Atténuation passive de vibrations auto-entretenues au moyen d'un NES bistable

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Plan

Nonlinear Energy Sink (NES)

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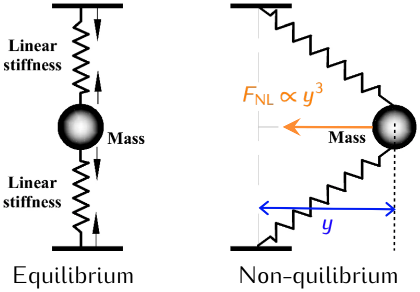

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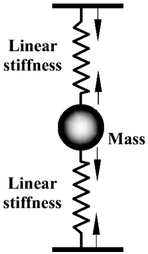
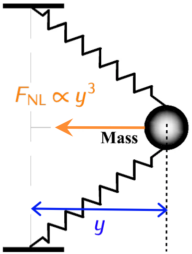
  - can **adjust its frequency** to that of the PS (relation amplitude/frequency)
  - **irreversibly absorbs** the energy of the SP (under certain conditions)



Equilibrium



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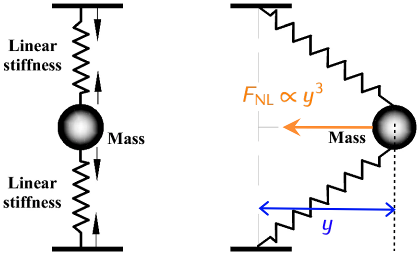

Equilibrium Non-quilibrium

Energy pumping

(*Targeted Energy Transfer* - TET)

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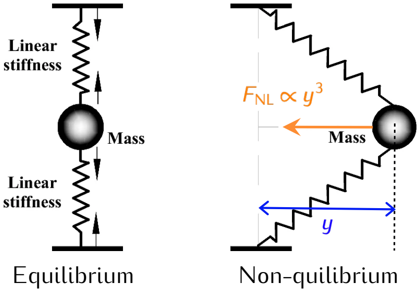

Energy pumping

(*Targeted Energy Transfer* - TET)

- Used for passive vibration control in mechanical and acoustic systems:
  - Free vibrations
  - Forced vibrations
  - **Self-sustained vibrations**

Plan

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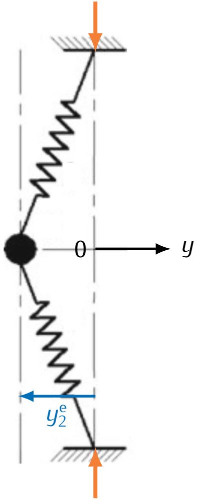
Bistable Nonlinear Energy Sink (BNES)

BNES = **cubic NES** with a **negative linear stiffness** component:

$$\ddot{y} + \mu\dot{y} - \beta y + \alpha y^3 = 0$$

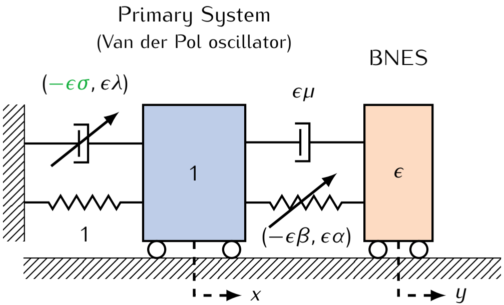
- Trivial equilibrium position $y_0^e = 0$ **unstable**
- **2 stable non trivial equilibrium positions** :
  - Right equilibrium position: $y_1^e = \sqrt{\frac{\beta}{\alpha}}$
  - Left equilibrium position: $y_2^e = -\sqrt{\frac{\beta}{\alpha}}$



Left equilibrium position

Van der Pol (VdP) oscillator coupled to a BNES

Diagram of the dimensionless system



σ : bifurcation parameter

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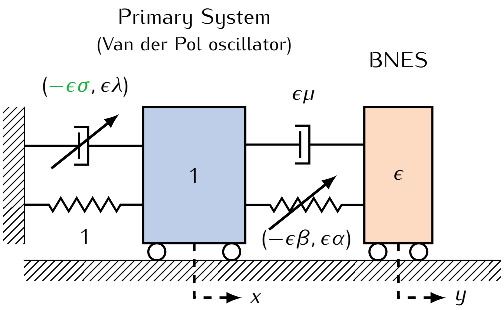

σ : bifurcation parameter

Stable equilibrium solutions of the coupled system

$$p_1^e = (x_1^e, y_1^e) = \left(0, \sqrt{\frac{\beta}{\alpha}}\right) \quad ; \quad p_2^e = (x_2^e, y_2^e) = \left(0, -\sqrt{\frac{\beta}{\alpha}}\right)$$

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Comparison: VdP alone vs VdP + NES vs VdP + BNES

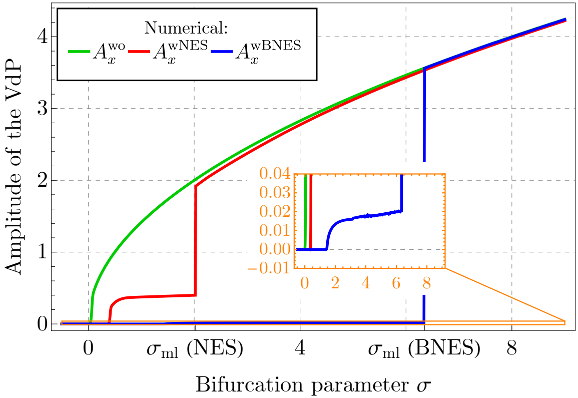


Figure. Bifurcation diagrams: A_x vs σ

A_x: amplitude of the VdP displacement x

σ : bifurcation parameter

Important values of σ

- $\sigma = 0$: Hopf bifurcation of the VdP alone
- σ_{ml} (NES): mitigation limit of the NES
- σ_{ml} (BNES): mitigation limit of the BNES

Identification of of response regimes

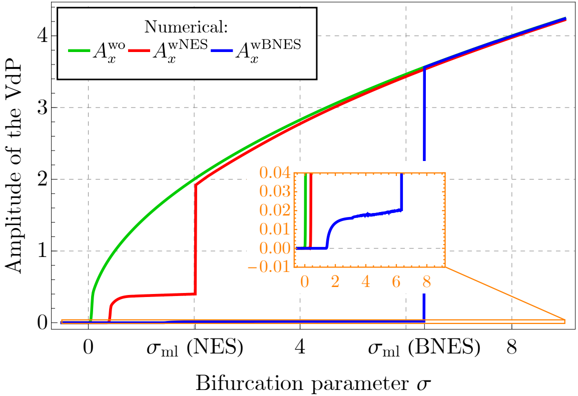
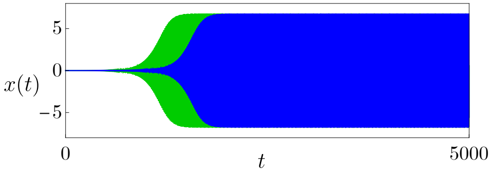


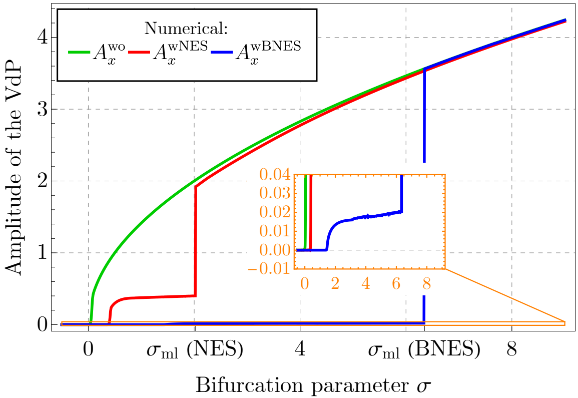
Figure. Bifurcation diagrams

$\sigma > \sigma_{\mathrm{ml}}$



without BNES — with BNES

Identification of of response regimes



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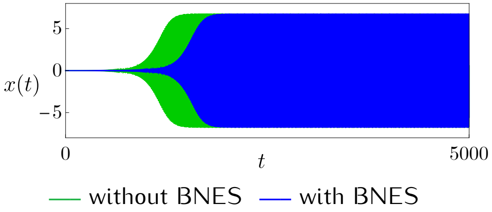

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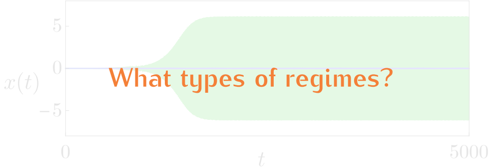
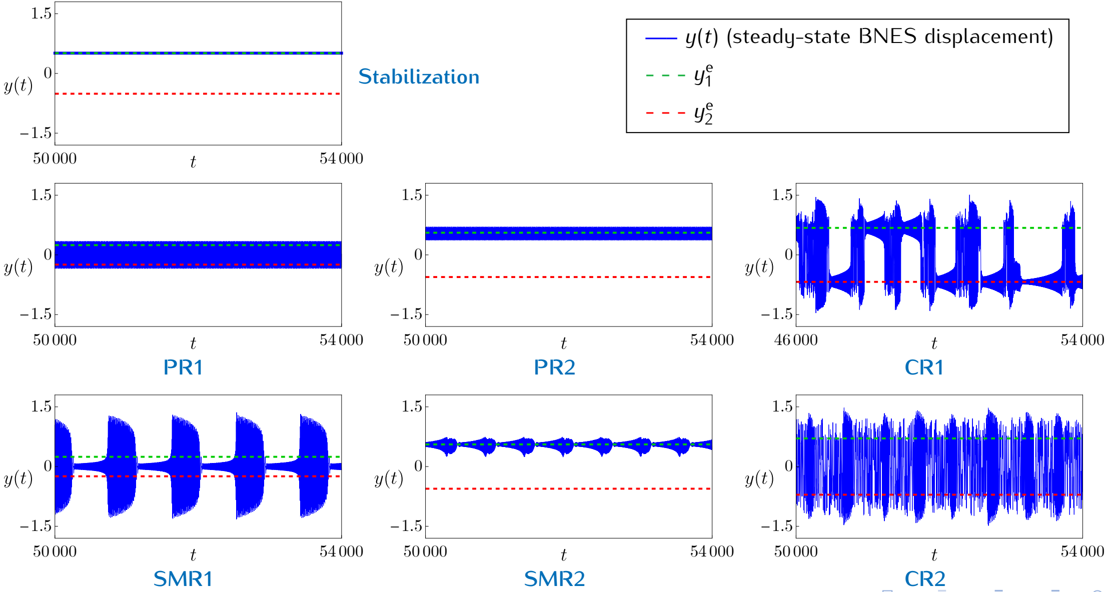


What types of regimes?

Identification of of response regimes

7 regimes identified



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- Change of variables : x (VDP) and y (BNES) $\Rightarrow$ $\boxed{u = x + \epsilon y}$ and $\boxed{v = x - y}$

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- **1 : 1 resonance capture**
  ↪ Use of the **Multiple Scale/Harmonic Balance Method (MSHBM)***, modified to consider that:
  - u (VdP motion): zero-mean periodic-like motion
  - v (BNES motion): nonzero-mean periodic-like motion

  $\Rightarrow$ Amplitude-Phase Modulation dynamics (APMD) (Flot de modulation) $\equiv$ *slow flow*

* [Lungo & Zulli (2012), Nonlinear Dynamics]

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a: amplitude of u
b: non oscillating part of v
c: amplitude of the oscillating part of v
δ: phase difference between u and v

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Original dynamics:		**APMD:**
Periodic solution	≡	Equilibrium
Quasiperiodic (SMR)		Periodic solutions

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Original dynamics:		APMD:
Periodic solution	$\equiv$	Equilibrium
Quasiperiodic (SMR)		Periodic solutions

$\Rightarrow$ APMD $\equiv$ (3,1)-fast-slow : **1 slow variable** a **and 3 fast variables** b, c **ans** δ

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APMD ≡ FAST-SLOW SYSTEM

- Time evolution characterized by possible succession **fast epochs** and **slow epochs**
- **Dynamic behavior of the system** investigated by means of **fast-slow analysis**

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when $\epsilon = 0$ one has

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$\hookrightarrow$ **fast subsystem**

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Critical Manifold

*Also called **Slow Invariant Manifold** (SIM)*

$$\mathcal{M}_0 = \left\{ (a,b,c,\delta) \in {\mathbb{R}^+}^3 \times [-\pi,\pi] \,\middle|\, g_1(b,c,0) = 0\ ,\ g_2(a,b,c,\delta) = 0\ ,\ g_3(a,b,c,\delta,0) = 0 \right\}$$

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- 1-dimensional manifold evolving in the 4-dimensional phase space of the APMD
- **In the vicinity of $\mathcal{M}_0$:** **slow evolution** of the APMD described by the **slow subsystem**
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From the fast subsystem: stability of $\mathcal{M}_0$: **does it attract or repel the fast dynamics?**

$\Rightarrow$ **Attracting** parts: $\mathcal{M}_0^{a}$ (they attract) and **Saddle-type** parts: $\mathcal{M}_0^{st}$ (they repel)

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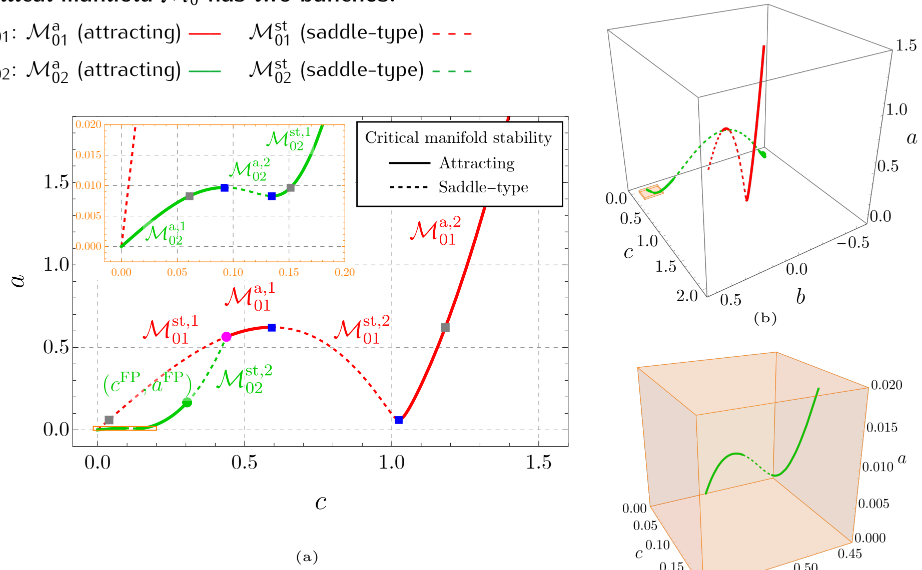
FROM THE SLOW SUBSYSTEM: fixed points (on $\mathcal{M}_0$)

⇒ Describes the slow dynamics in the vicinity of $\mathcal{M}_0$

The critical manifold $\mathcal{M}_0$ has two banches:

$\mathcal{M}_{01}$: $\mathcal{M}^{a}_{01}$ (attracting) —— $\mathcal{M}^{st}_{01}$ (saddle-type) - - -

$\mathcal{M}_{02}$: $\mathcal{M}^{a}_{02}$ (attracting) —— $\mathcal{M}^{st}_{02}$ (saddle-type) - - -



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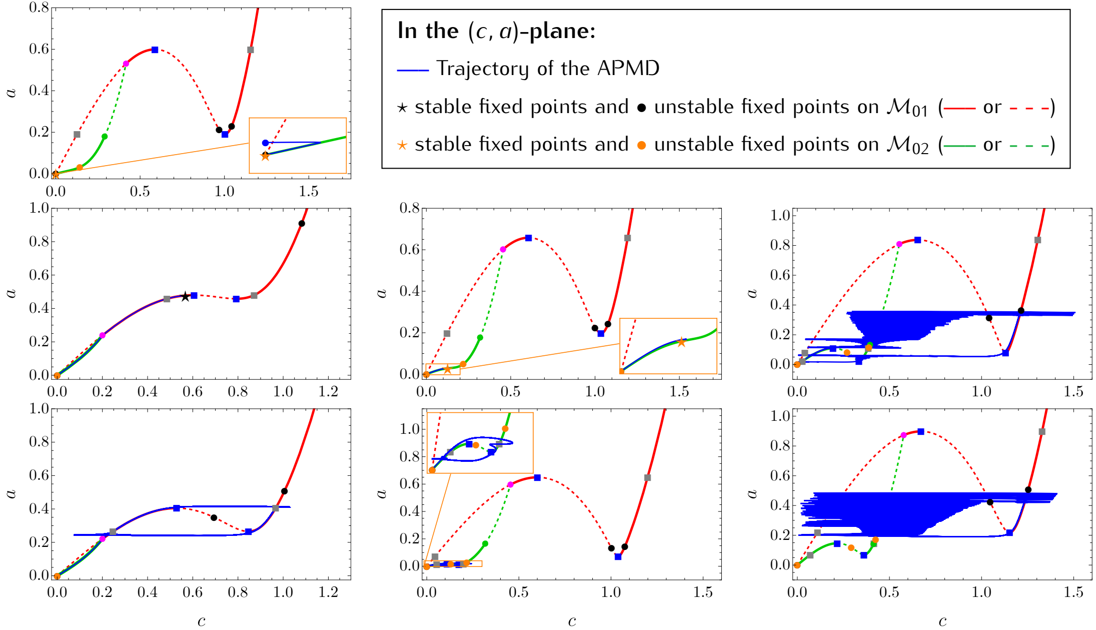
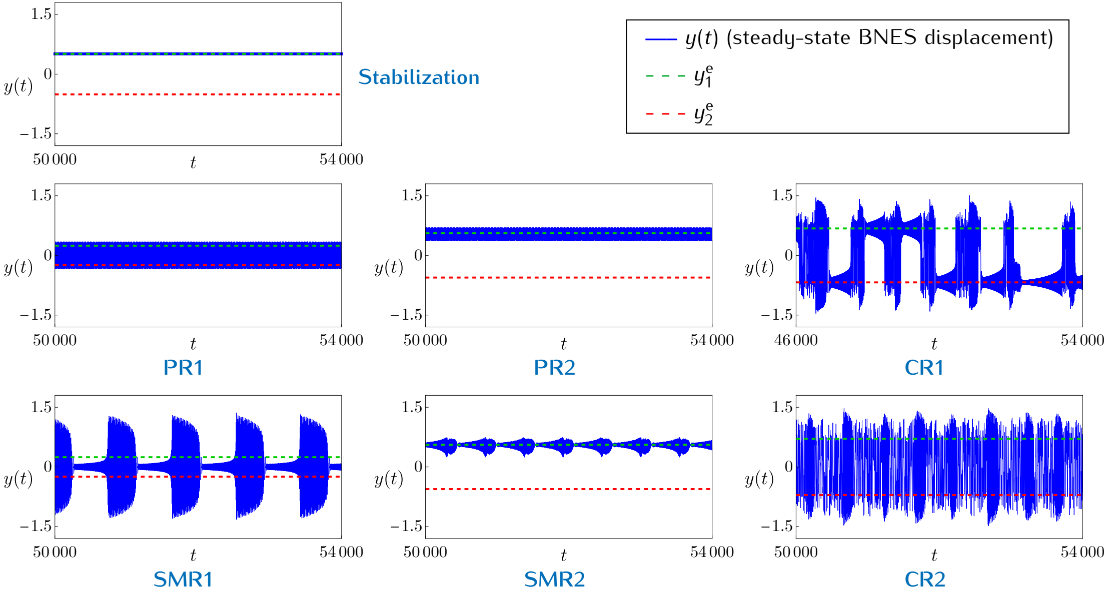

In the (c, a)-plane:
Trajectory of the APMD
$\star$ stable fixed points and $\bullet$ unstable fixed points on $\mathcal{M}_{01}$ (— or - - -)
$\star$ stable fixed points and $\bullet$ unstable fixed points on $\mathcal{M}_{02}$ (— or - - -)
a
c

Original dynamics



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Perspectives

- Finding and studying other solutions solutions of the fast subsystem (such as periodic, quasiperiodic or even chaotic motions)
- Computing the invariant manifolds tracking these solutions