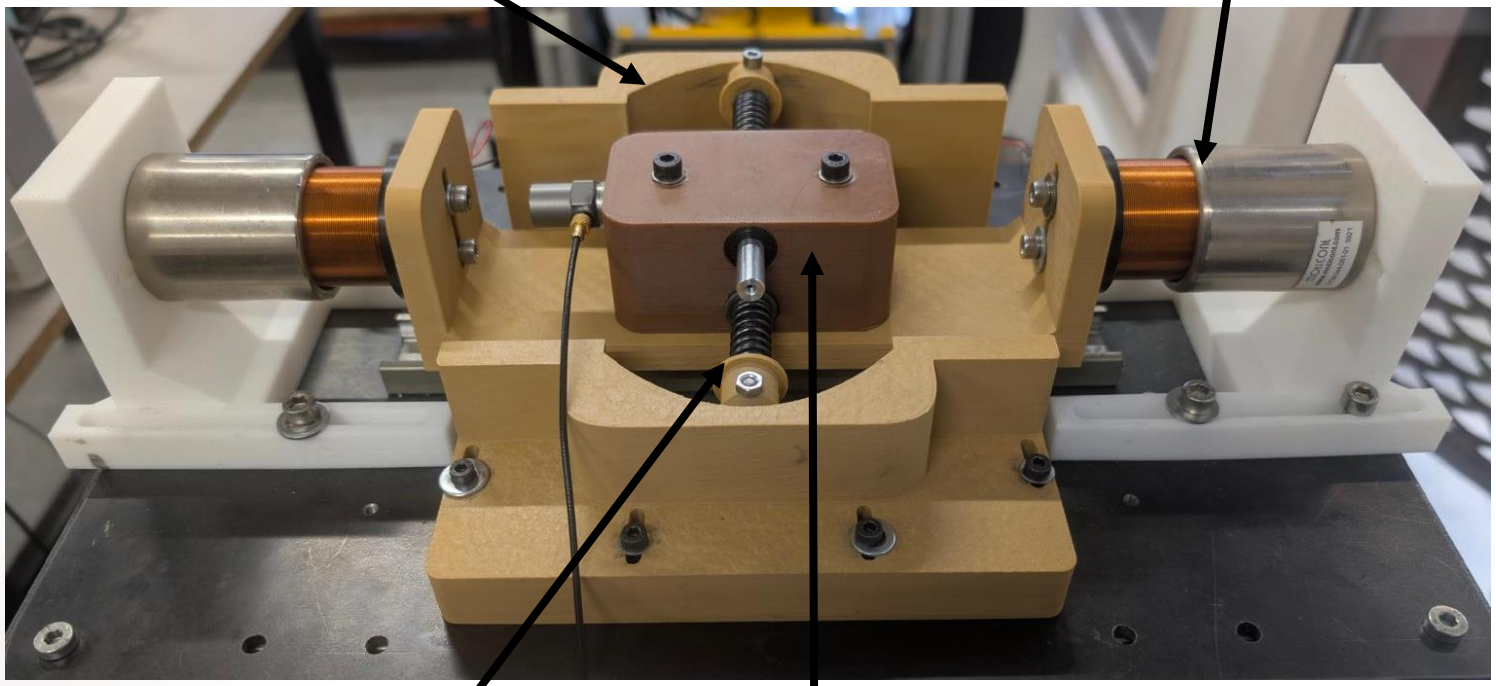


Force profile : $f(x) = 14.1x^2$
Spring stiffness $k_l = 16700\ \mathrm{N/m}$



$$F_v = kx + k_{nl}x^3$$

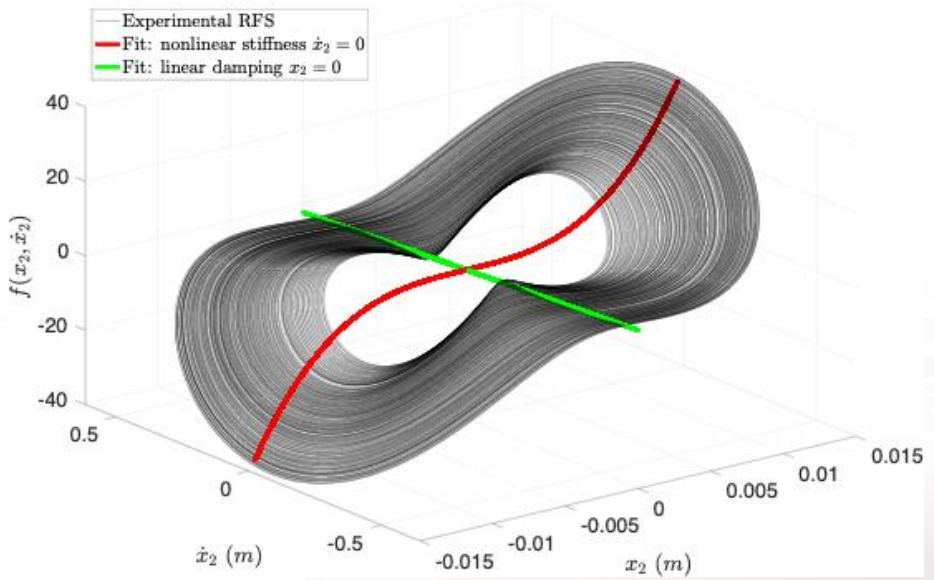
Experimental setup



$$\epsilon = \frac{m_2}{m_1} \approx 6\%$$

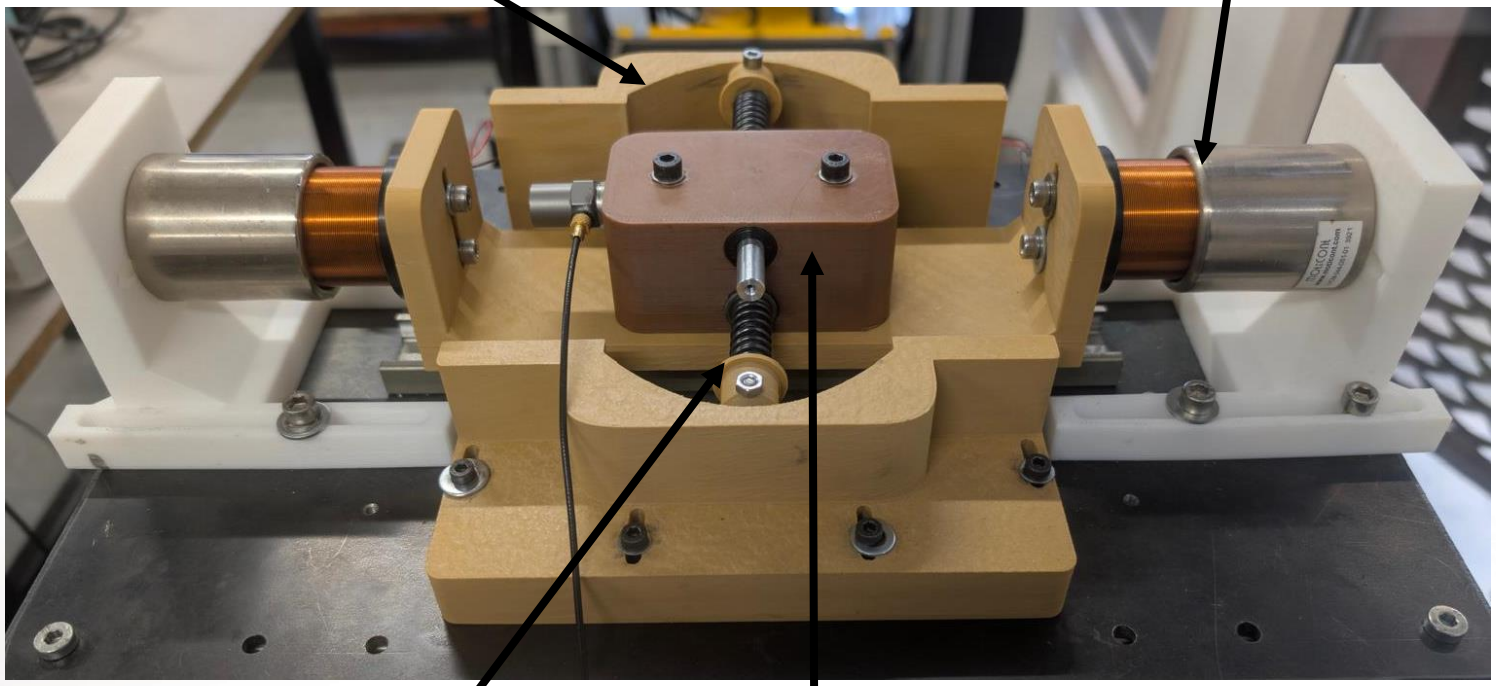
Restoring force surface method [Kerschen et al., 2006] :

$$f(x, \dot{x}) = P\cos(\omega\, t) - m_{abs}\ddot{x}$$



Kerschen, G., Worden, K., Vakakis, A. F., & Golinval, J. C. (2006). Past, present and future of nonlinear system identification in structural dynamics. Mechanical systems and signal processing, 20(3), 505-592.

Experimental setup



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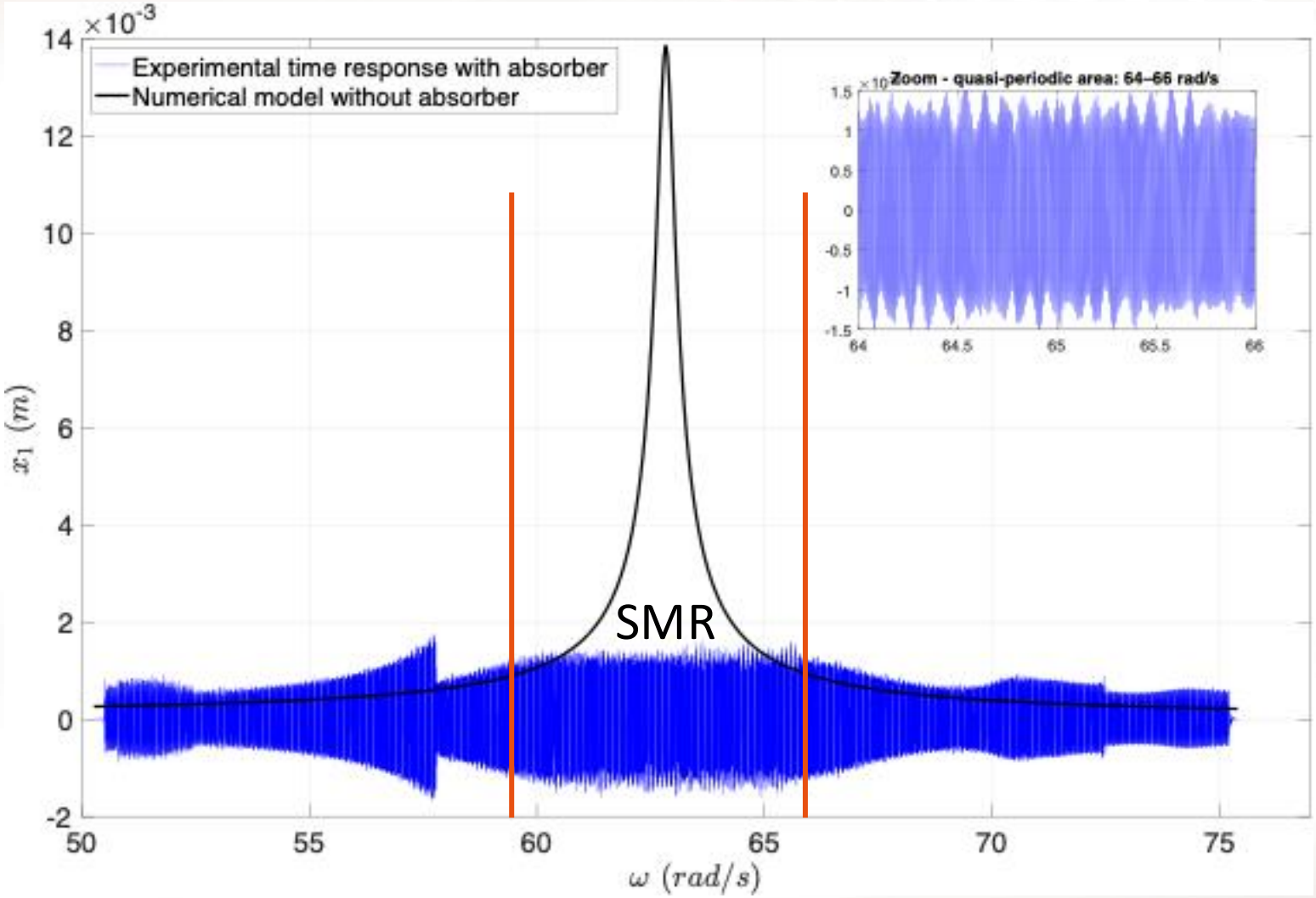
$$f(x, \dot{x}) = Pcos(\omega\, t) - m_{abs}\ddot{x}$$

Primary structure	HNES
$m_1 = 9.43$ kg $\omega_0 \approx 80.42$ rad.s^{-1} $\xi = 0.032$	$m_2 = 0.7$ kg $k_2 = 1.5 \times 10^3$ N/m $c_2 = 8$ N.s/m $k_{nl2} = 6.7 \times 10^6$ N/m^3

Kerschen, G., Worden, K., Vakakis, A. F., & Golinval, J. C. (2006). Past, present and future of nonlinear system identification in structural dynamics. Mechanical systems and signal processing, 20(3), 505-592.

Experimental results

Case without control



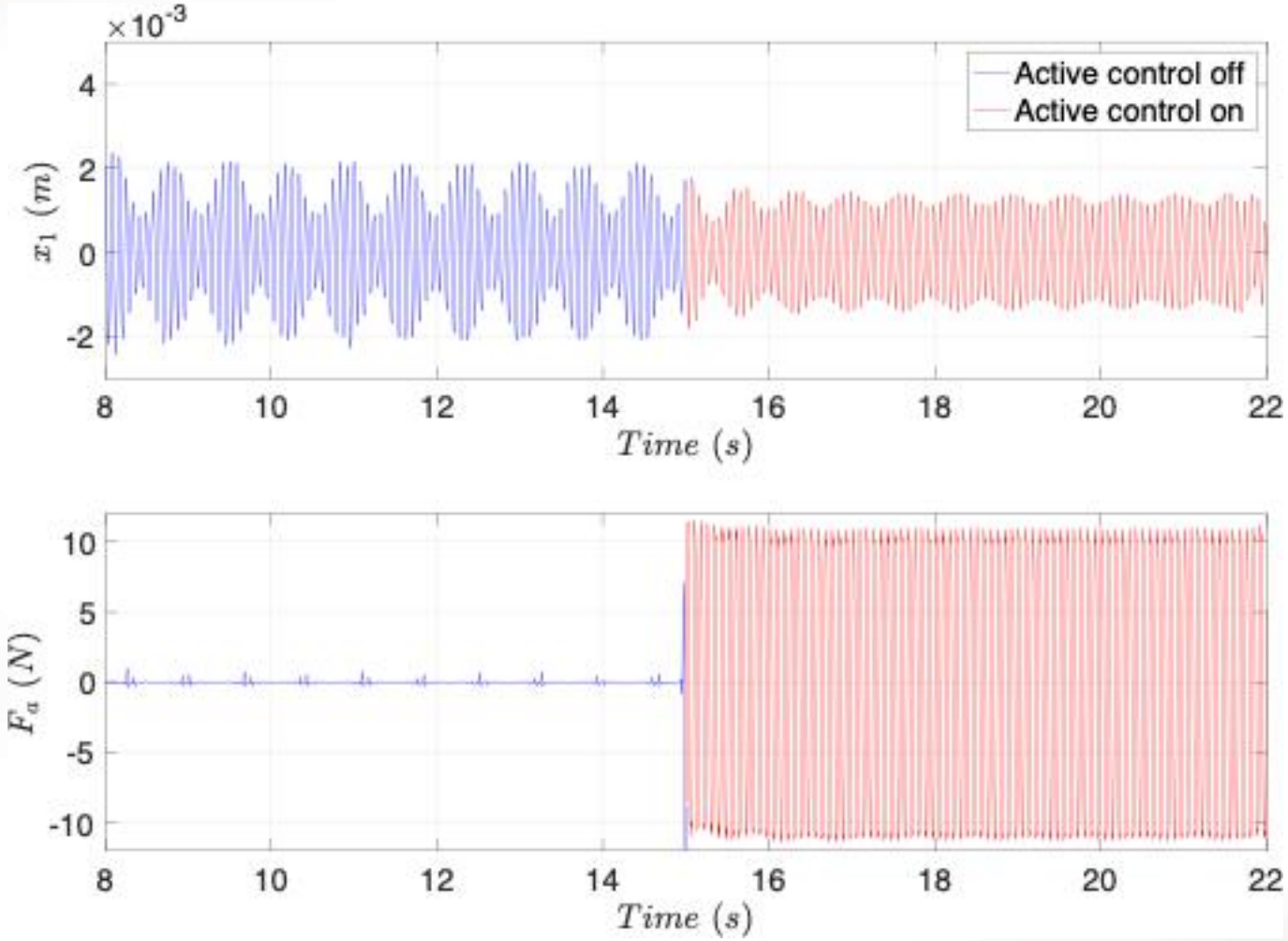
Experimental frequency response of the passive NES for moderate forcing amplitudes

Experimental results

Shape of the control law: $\mathrm{F_a} = K_{nl}(x_2 - x_1)^3$

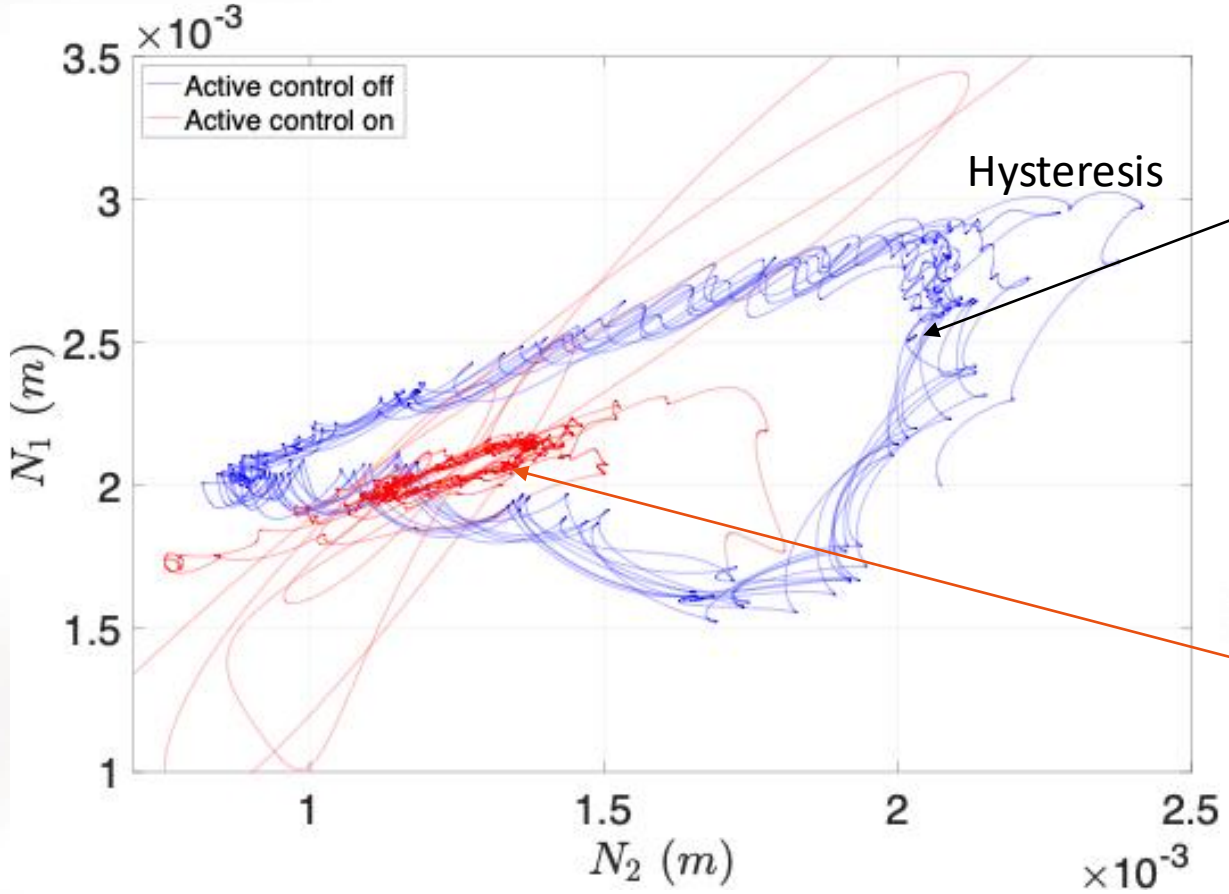
A video is worth a thousand words ☺

Experimental results

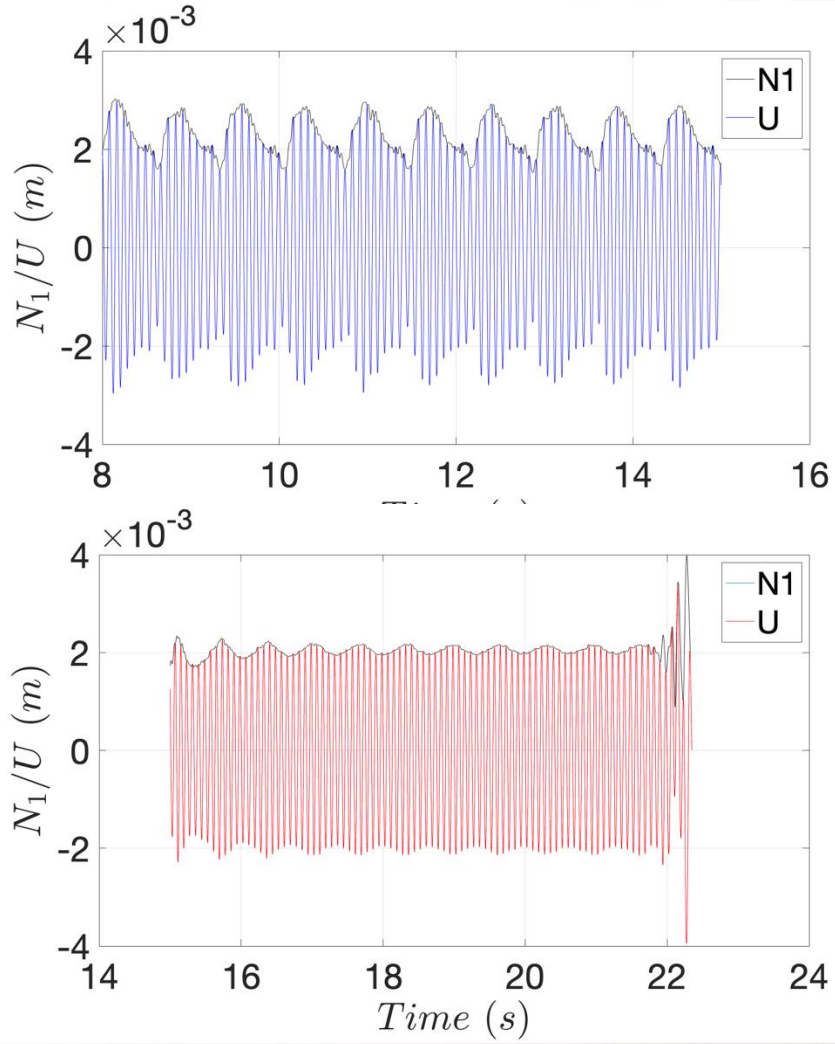


Time response of the first-floor displacement and the active control force for a high level.

Experimental results



Hysteresis



Experimental Slow Invariant Manifold with and without active control

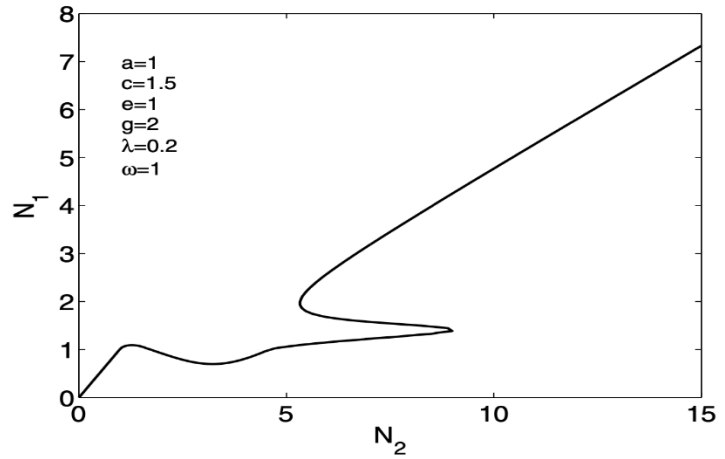
Synthesis, perspectives and improvements

Synthesis

- Experimental hybrid non-linear energy sink (HNES)
- Non-linearity generated by wheel on a nonlinear profile
- Active part generated by voice coil
- Threshold real-time adaptation thanks to the active control

Perspectives and improvements

- Energy consumption and stability analysis
- **Master isolated responses**
- SIM with non-polynomial or multiple non-linearities

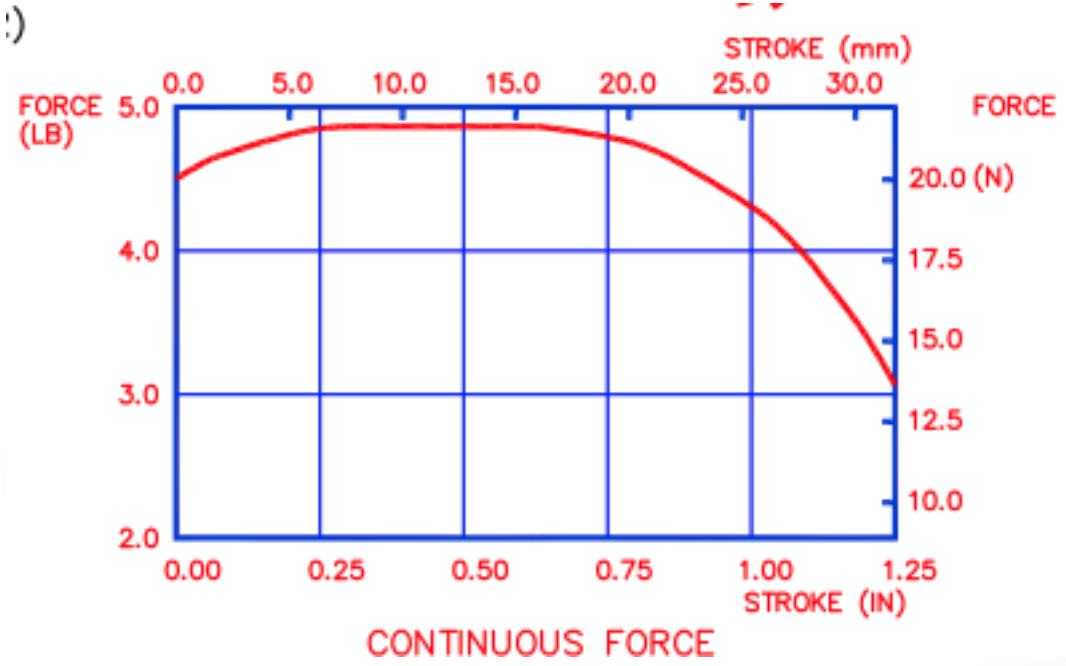


Lamarque et al 2012

Electromagnetic transducers

INTERMITTENT FORCE @10% DUTY CYCLE	68.4 N	15.4 LB
CONTINUOUS FORCE	21.6 N	4.9 LB
FORCE CONSTANT	10.3 N/A	2.3 LB/A
BACK EMF CONSTANT	10.3 V/M/S	0.26 V/IN/S
STROKE	31.8 mm	1.25 IN
COIL CLEARANCE PER SIDE	0.38 mm	0.015 IN
COIL ASSY MASS	165 GR	5.8 OZ
BODY MASS	405 GR	14.3 OZ
COIL RESISTANCE	6.3 Ohms	
COIL INDUCTANCE @120 Hz	11.9 mH	
MAX CONTINUOUS POWER	28.0 W	

ALL VALUES AT 25°C



Manevitch variables

$$\psi_1 = \phi_1 e^{i\Omega\tau} = \dot{V} + \mathrm{i}\Omega V,$$
$$\psi_2 = \phi_2 e^{i\Omega\tau} = \dot{Y} + i\Omega Y$$
with $i^2 = -1$

$$\sqrt{\left(\frac{N_2\lambda_2}{2} - \alpha\Re(F_a\,(N_2))\right)^2 + \left(\left(\frac{\Omega}{2} - \frac{K_2}{2\Omega} - \frac{3K_{nl2}}{8\Omega^3}N_2^2\right)N_2 - \alpha\Im(F_a\,(N_2))\right)^2} = \sqrt{\left(\frac{N1}{2\Omega} + \alpha\Im(F_a\,(N_1))\right)^2 + (\alpha\Re(F_a\,(N_1)))^2}$$