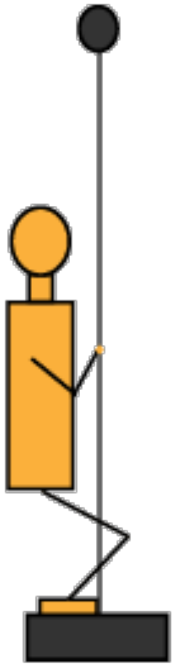
New physical insights in passive dynamical stabilization in time periodic systems

A. Anzoleaga Grandi, S. Protière and A. Lazarus

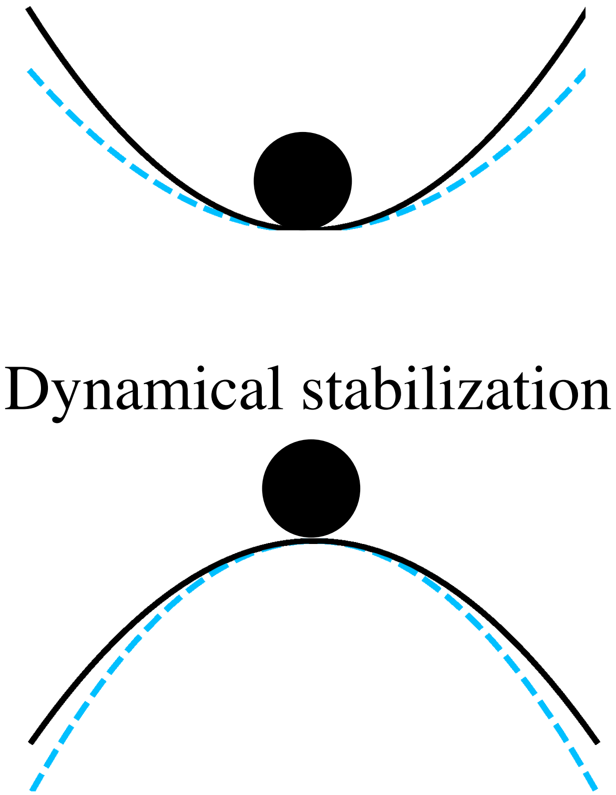
Time-periodic systems

Child on a swing



2 parameters { Period of modulation; Amplitude of modulation

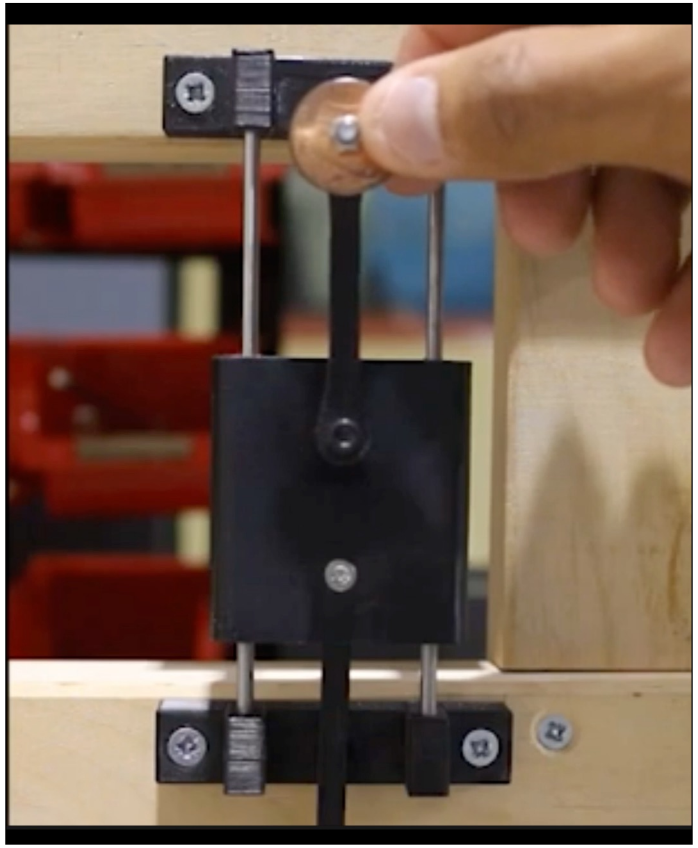
Parametric instabilities



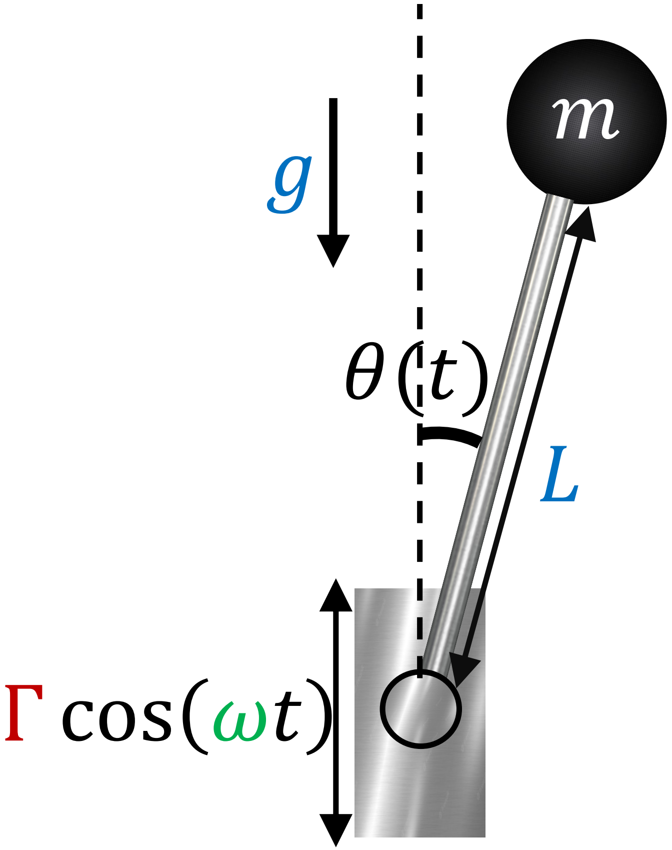
Dynamical stabilization

[1] J. A Richards. *Springer Science & Business Media*, 2012.

Dynamical stabilization



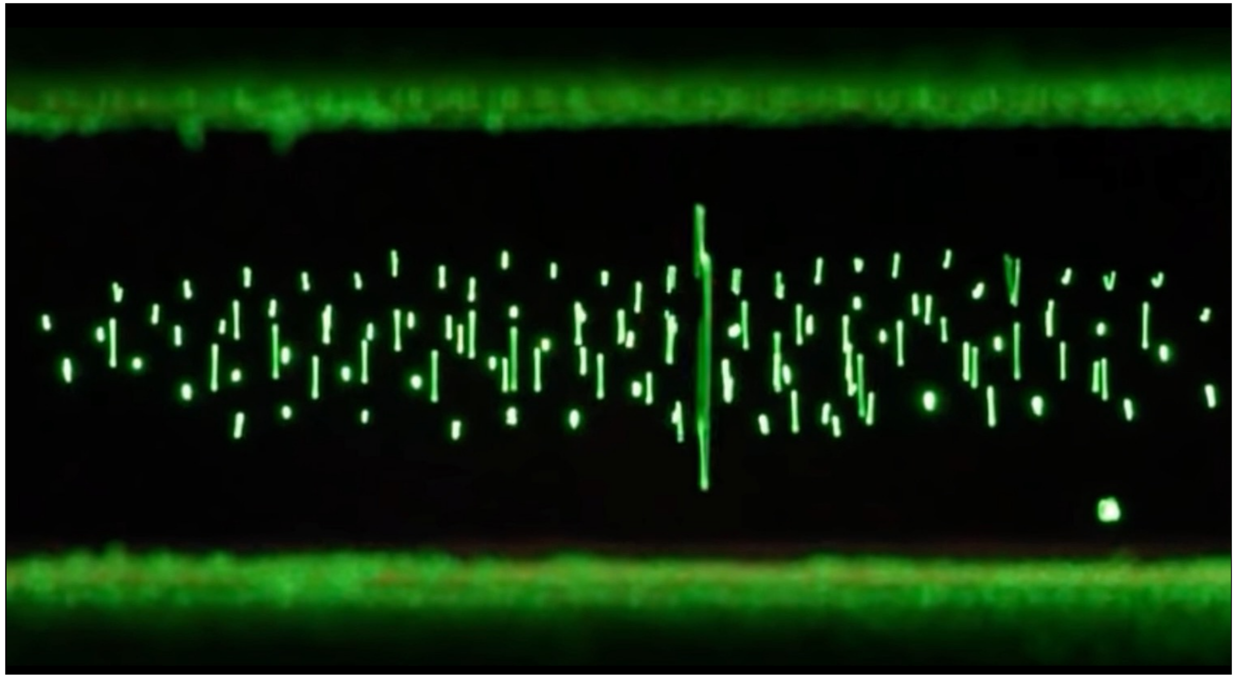
Dynamical stabilization of the upward position of a pendulum
Video credit: University of California



Kapitza limit: $\Gamma^2\omega^2 \geq 2gL$

P.L. Kapitza, *Sov. Phys. JETP*, 588–597, 1951.

Dynamical stabilization

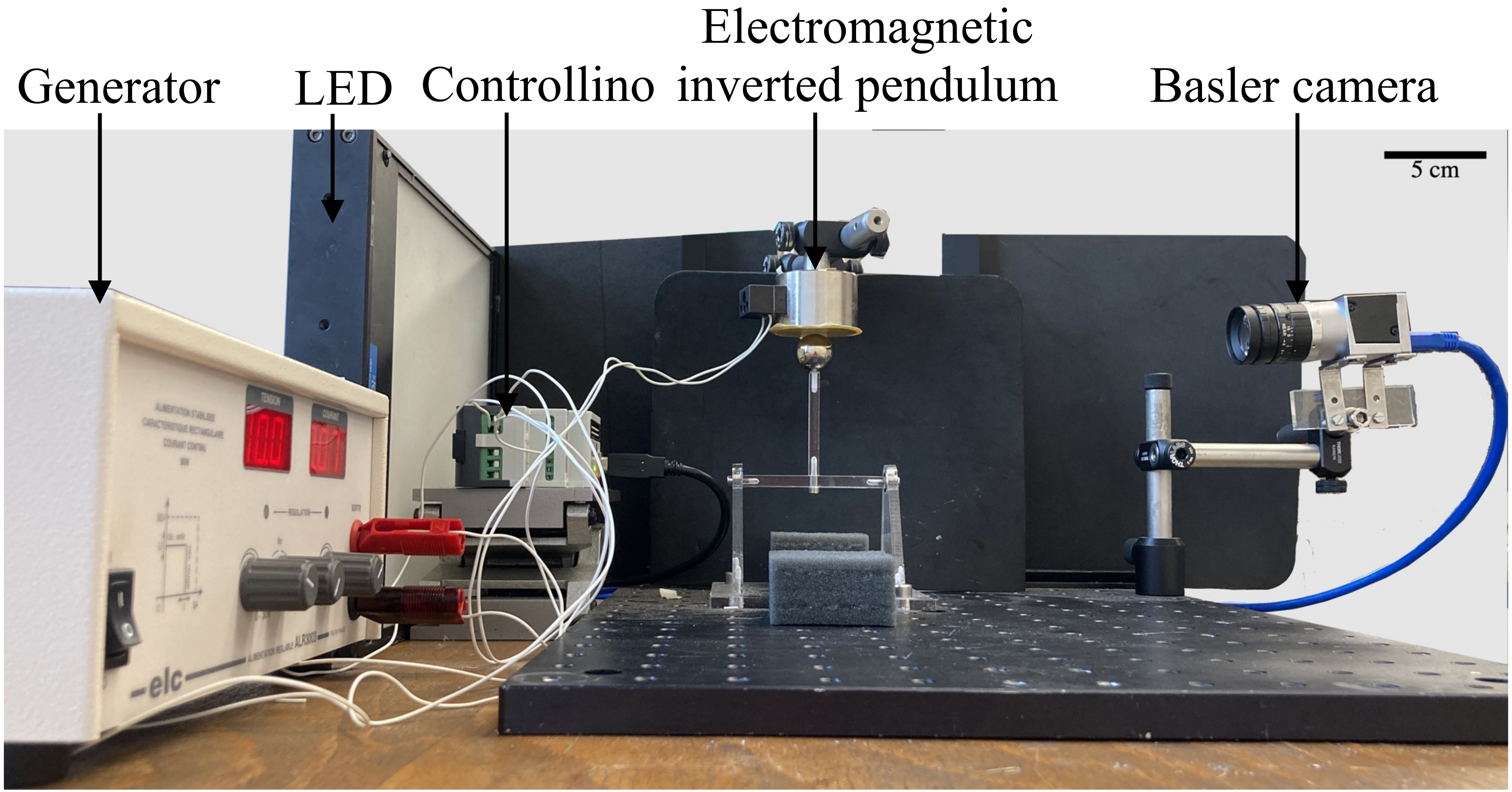


Paul’s Trap: trapping particles in an electric field [1]
Video credit: Newtonian Labs

Making two liquid layer levitate [2]

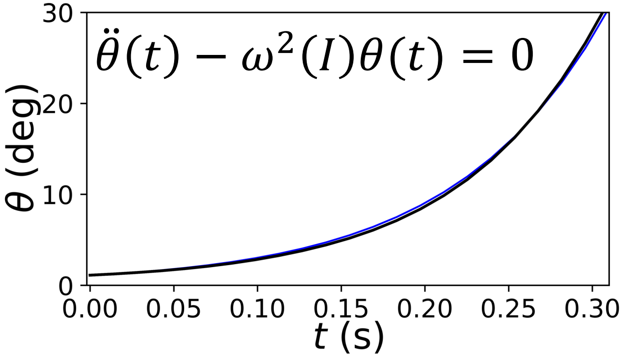
[1] W. Paul. *Reviews of modern physics*, **62**, 1990.
[2] B.Apffel *et al*. *Nature*, **585**, 2020.

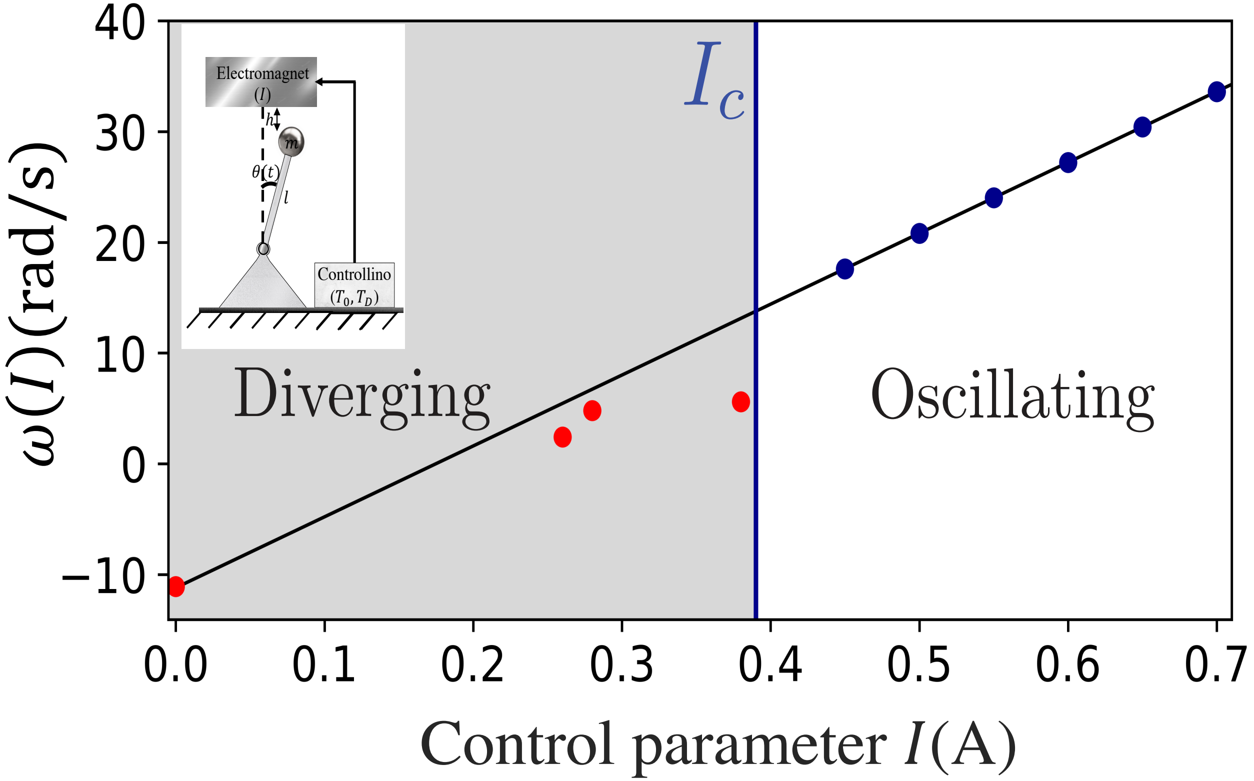
Electromagnetic inverted pendulum



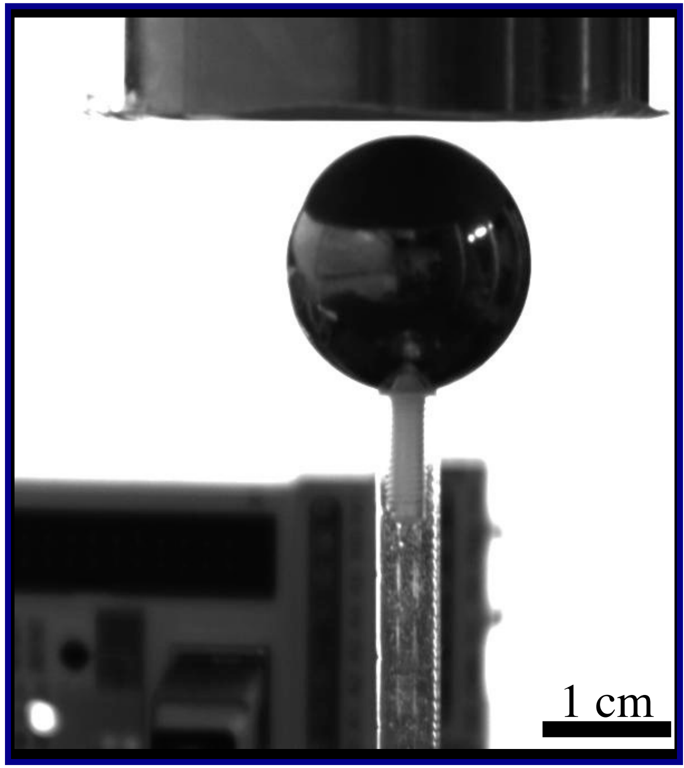
Large modulations at the macroscopic scale

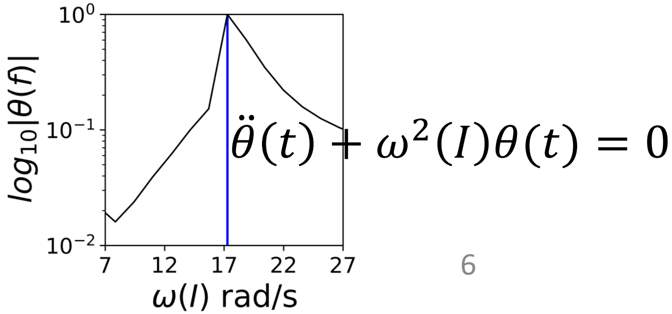
$I = 0\ A$





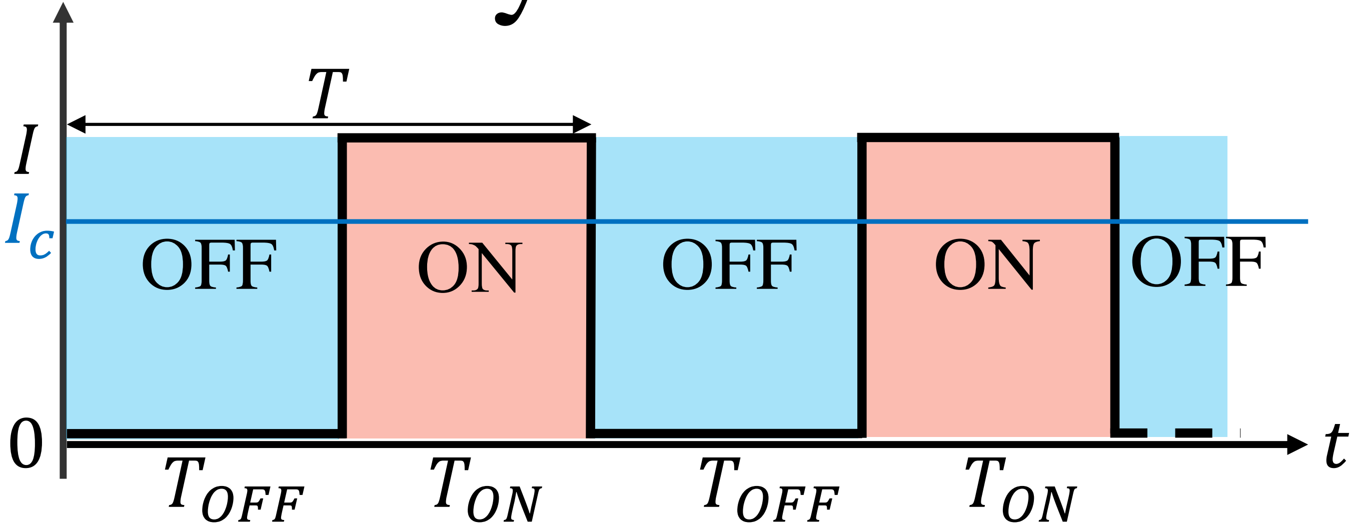
$I = 0.45\ A$

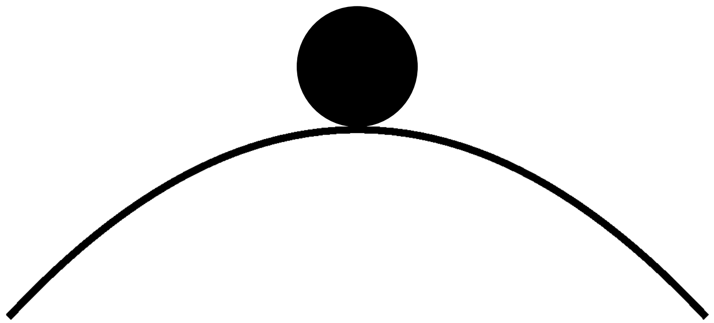


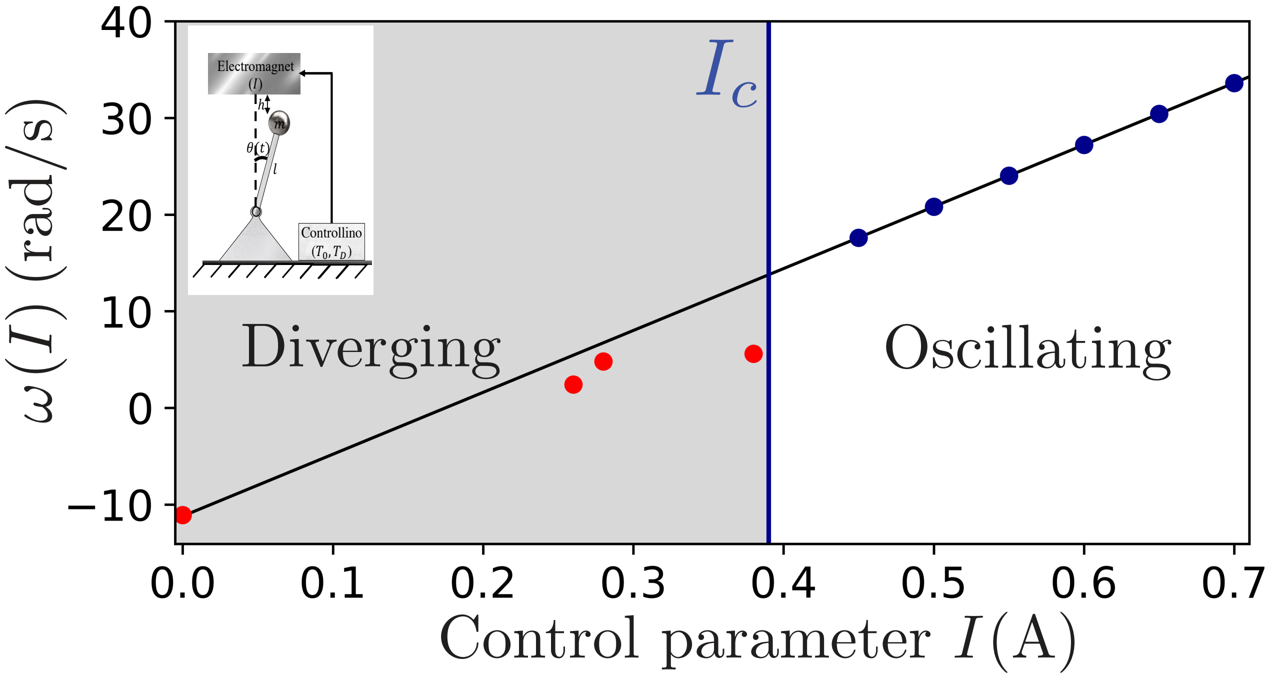


White light = Electromagnets ON

Experimental dynamical stabilization







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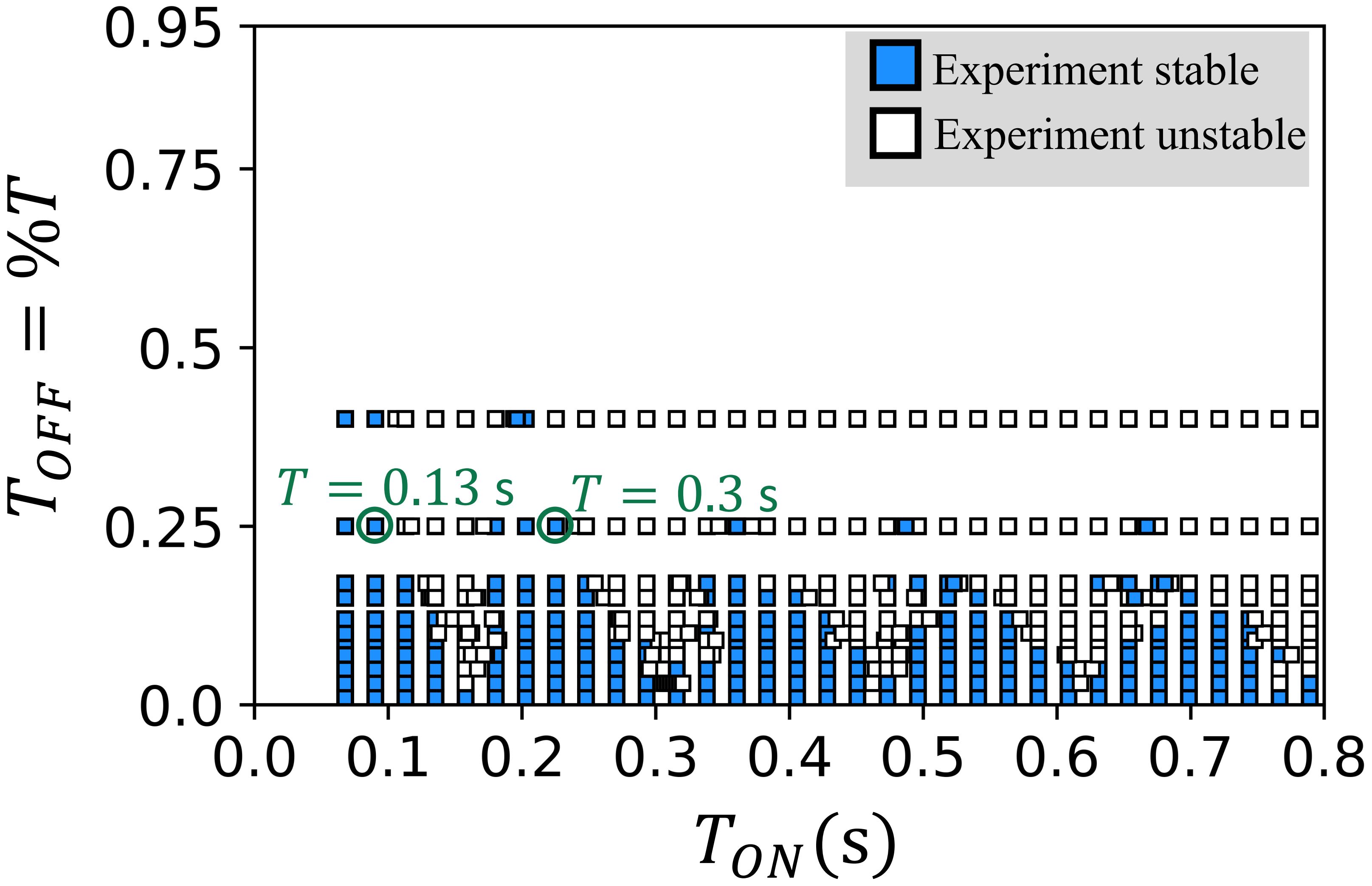
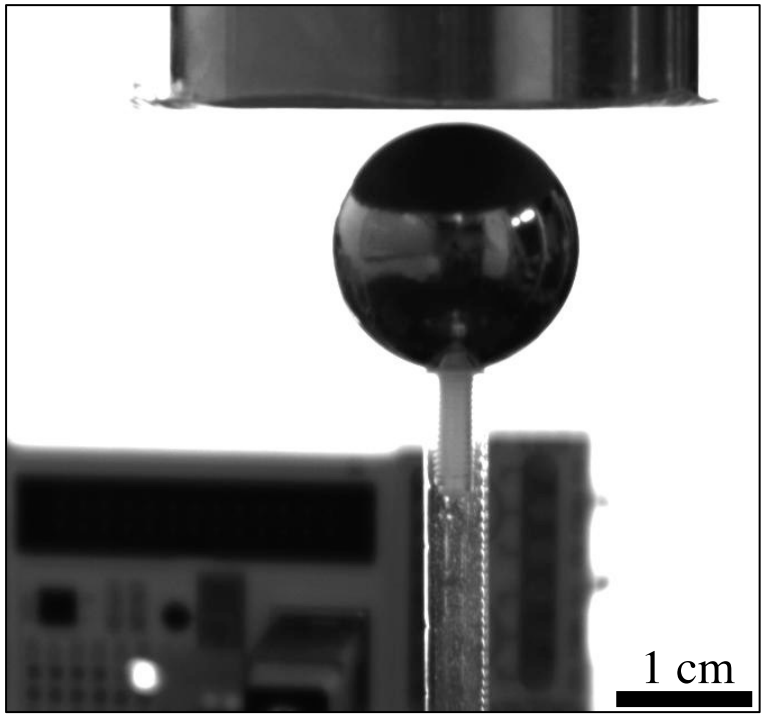
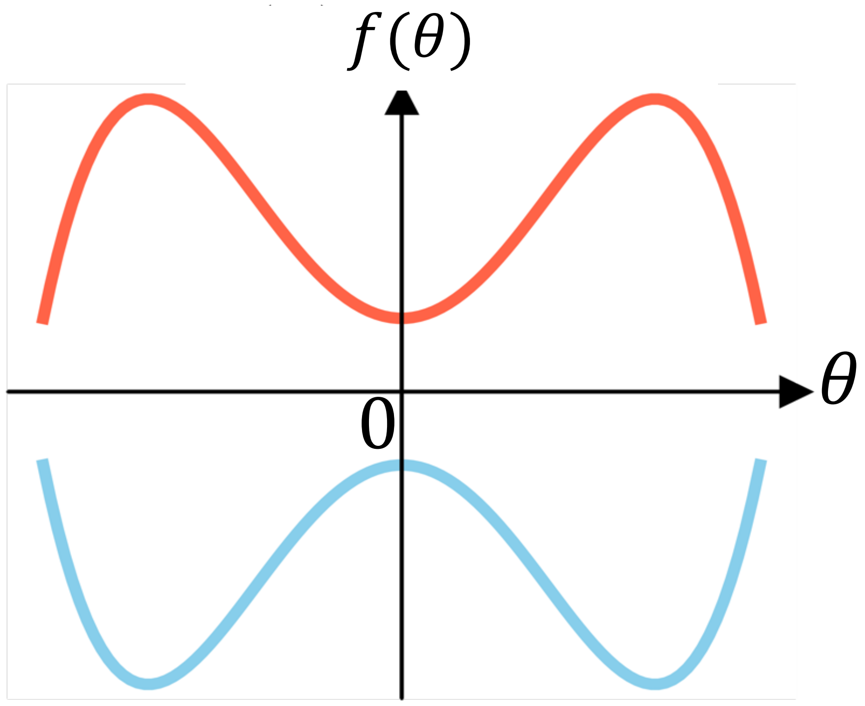


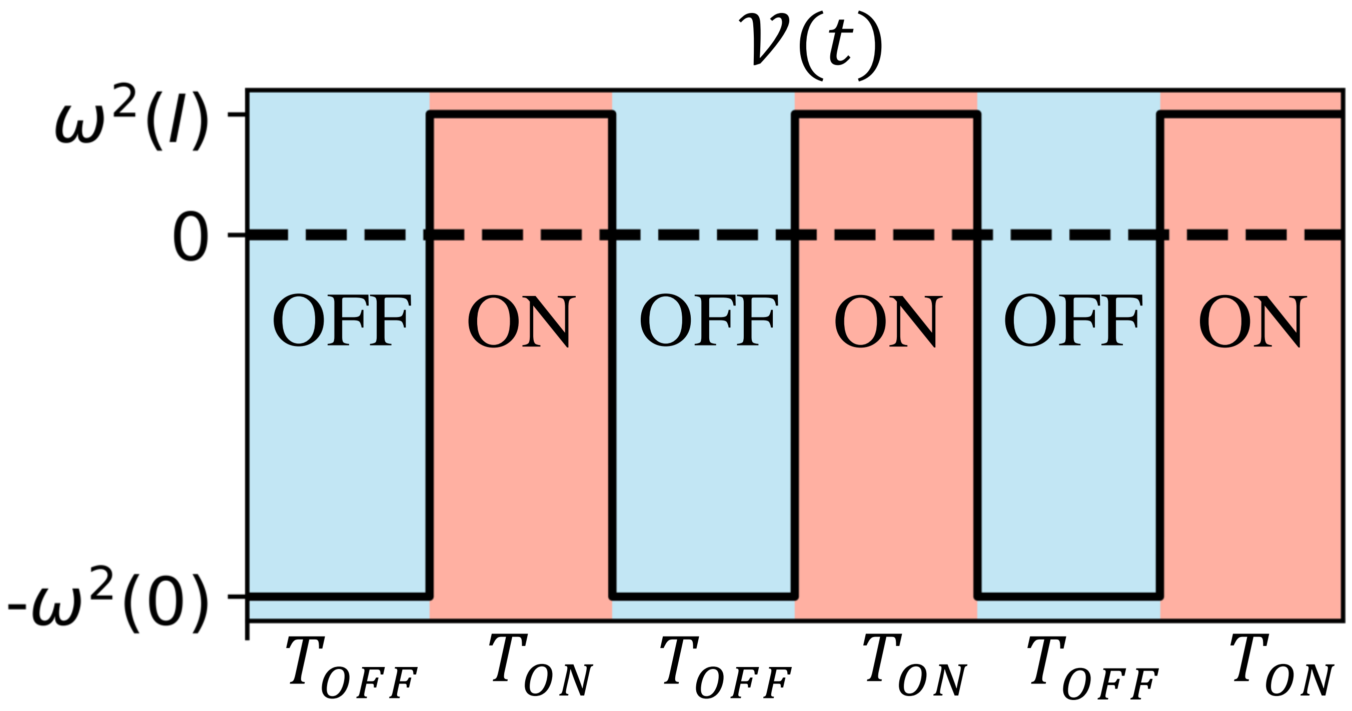


Characteristic time:
$T_0 = 0.57$ s

Inverted pendulum: Smith et. al (1992):
$T_0 = 0.18$ s
$T = 6.4{\times}10^{-3}$ s

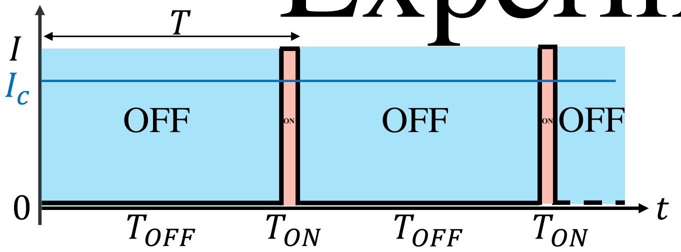
Experimental dynamical stabilization

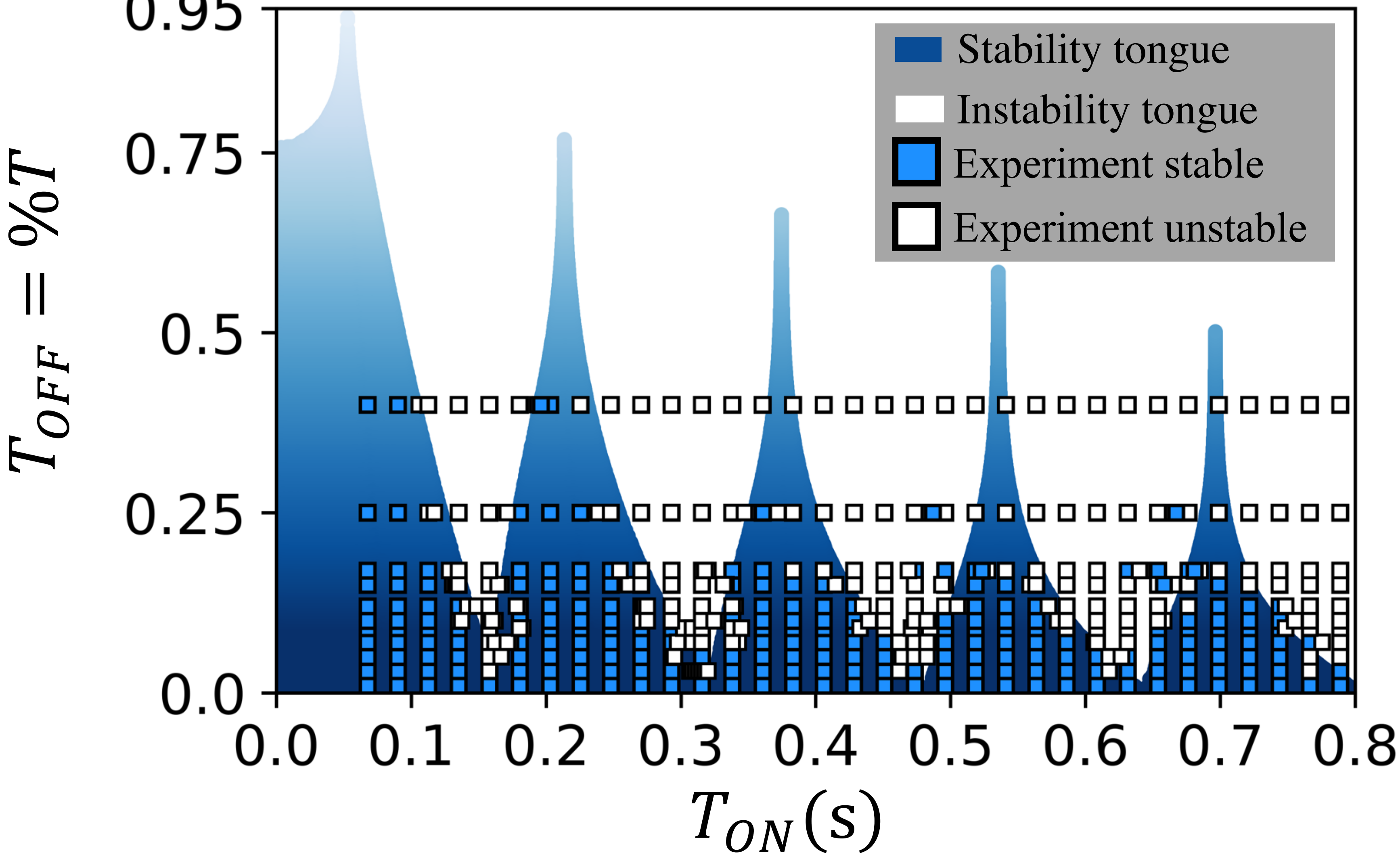




$$\begin{cases} \dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2} - \omega^2(0)\theta(t) = 0 \text{ during } T_{OFF} \\ \dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \omega^2(I)\theta(t) = 0 \text{ during } T_{ON} \end{cases}$$

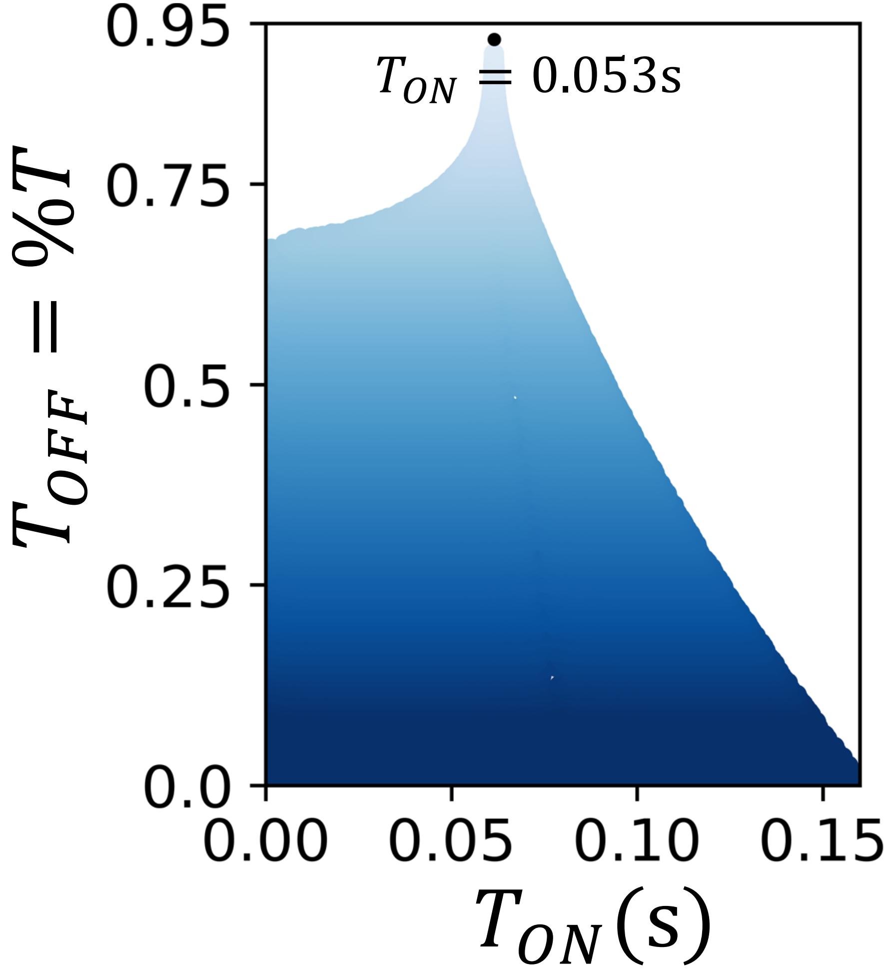
Experimental dynamical stabilization





Numerical study of the tip of the stability regions

$\omega(0) = 11.1$ rad/s and $\omega(I) = 19.5$ rad/s
$T_{OFF} = 0.95\%T$



Numerical study of the tip of the stability regions

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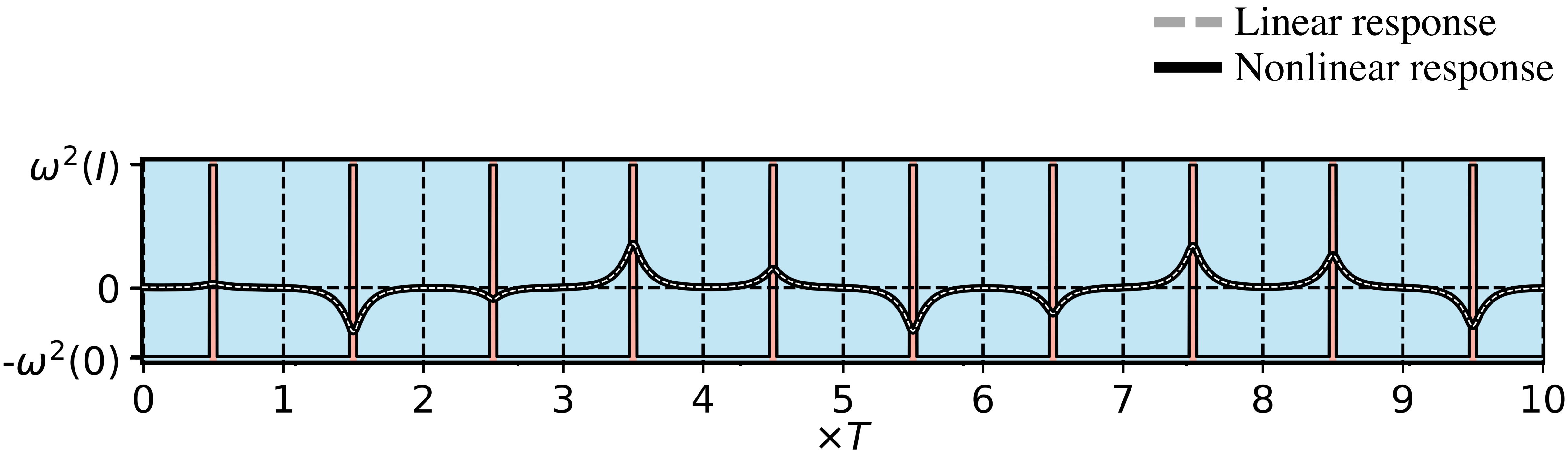

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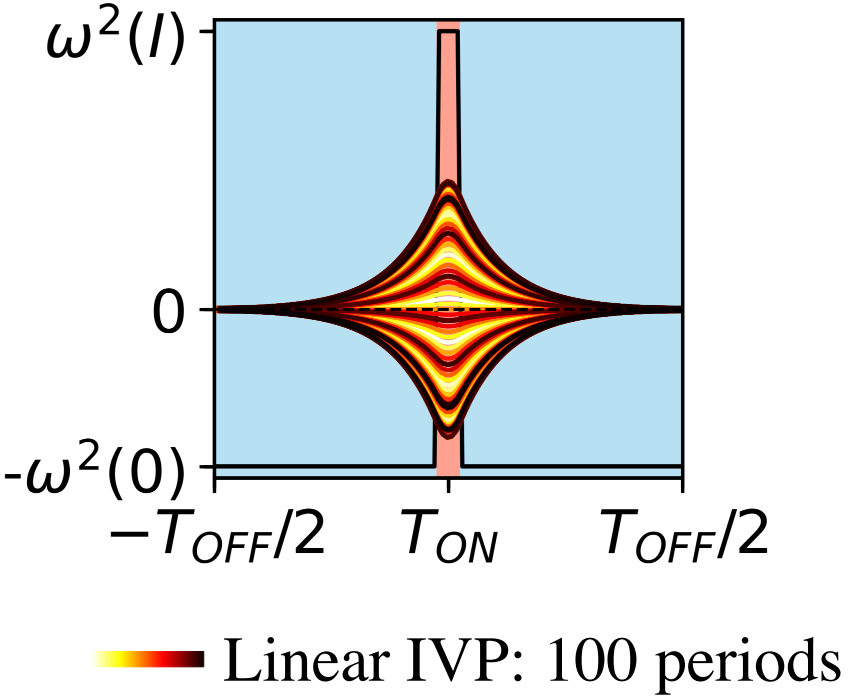

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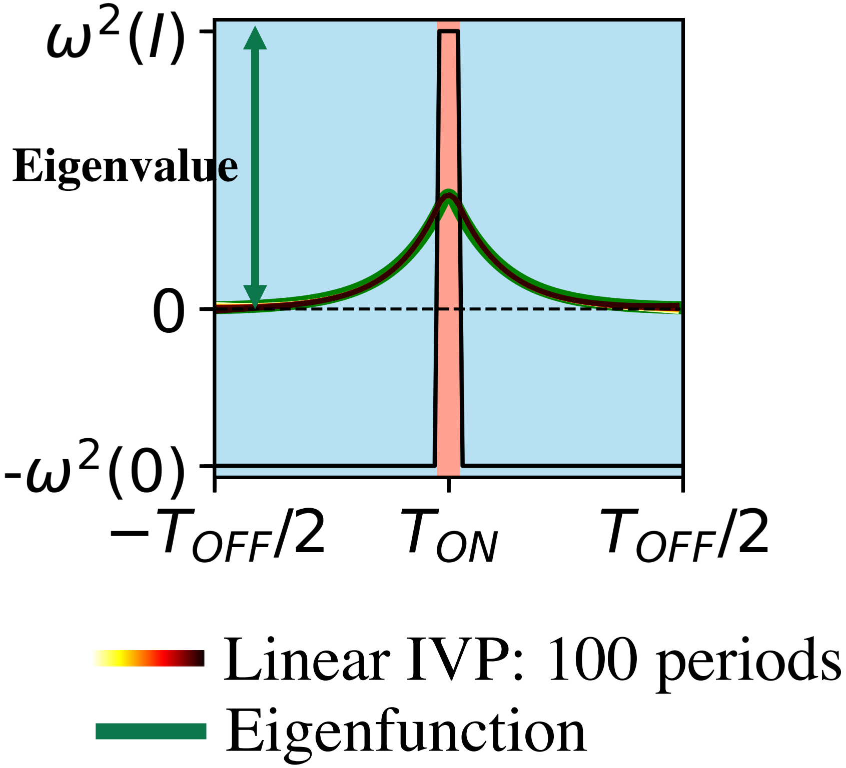

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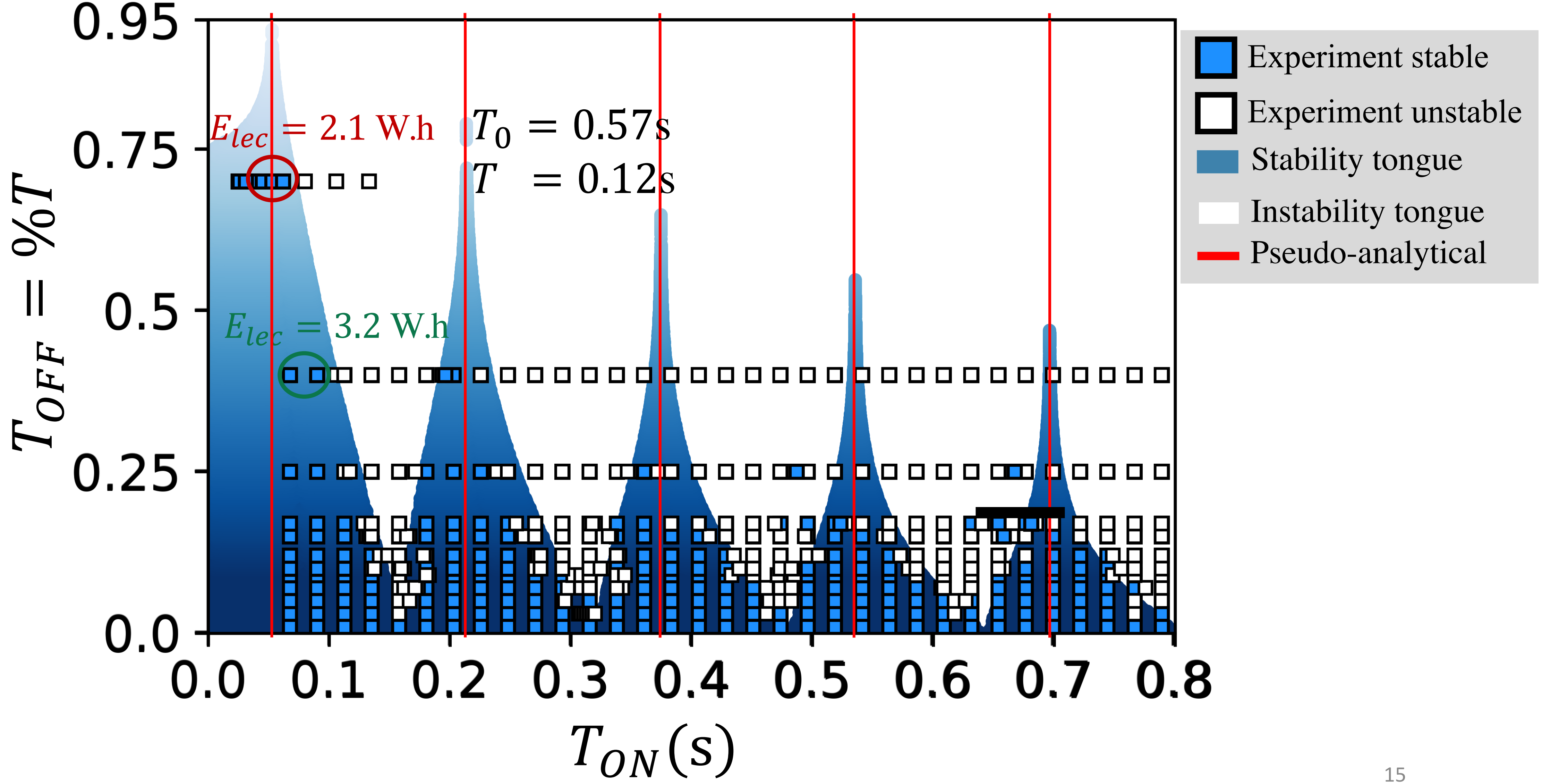
Boundary value problem [1]

$$\begin{cases} -\left[\dfrac{d^2}{dt^2} - \Delta\Omega^2\right]\theta(t) = \omega^2(I)\theta(t), t \in \text{Outside} \\ -\left[\dfrac{d^2}{dt^2} + \ 0\ \right]\theta(t) = \omega^2(I)\theta(t), t \in [-T_{ON}/2, T_{ON}/2] \\ \theta(-\infty) = \theta(\infty) = 0 \\ \text{Outside} = [-\infty, -T_{ON}/2] \cup [T_{ON}/2, +\infty] \text{ and } \Delta\Omega^2 = \omega^2(I) + \omega^2(O) \end{cases}$$

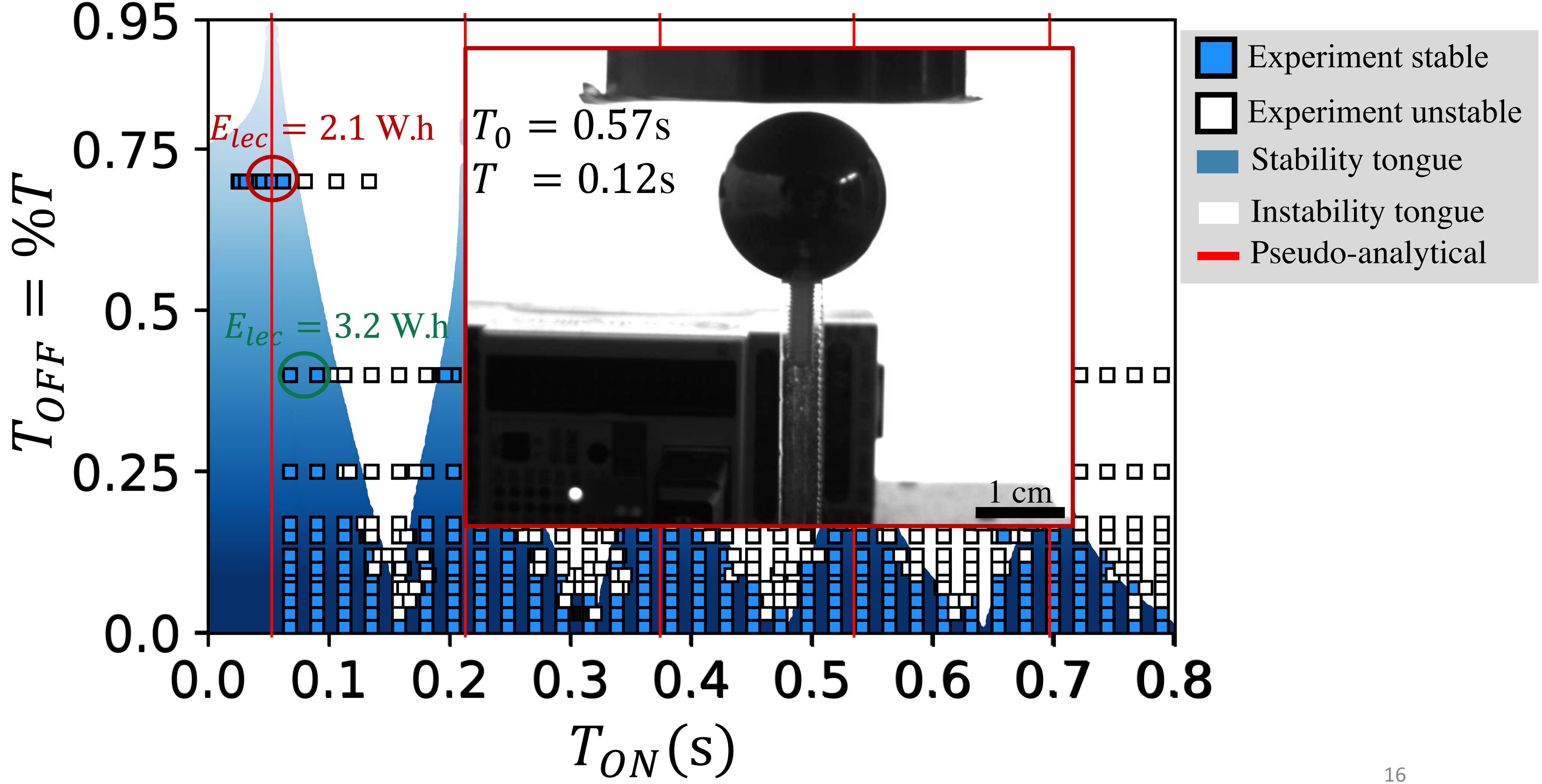
[1] A.Messiah. *Quantum mechanics*, **1**, 1961.

Experimental validation of the master curves



A. A Grandi *et al*. *Nonlinear Dynamics*, 111:13, 2023.

Experimental validation of the master curves



A. A Grandi *et al*. *Nonlinear Dynamics*, 111:13, 2023.

Thank you for your attention