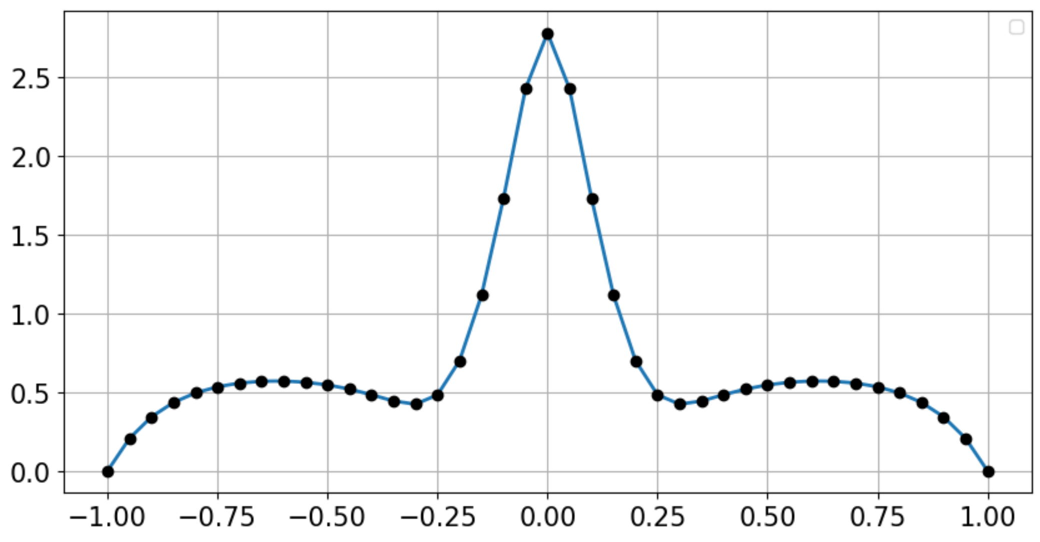
NONLINEAR STANDING WAVE FORMATION IN DAMPED OSCILLATOR CHAINS



Arthur Barbosa, Ph.D. student

Dr. Najib Kacem

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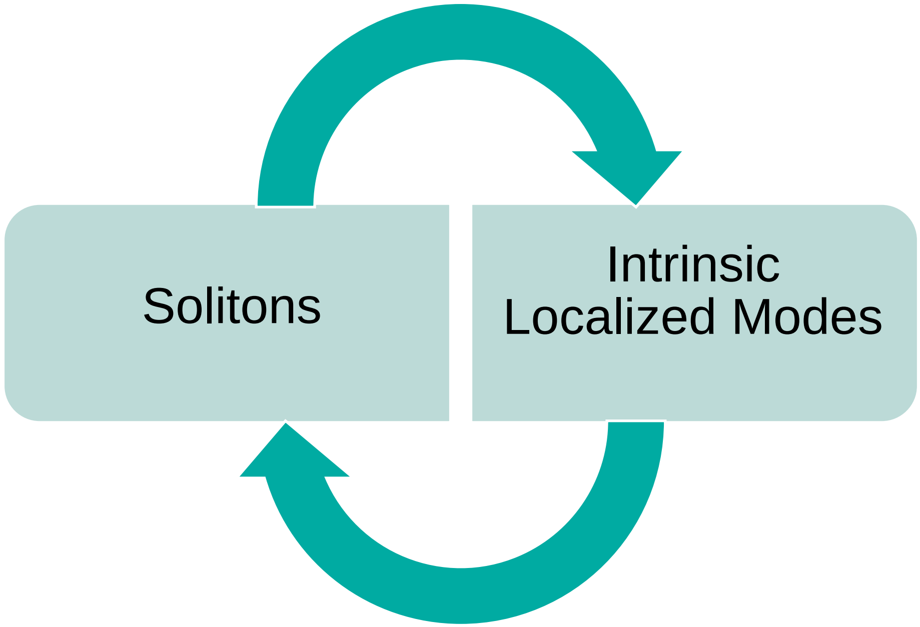
Introduction and motivation of work

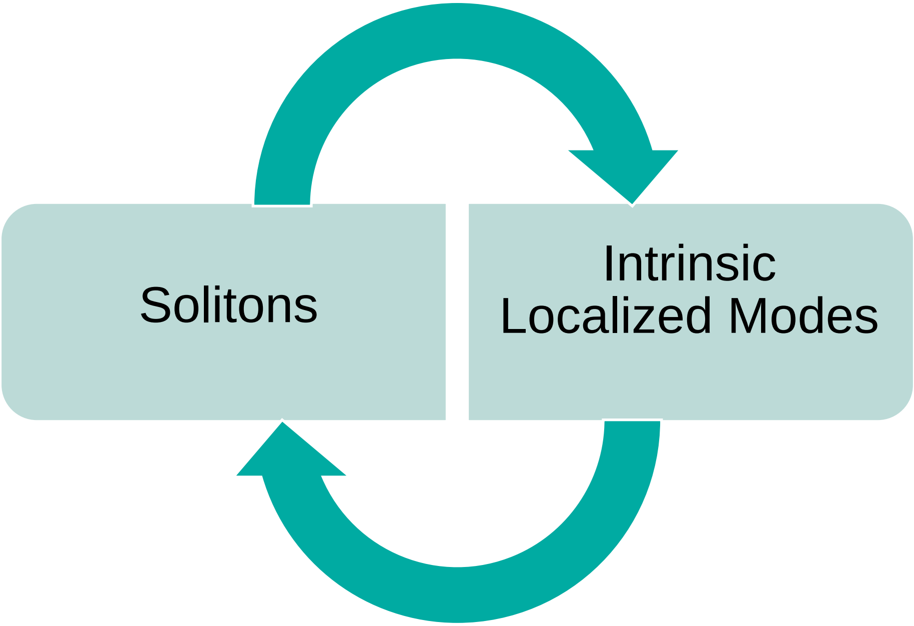
- non-dispersive waves that **maintain their shape and speed** over time and space
- arise due to a balance between **nonlinear effects and dispersion**
- **localized energy** that is concentrated in a **compact region** of space



- non-linear **vibration** localization phenomenon
- spatially **localized** and temporary periodic solution
- **homogeneous** lattice

Introduction and motivation of work

- Mechanics
- Optics
- Plasma Physics
- Hydrodynamic
- ...

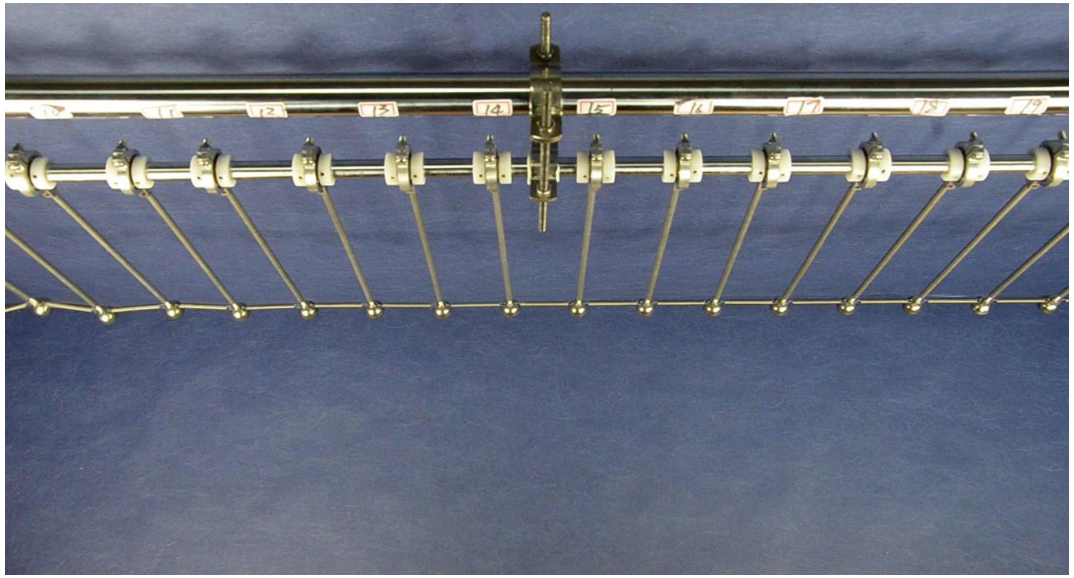


- Beams
- Pendulums
- Point Masses
- Rotors
- Coupled nonlinear oscillators in general

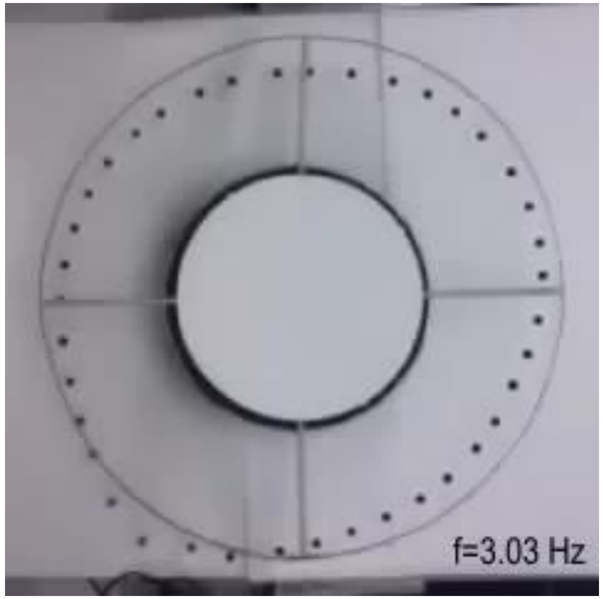
Introduction and motivation of work

Physics Department of the University of Burgundy,
Institut CARNOT de Bourgogne, Équipe Solitons, Laser et Communications optiques
Video by Julien FATOME, Stéphane PITOIS and Guy MILLOT
([Collision of KdV solitons - YouTube])
([File:Soliton hydro.jpg - Wikimedia Commons])

Liu, Xinyun, et al. "Experimental demonstration of emission of solitons from a resonant localized wave." *Physical Review E* 102.5 (2020): 052201.

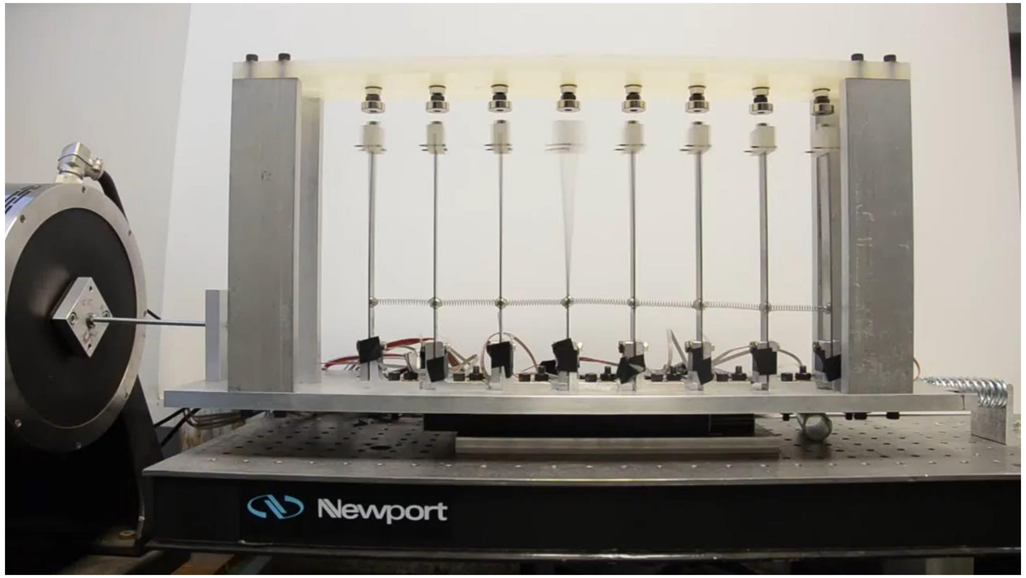


Introduction and motivation of work



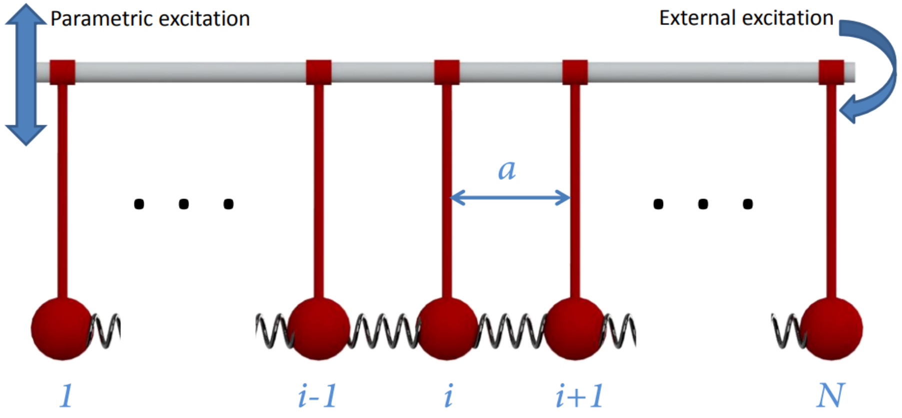
Sánchez-Morcillo, V. J., et al. "Spatio-temporal dynamics in a ring of coupled pendula: Analogy with bubbles." *Localized Excitations in Nonlinear Complex Systems: Current State of the Art and Future Perspectives* (2014): 251-262.

Perkins, J. Edmon. *Noise-influenced dynamics of nonlinear oscillators*. Diss. University of Maryland, College Park, 2015.



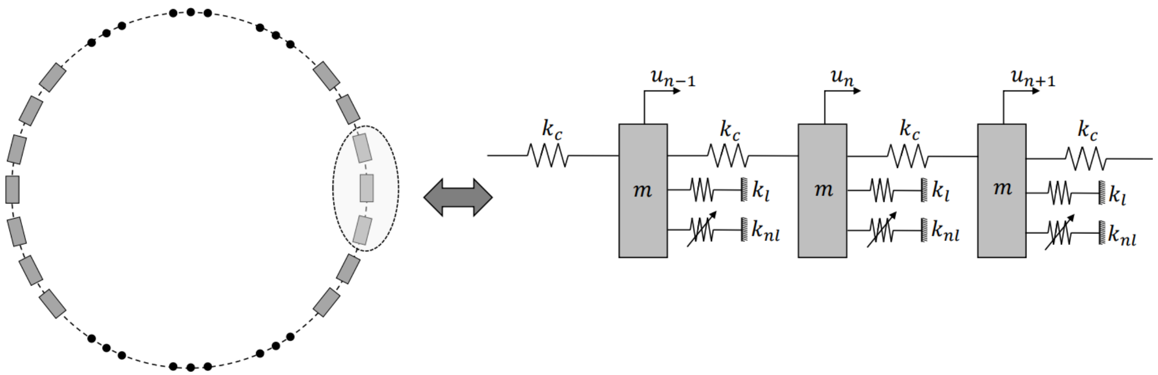
Introduction and motivation of work

Continuous Domain ⟷ Discreet Domain



Jallouli, Aymen, Najib Kacem, and Noureddine Bouhaddi. "Stabilization of solitons in coupled nonlinear pendulums with simultaneous external and parametric excitations." *Communications in Nonlinear Science and Numerical Simulation* 42 (2017): 1-11.

Fontanela, Filipe, et al. "Dissipative solitons in forced cyclic and symmetric structures." *Mechanical Systems and Signal Processing* 117 (2019): 280-292.



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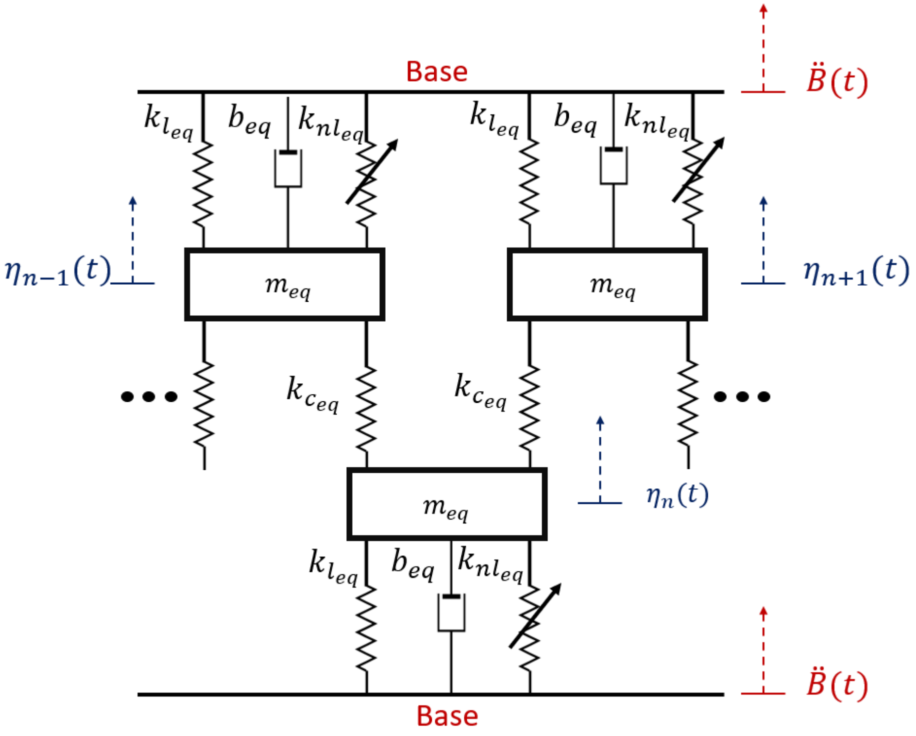
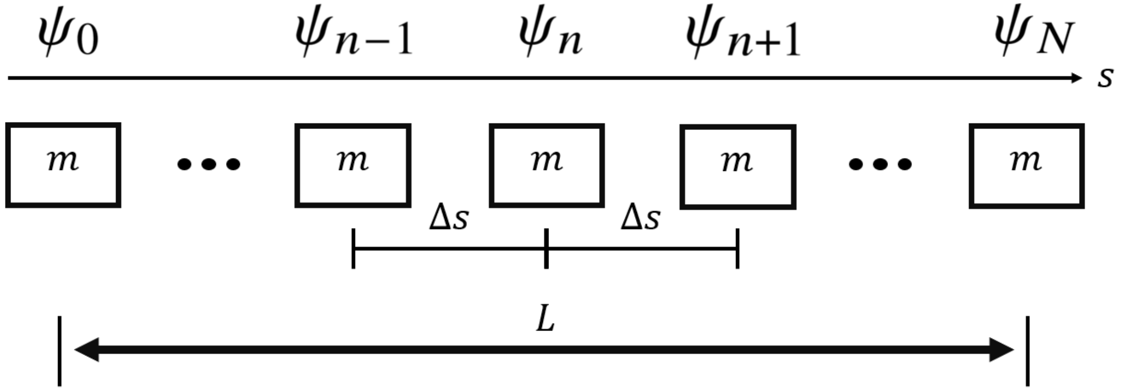
Mathematical Model for Nonlinear Oscillator Chains

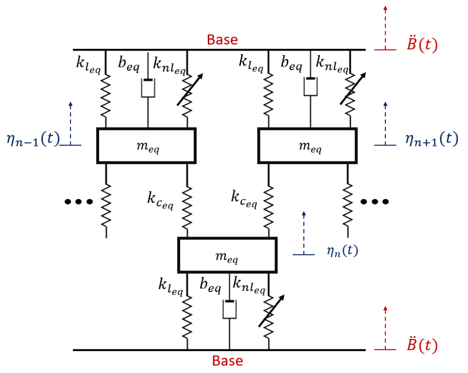
Equivalent System



Continuity hypothesis



Mathematical Model for Nonlinear Oscillator Chains



$$\ddot{\eta}_n + 2\zeta\omega_0\dot{\eta}_n + \omega_0^2\eta_n - \frac{k_{nl_{eq}}}{m_{eq}}\eta_n^3 + \frac{k_{c_{eq}}}{m_{eq}}\left(2\eta_n - \eta_{n+1} - \eta_{n-1}\right) = -0.5B_A\left(e^{i\omega t} + e^{-i\omega t}\right)$$

Perturbation method

$$B_A = \varepsilon B_{Ae},\; k_c = \varepsilon k_{ce}$$

$$k_{nl} = \varepsilon k_{nle},\quad \zeta = \varepsilon\zeta_e$$

Multiple scale method

$$\eta_n = -\left(\psi_n(T)\, e^{-it\omega_0(1-\varepsilon)} + e^{it\omega_0(1-\varepsilon)}\overline{\psi}_n(T)\right) + O(\epsilon)$$

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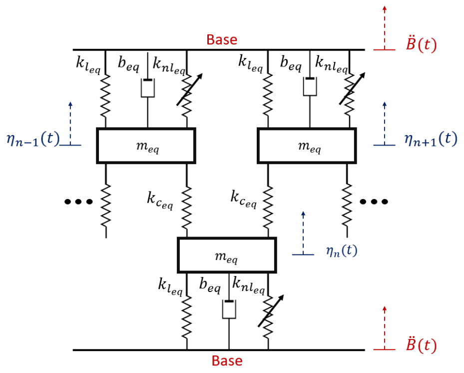
Mathematical Model for Nonlinear Oscillator Chains



$$\frac{0.25mB_{Ae}}{k_l} + i\zeta_e\psi_n(T) + \frac{3k_{nle}}{2k_l}\psi_n^2(T)\overline{\psi}_n(T)$$

$$+$$

$$\left(\frac{k_{ce}}{k_l}\psi_n(T) - \frac{k_{ce}}{2k_l}\psi_{n+1}(T) - \frac{k_{ce}}{2k_l}\psi_{n-1}(T)\right) - \psi_n(T)$$

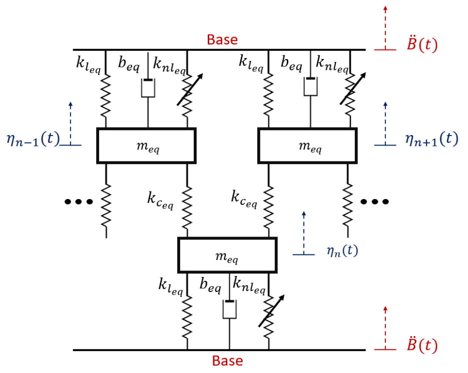
$$=$$

$$-\frac{i}{\omega_0}\frac{d\psi_n(T)}{dT}.$$

Nonlinear Schrödinger Equation:

- Discretized
- Dimensional

Mathematical Model for Nonlinear Oscillator Chains



$$\frac{0.25mB_{Ae}}{k_l} + i\zeta_e\psi_n(T) + \frac{3k_{nle}}{2k_l}\psi_n^2(T)\overline{\psi}_n(T)$$
$$+$$
$$\left(\frac{k_{ce}}{k_l}\psi_n(T) - \frac{k_{ce}}{2k_l}\psi_{n+1}(T) - \frac{k_{ce}}{2k_l}\psi_{n-1}(T)\right) - \psi_n(T)$$
$$=$$
$$-\frac{i}{\omega_0}\frac{d\psi_n(T)}{dT}.$$

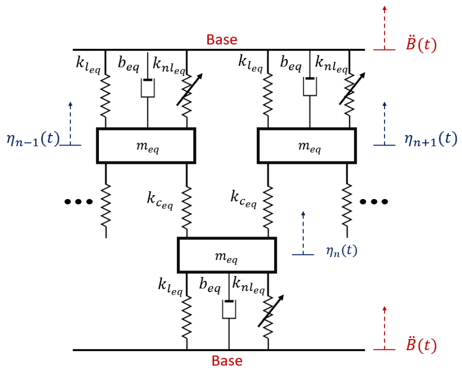
Adjacent Oscillators

$$\psi_n(T) = \psi\left(n\Delta s, T\right) = \psi\left(s_n, T\right)$$
$$\psi_{n\pm1} \approx \psi\left(s_n, T\right) \pm \Delta s\frac{\partial}{\partial s}\psi\left(s_n, T\right) + 0.5\Delta s^2\frac{\partial^2}{\partial s^2}\psi\left(s_n, T\right)$$

Adimensio-nalization

$$\hat{\psi} = \sqrt{\frac{3k_{nle}}{4k_l}}\psi\left(s, T\right), \qquad \hat{T} = T\omega_0, \qquad \hat{s} = \frac{s}{\delta},$$
$$\Delta\hat{s} = \frac{\Delta s}{\delta}, \qquad k_{ce} = \frac{2\delta^2}{\Delta s^2}k_l, \quad h = 0.125\sqrt{3}mB_{Ae}\sqrt{\frac{k_{nle}}{k_l^3}},$$

Mathematical Model for Nonlinear Oscillator Chains



$$-i\frac{\partial\hat{\psi}}{\partial\hat{T}} = h + (2|\hat{\psi}|^2 + i\zeta_e - 1)\hat{\psi} + \frac{\partial^2\hat{\psi}}{\partial\hat{s}^2}$$

Nonlinear Schrödinger Equation:

- Continuous Domain
- Dimensionless

Introduction and motivation of work

$$-i\frac{\partial\hat{\psi}}{\partial\hat{T}} = h + (2|\hat{\psi}|^2 + i\zeta_e - 1)\hat{\psi} + \frac{\partial^2\hat{\psi}}{\partial\hat{s}^2}$$

Time Factor

External Excitation

Nonlinearity

Damping

Space Factor

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Existence and Stability of Solitonic Solutions

$$-i\frac{\partial\hat{\psi}}{\partial\hat{T}} = h + (2|\hat{\psi}|^2 + i\zeta_e - 1)\hat{\psi} + \frac{\partial^2\hat{\psi}}{\partial\hat{s}^2}$$

$$\frac{\partial\hat{\psi}}{\partial\hat{T}} = 0$$

Intrinsic Localized Modes

Existence and Stability of Solitonic Solutions

$$-i\frac{\partial\hat{\psi}}{\partial\hat{T}} = h + (2|\hat{\psi}|^2 + i\zeta_e - 1)\hat{\psi} + \frac{\partial^2\hat{\psi}}{\partial\hat{s}^2}$$

$\zeta_e = 0$ ⟷ Damping = 0

Analytical solutions

$$\psi(x) = \psi_0\left(1 + \frac{2\sinh^2\alpha}{1 - \cosh\alpha\cosh(Ax)}\right)$$

$$h = \frac{\sqrt{2}\cosh^2\alpha}{(1+2\cosh^2\alpha)^{3/2}}, \quad \psi_0 = \frac{1}{\sqrt{2(1+2\cosh^2\alpha)}}, \quad A = 2\psi_0\sinh\alpha$$

Barashenkov, I. V., and Yu S. Smirnov. "Existence and stability chart for the ac-driven, damped nonlinear Schrödinger solitons." *Physical Review E* 54.5 (1996): 5707.

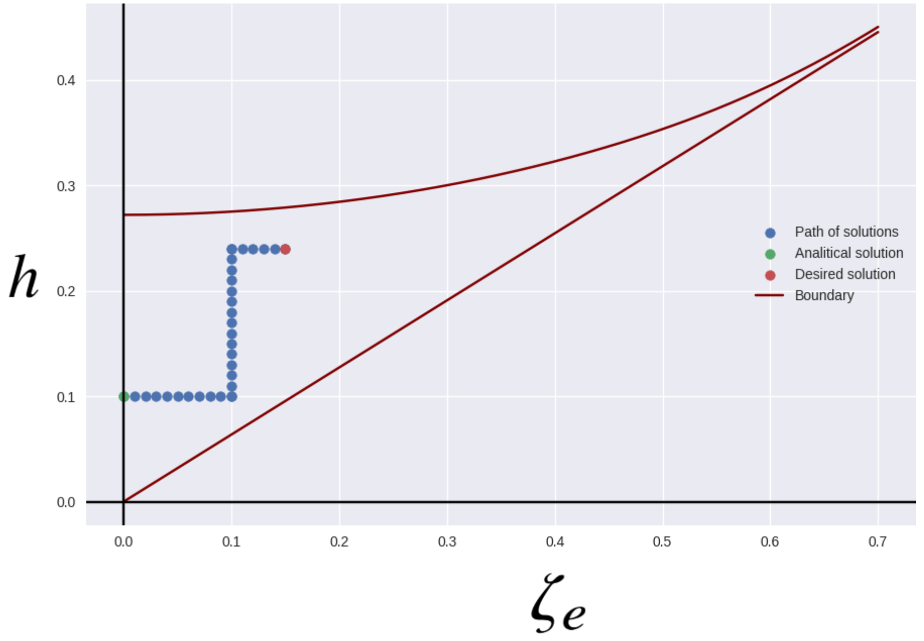
Existence and Stability of Solitonic Solutions

$$-i\frac{\partial\hat{\psi}}{\partial\hat{T}} = h + (2|\hat{\psi}|^2 + i\zeta_e - 1)\hat{\psi} + \frac{\partial^2\hat{\psi}}{\partial\hat{s}^2}$$

External Excitation (h)

Damping ($i\zeta_e$)

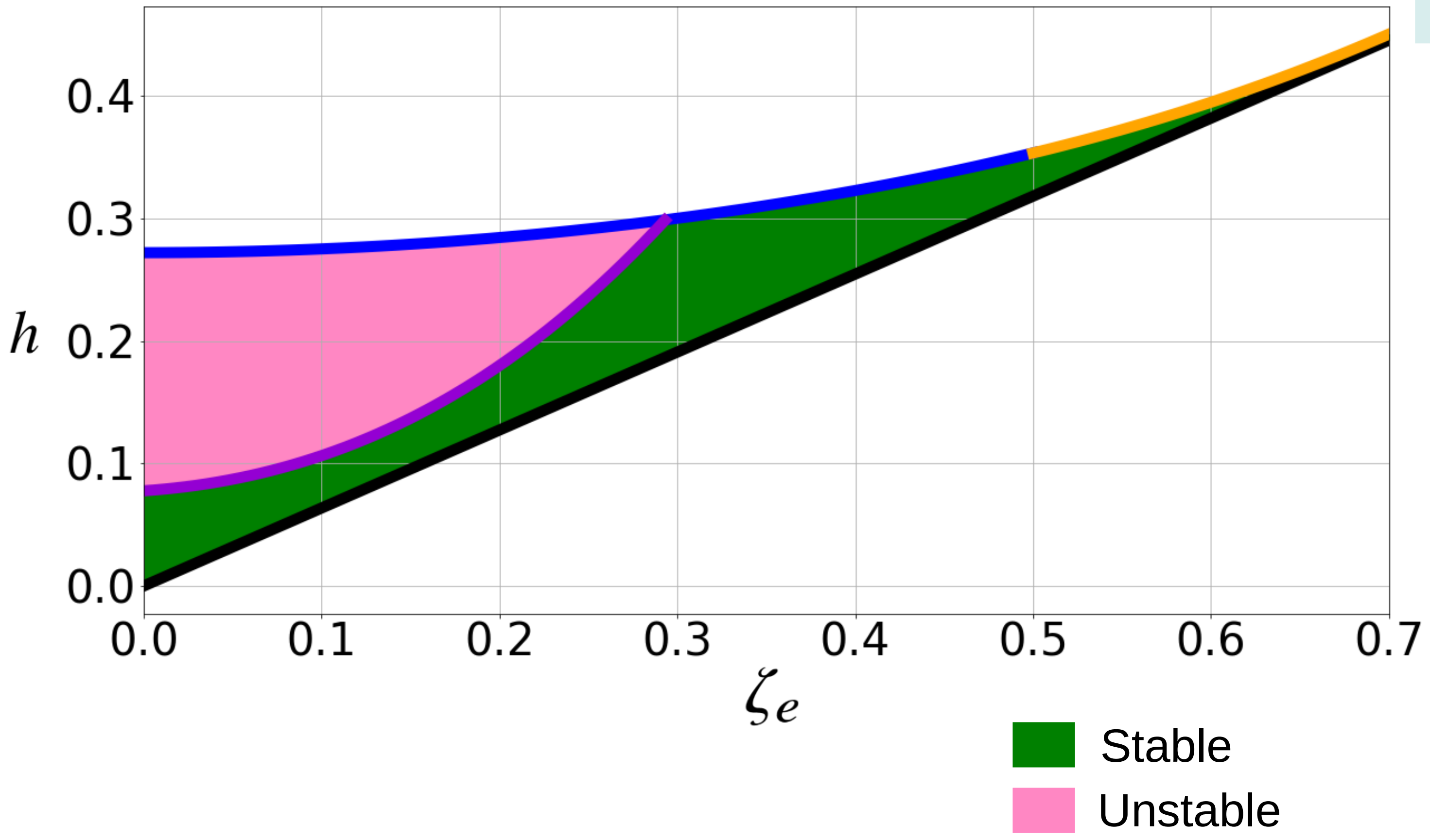
No analytical solutions



Existence and Stability of Solitonic Solutions

$$-i\frac{\partial\hat{\psi}}{\partial\hat{T}} = h + (2|\hat{\psi}|^2 + i\zeta_e - 1)\hat{\psi} + \frac{\partial^2\hat{\psi}}{\partial\hat{s}^2}$$

$$\frac{\partial\hat{\psi}}{\partial\hat{T}} = 0$$



Design methodology

$$\zeta = \varepsilon \zeta_e$$

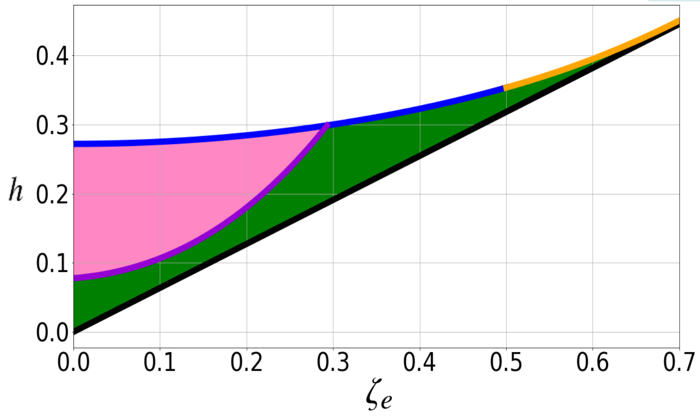
$$\hat{L} = \frac{L}{\delta}$$

$$k_c = \frac{2\varepsilon \delta^2}{\Delta s^2} k_l$$

$$h = 0.125\sqrt{3} m \frac{B_A}{\varepsilon} \sqrt{\frac{k_{nl}}{\varepsilon k_l^3}}$$

Design Variables

Stability Conditions



Design methodology

1º Physical variables of the oscillators $\omega_0^2 = \frac{k_{leq}}{m_{eq}}$ $\frac{b_{eq}}{m_{eq}} = 2\zeta\omega_0$

2º Perturbation factor

$$\zeta = \varepsilon\zeta_e$$

3º

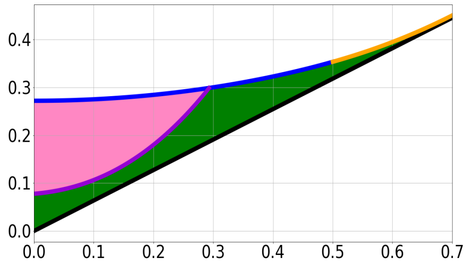
Domain construction

$$\hat{L} = \frac{L}{\delta}$$

Number of oscillators $\Delta s = \frac{L}{N-1}$

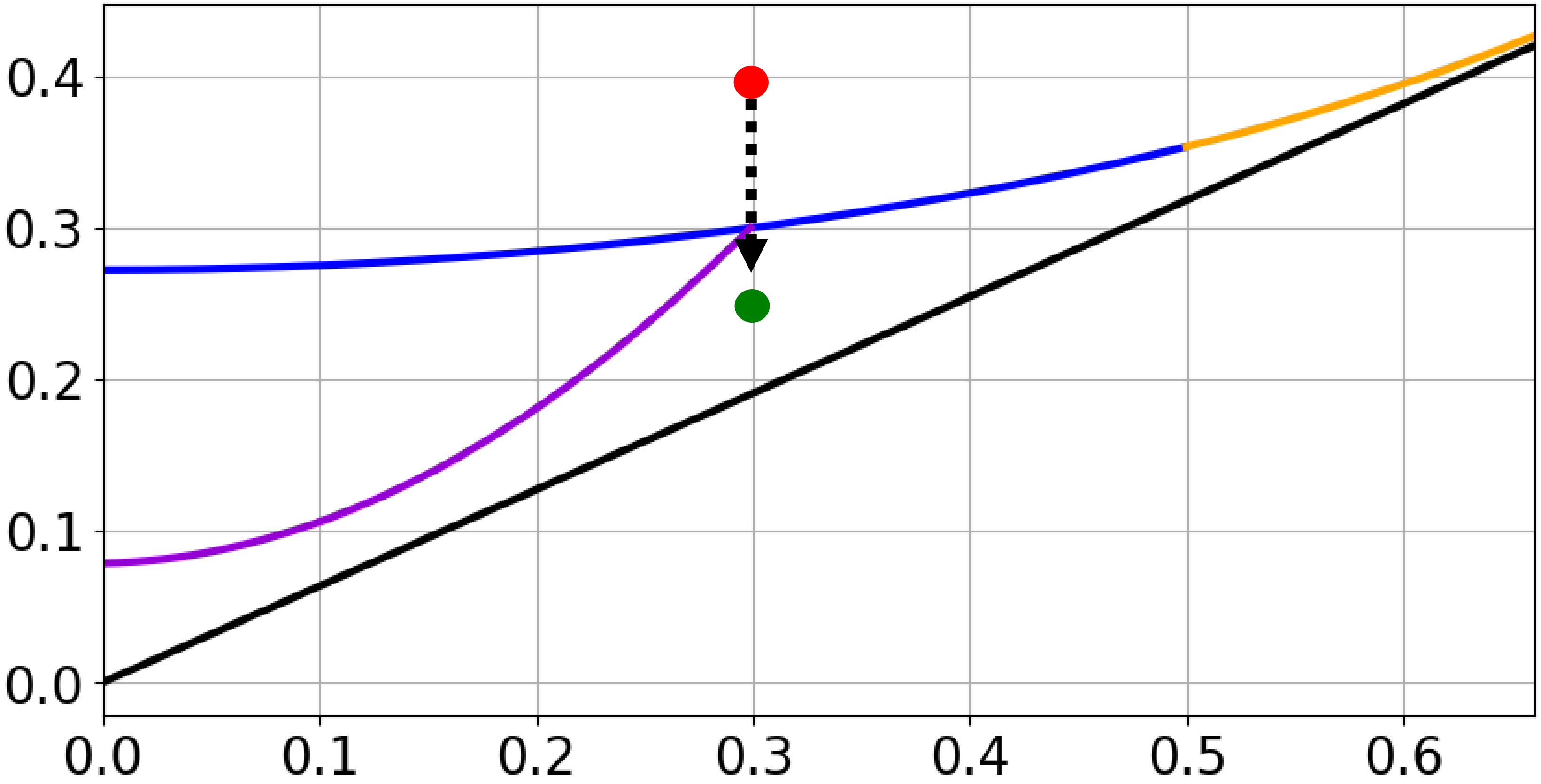
Coupling stiffness $k_c = \frac{2\varepsilon\delta^2}{\Delta s^2} k_l$

4º Stability Conditions

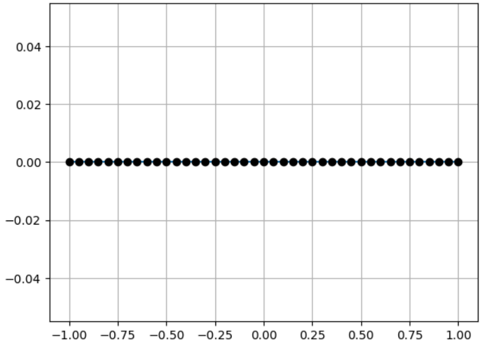


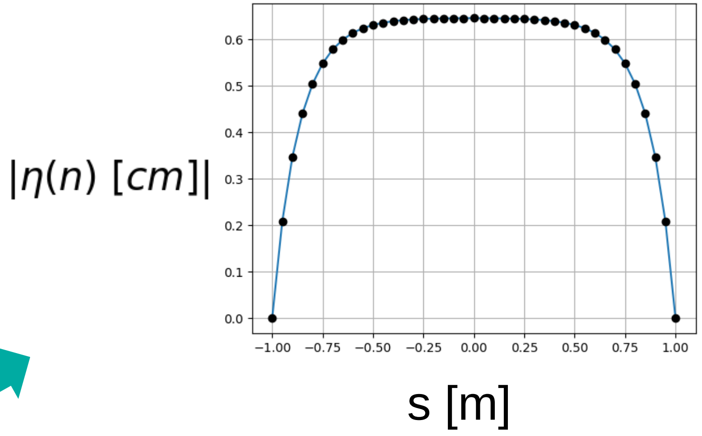
5º Nonlinearity $h = 0.125\sqrt{3}m\frac{B_A}{\varepsilon}\sqrt{\frac{k_{nl}}{\varepsilon k_l^3}},$

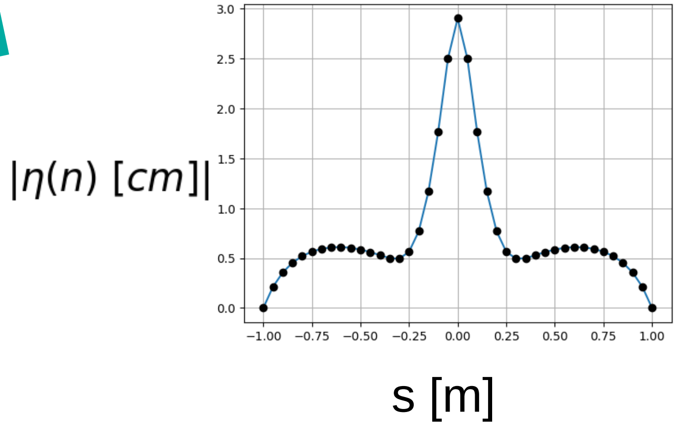
Numerical Example



Numerical Example







Parameters obtained by the developed methodology

Parameter	value	Unit
m	0.5	kg
k_l	24180.530	N/m
ω_0	70π	rad/s
ζ	0.3%	dimensionless
B_A	2.943	m/s^2
N	41	dimensionless
L	2	m
Δs	0.05	m
ε	0.01	dimensionless
δ	0.118	dimensionless
k_c	2693.518	N/m

Parameter	value	Unit
Total simulation time	25	s
Time step between iterations	0.0005	s
h for time $< 5s$	0.4	dimensionless
k_{nl} for time $< 5s$	22287207.593	N/m^3
h for time $\geq 5s$	0.25	dimensionless
k_{nl} for time $\geq 5s$	8705940.466	N/m^3

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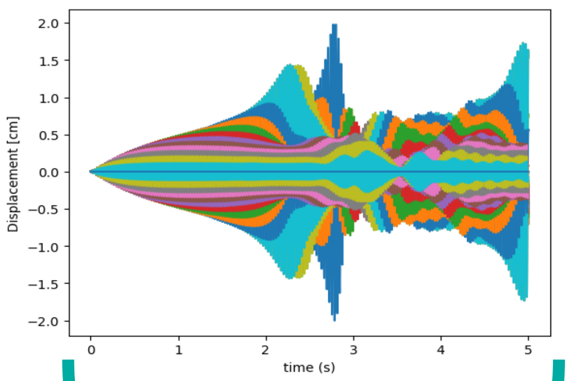
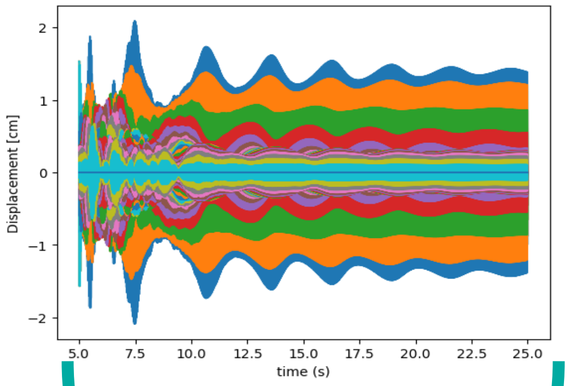
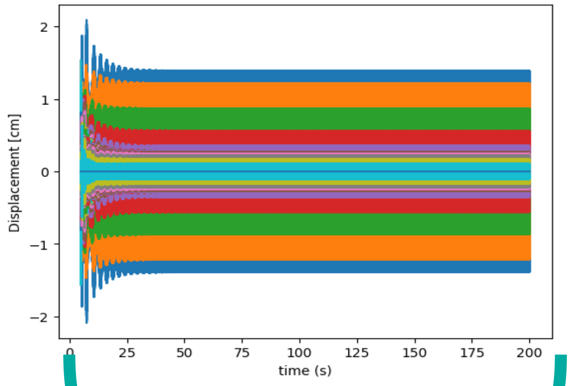
Numerical Example



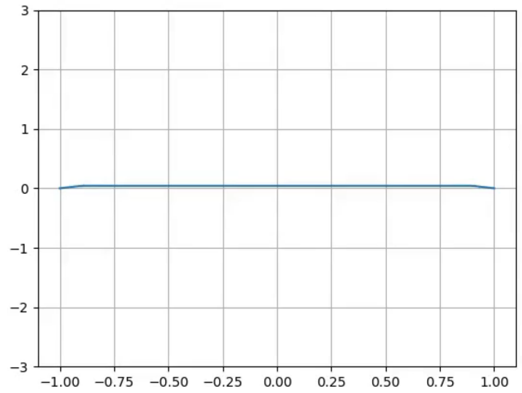
Initial Unstable Condition

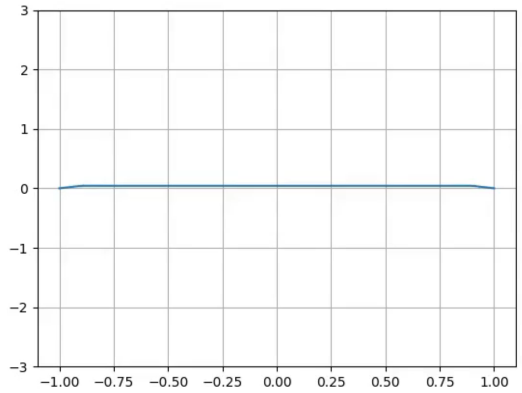


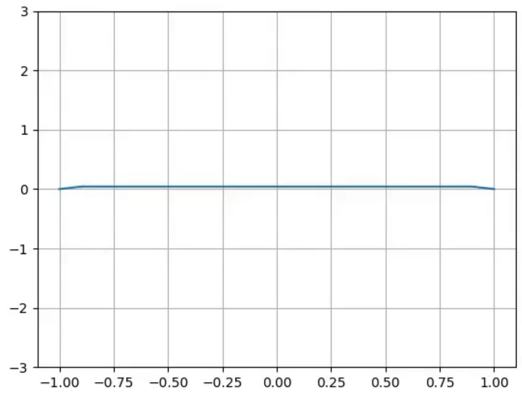
Initial Stable Condition



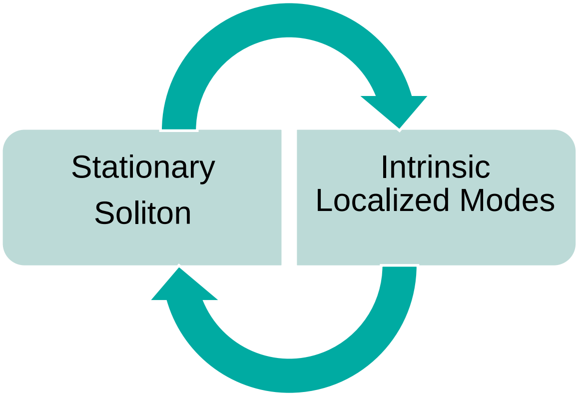
Temporal Stability

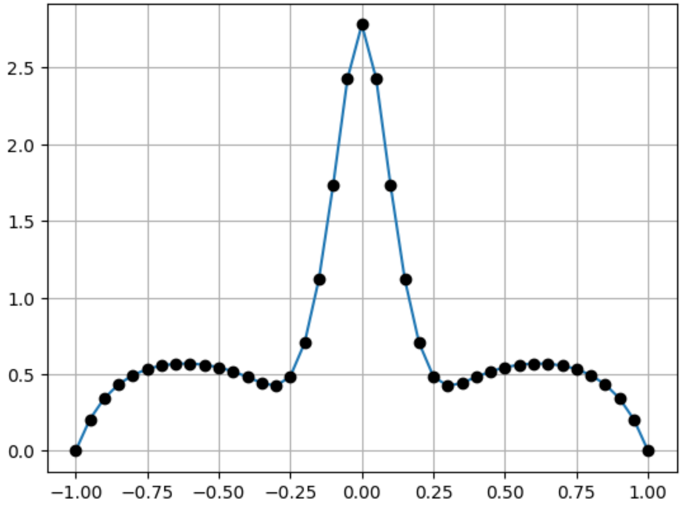






Discussion and Conclusion





Topics requiring additional investigation:

- Number of Oscillators
- Initial conditions of instability
- Numerical Integration Algorithms

Thank you very much for your attention

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