Dynamique non linéaire de structures élancées hautement flexibles

Calcul des modes non linéaires avec une stratégie éléments finis dans le domaine fréquentiel

09 novembre 2023
Journées annuelles du GdR EX-MODELI | Besançon

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2: THREAD European Training Network, Marie Skłodowska-Curie Grant Agreement No. 860124, Horizon 2020

Joint Training on Numerical Modeling of Highly Flexible Structures for Industrial Applications

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 860124.

Introduction

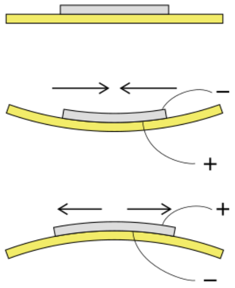
The dynamical study of **highly flexible beam structures** represents a **current and important subject of research**.

- Highly flexible beam structures are found in many industrial applications:

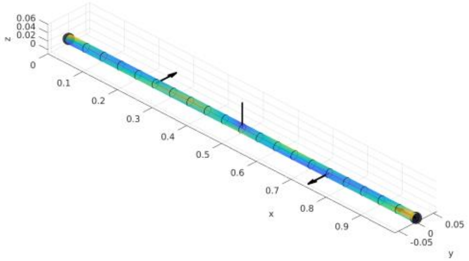





- Robotics and soft robotics
- Cable car and elevator systems
- Cabling and cable harnesses
- Flexible structures in aerospace
- Automotive industry
- Biomedical applications
- Micro/Nano-electromechanical systems (MEMS/NEMS)



“Highly flexible” indicates → an extreme capacity for bending
implying → **geometrical nonlinearities**
meaning that → no analytical solutions are available at very high bending amplitudes



Conclusion: **efficient numerical solutions** are needed to simulate the dynamics of these structures at extreme amplitudes.

Motivation

- General study of geometrically nonlinear systems (esp. nonlinear dynamics)
  (M.Debeurre – Thursday 09.11 – 15:30)
- Efficient numerical simulations and nonlinear reduced order models
  (A.Grolet – Thursday 09.11 – 14:45)
- Experimental validation

Debeurre, M., Grolet, A., Cochelin, B., Thomas, O.: Finite element computation of nonlinear modes and frequency response of geometrically exact beam structures. *Journal of Sound and Vibration* **548** (117534) (2023).

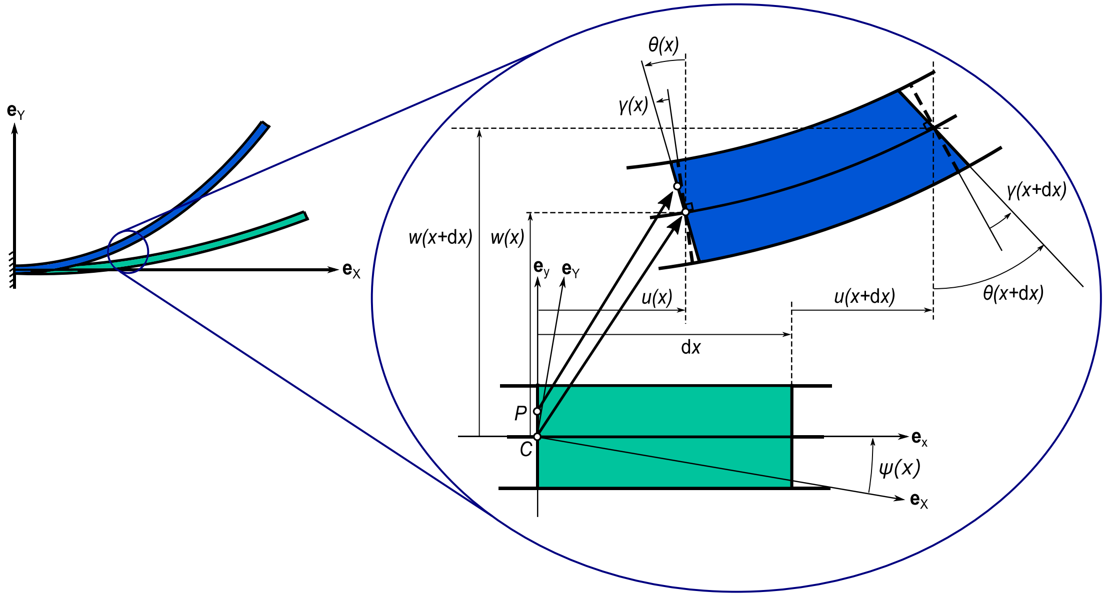
2D Geometrically exact beam model and resolution in the frequency domain

1. Geometrically exact beam model

reduce to 1D kinematics – displacement of the centerline

no truncation of the rotation terms – "exact" at any amplitude

- **2D *in-plane* motion:** 3 degrees of freedom: $u(x)$ (axial), $w(x)$ (transverse), $\theta(x)$ (rotation of x-section)

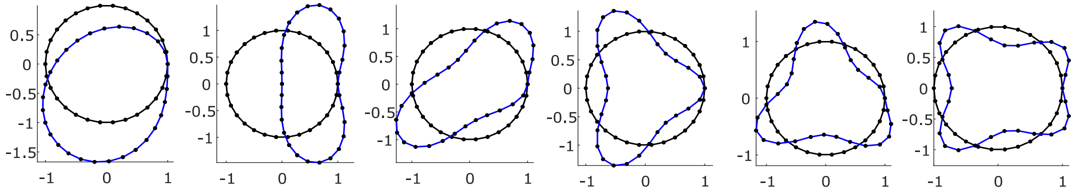


2. Geometrical nonlinearities: how to parametrize rotations?

- **2D:** rotation matrices ($\sin(\theta)$ and $\cos(\theta)$)

3. Finite element (FE) discretization

- Structure discretized into (linear†) shear-deformable beam elements
- **e.g. Flexible ring** (first six mode shapes)



4. Solved in the frequency domain

- **FE model** + harmonic balance + continuation (MANLAB) [1]:

Frequency responses → $\mathbf{M}\ddot{\mathbf{u}} + \mathbf{D}\dot{\mathbf{u}} + \mathbf{f}_{\text{int}}(\mathbf{u}) = \mathbf{f}_{\text{ext}}$

Nonlinear modes / backbone curves → $\mathbf{M}\ddot{\mathbf{u}} + \mathbf{f}_{\text{int}}(\mathbf{u}) = \mathbf{0}$

$\mathbf{M}$: mass, $\mathbf{D}$: damping, $\mathbf{f}_{\text{int}}(\mathbf{q})$: nonlinear internal force, $\mathbf{f}_{\text{ext}}$: external force, $\mathbf{u}$: degrees of freedom

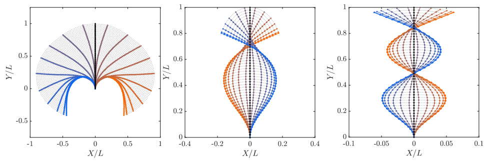
[1] MANLAB: an interactive path-following and bifurcation analysis software, available at http://manlab.lma.cnrs-mrs.fr/.

Debeurre, M., Grolet, A., Cochelin, B., Thomas, O.: Finite element computation of nonlinear modes and frequency response of geometrically exact beam structures. *Journal of Sound and Vibration* **548** (117534) (2023).

2D Geometrically exact beam model and resolution in the frequency domain

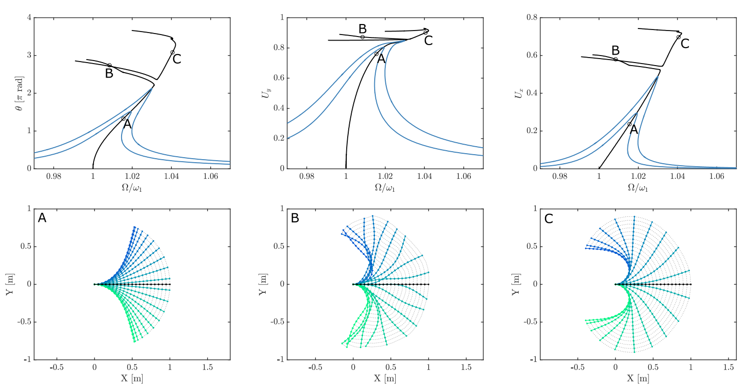
Nonlinear phenomena:

- frequency Ω dependence on amplitude
- internal resonance: transfers of energy between nonlinear modes
- Bifurcations, stability analysis, etc…

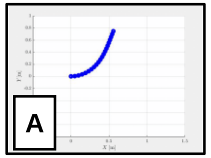


Cantilever

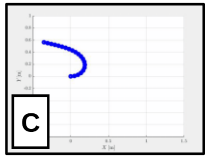
Mode 1



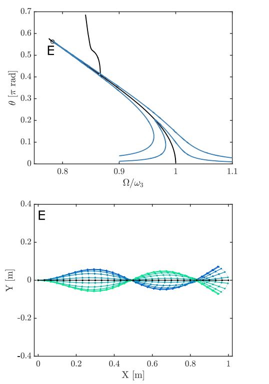
Low amplitude

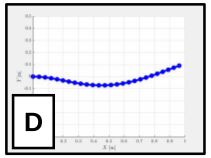


High amplitude

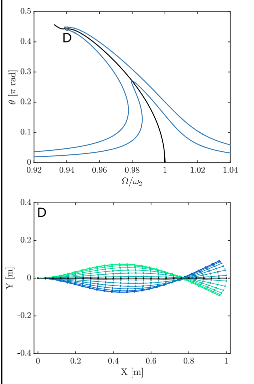


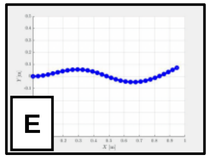
Mode 2





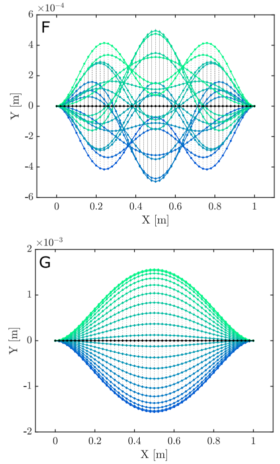
Mode 3

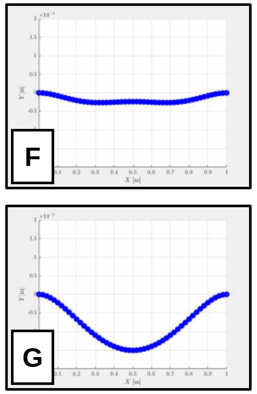




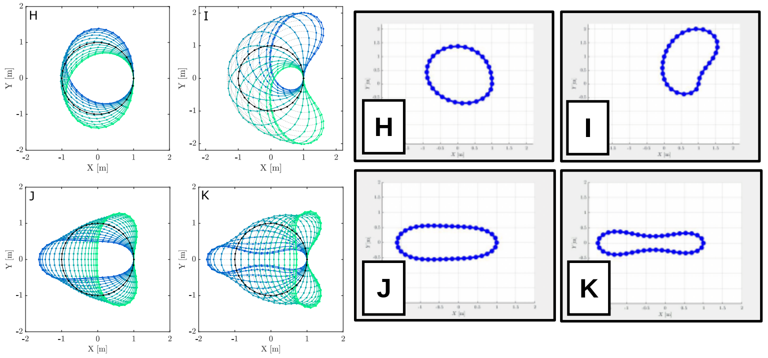
Other structures

Clamped-clamped beam (mode 1)

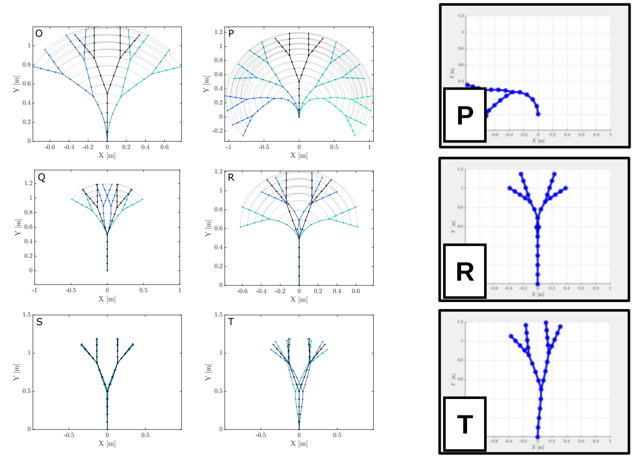




Flexible ring (modes 1 & 2)



Tree model (modes 1 – 3)

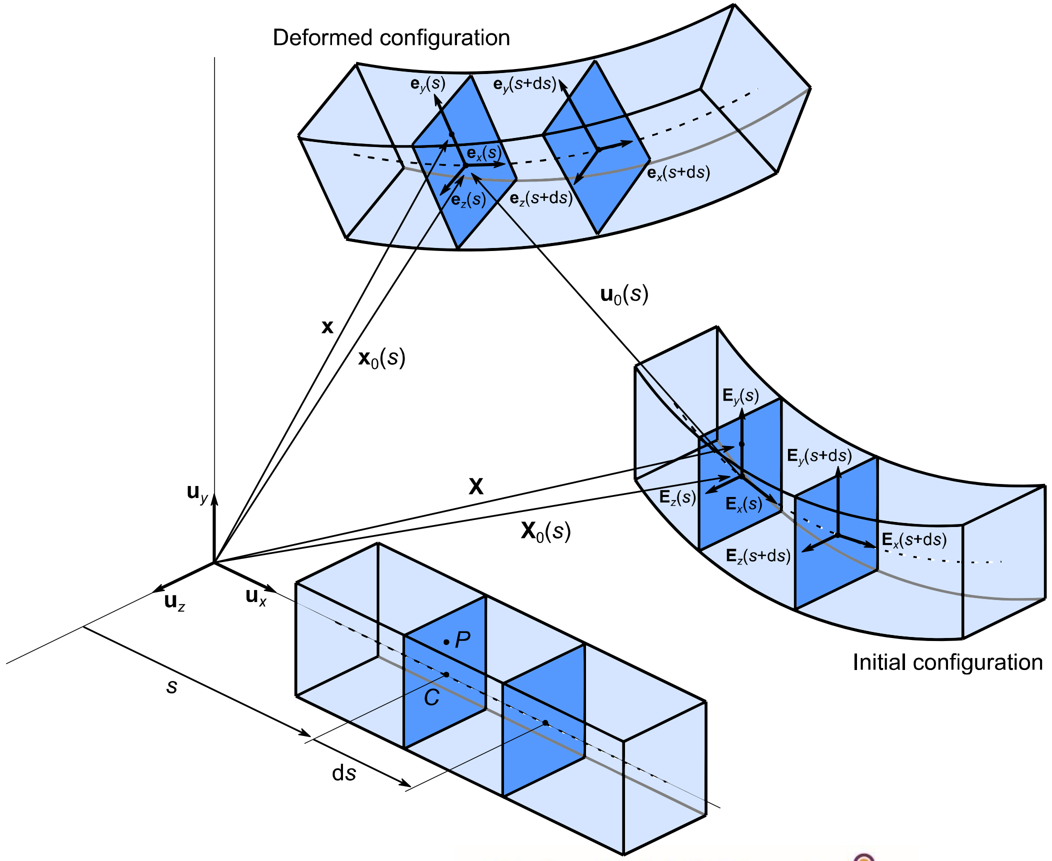


3D Geometrically exact beam model and resolution in the frequency domain

1. Geometrically exact beam model

- **3D motion:** 6 degrees of freedom:

  $\mathrm{u(x)}$ (axial), $\mathrm{w(x)}$ and $\mathrm{v(x)}$ (2x transverse),
  $\theta_x(\mathrm{x})$ (twist), $\theta_y(\mathrm{x})$ and $\theta_z(\mathrm{x})$ (2x rotation of x-section)



2. Geometrical nonlinearities: how to parameterize rotations?

- **3D**: quaternions – 4 dimensional complex numbers

$$\hat{a} = \underbrace{a_0}_{\text{scalar}} + \underbrace{ia_1 + ja_2 + ka_3}_{\text{vector}}$$

- To define rotations in 3D: *unit* quaternions

$$\hat{q} = \left[\cos\left(\frac{\theta}{2}\right) + \mathbf{n}\sin\left(\frac{\theta}{2}\right)\right]^{\mathrm{T}}, \ \mathbf{n}\text{: unit vector}$$

- Rewrite equations of motion with quaternion algebra

3. Finite element (FE) discretization

- Structure discretized into (quadratic†) shear-deformable beam elements

4. Solved in the frequency domain

- **FE model** + harmonic balance + continuation (MANLAB) [1]:

Frequency responses	Nonlinear modes / backbone curves
$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{D}\dot{\mathbf{u}} + \mathbf{f}_{\mathrm{int}}(\mathbf{u}) = \mathbf{f}_{\mathrm{ext}}$	$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{f}_{\mathrm{int}}(\mathbf{u}) = \mathbf{0}$

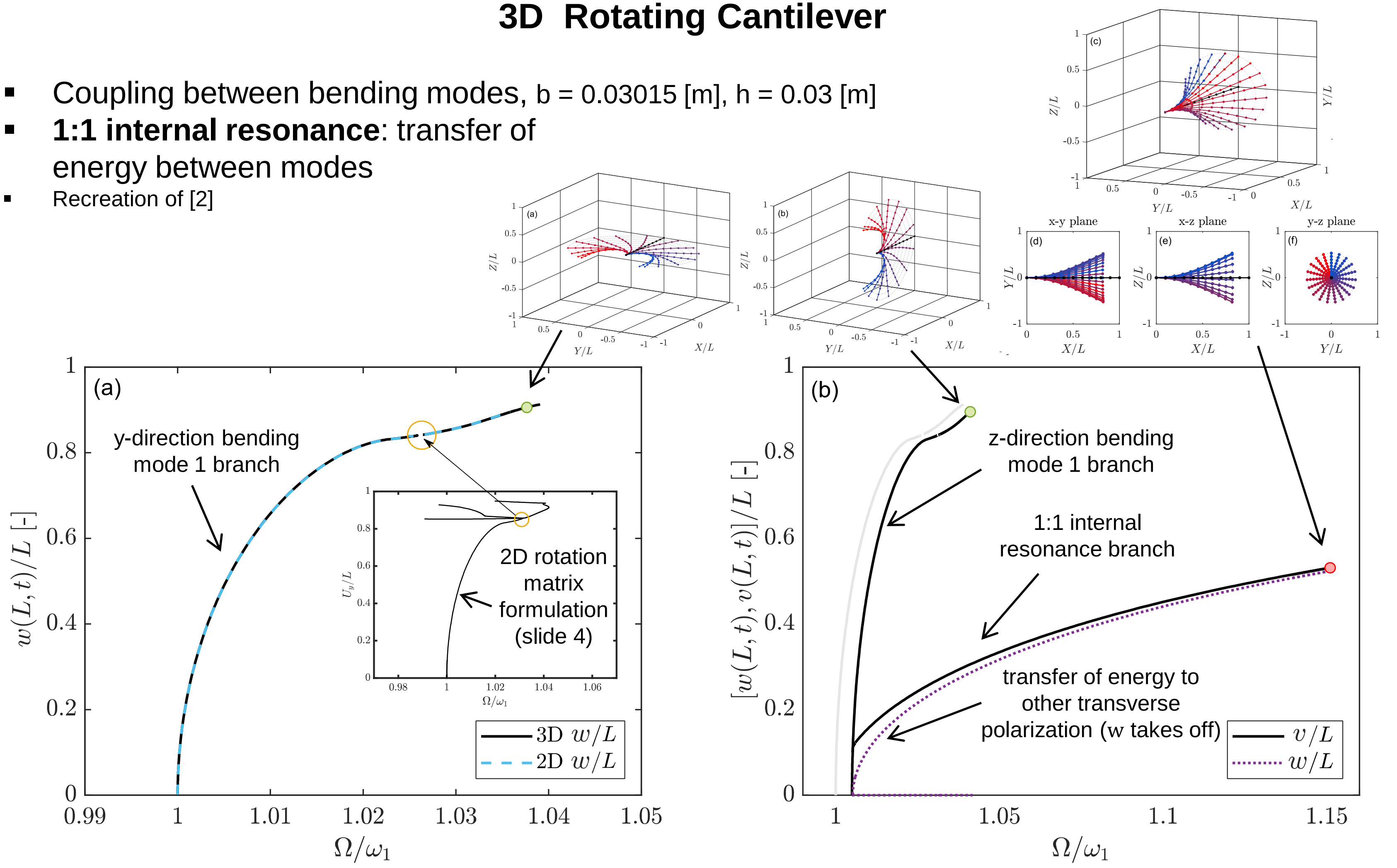
$\mathbf{M}$: mass, $\mathbf{D}$: damping, $\mathbf{f}_{\mathrm{int}}(\mathbf{q})$: nonlinear internal force, $\mathbf{f}_{\mathrm{ext}}$: external force, $\mathbf{u}$: degrees of freedom

† Polynomial shape functions …

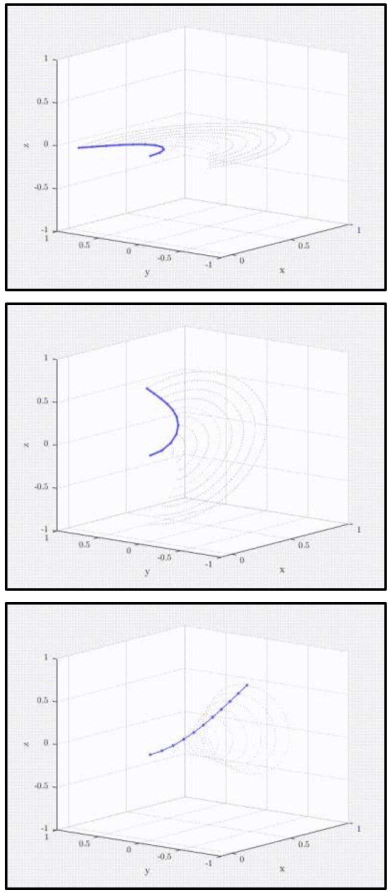
3D Geometrically exact beam model and resolution in the frequency domain

3D Rotating Cantilever

- Coupling between bending modes, b = 0.03015 [m], h = 0.03 [m]
- **1:1 internal resonance**: transfer of energy between modes
- Recreation of [2]



3D Rotating Cantilever

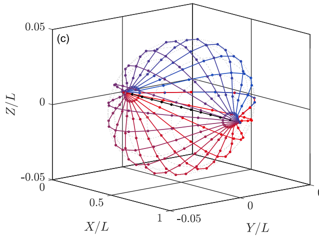


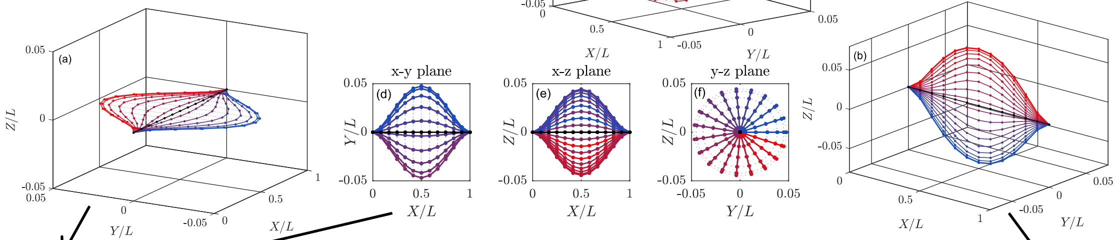
[2] Vincent *et al.*: Nonlinear polarization coupling in freestanding nanowire/nanotube resonators. *Journal of Applied Physics* **125** (044302) (2019).

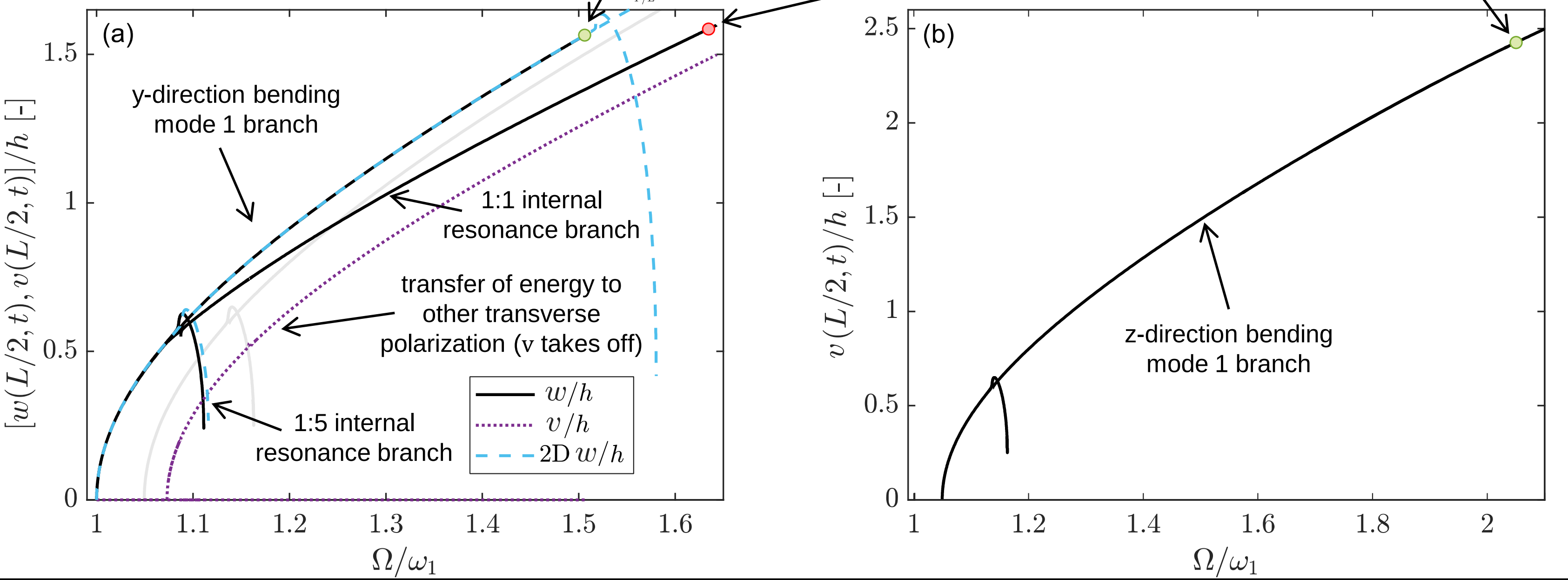
3D Geometrically exact beam model and resolution in the frequency domain

3D Rotating Clamped-Clamped

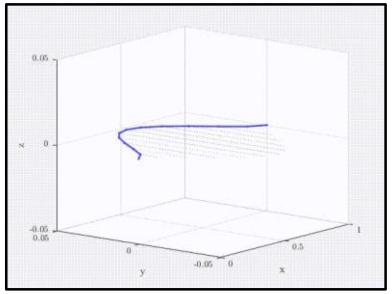
- Coupling between bending modes, b = 0.0315 [m], h = 0.03 [m]
- 1:5 internal resonance
- **1:1 internal resonance**: transfer of energy between modes
- Recreation of [3]



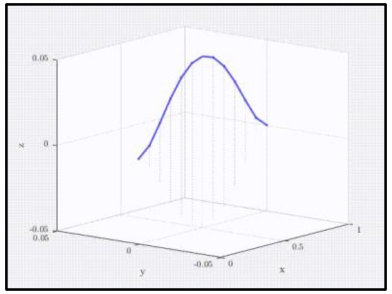




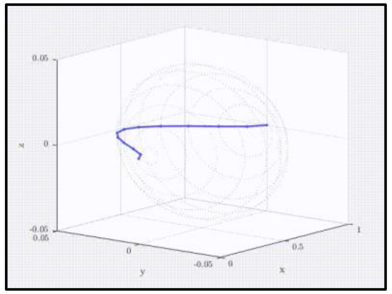
3D Rotating clamped-clamped



y-direction bending mode 1



z-direction bending mode 1



1:1 internal resonance

[3] Shen *et al.*: Comparison of Reduction Methods for Finite Element Geometrically Nonlinear Beam Structures. *Vibration* **4**, p. 175-204 (2021).

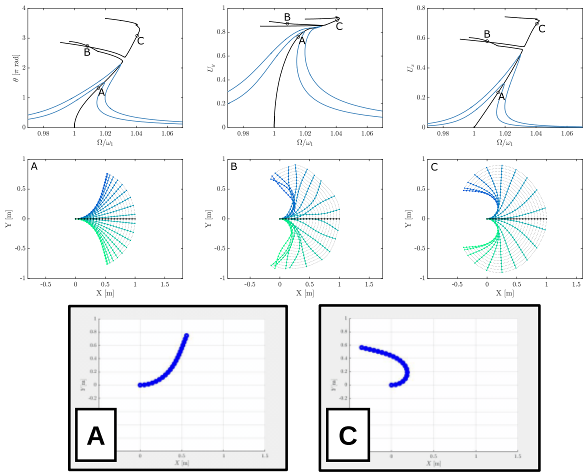
Validation of the quaternion-based formulation

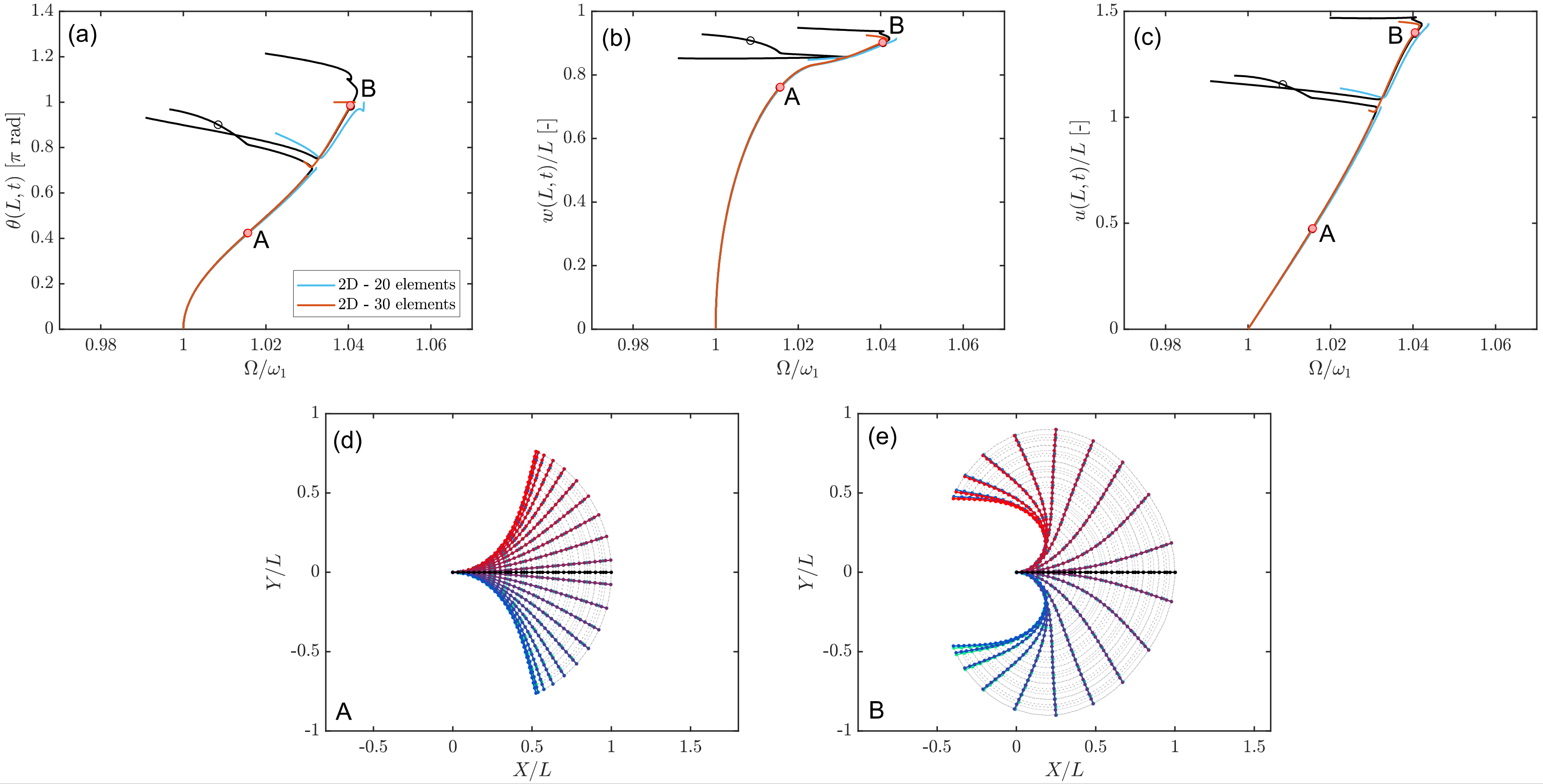
Cantilever – large rotations

- Comparison between formulations: 2D rotation matrix-based (20 elements) vs. 2D quaternion-based (20 or 30 elements)
- More computationally efficient formulation?

Observations:

- Quaternion formulation: more (linear) elements for convergence
- Main backbone curve
- Deformed shapes
- Internal resonance branches





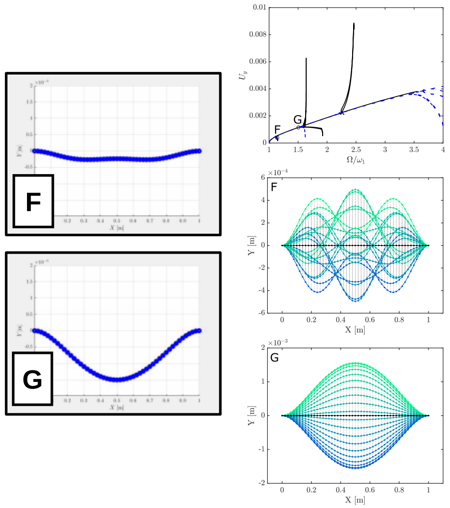
Validation of the quaternion-based formulation

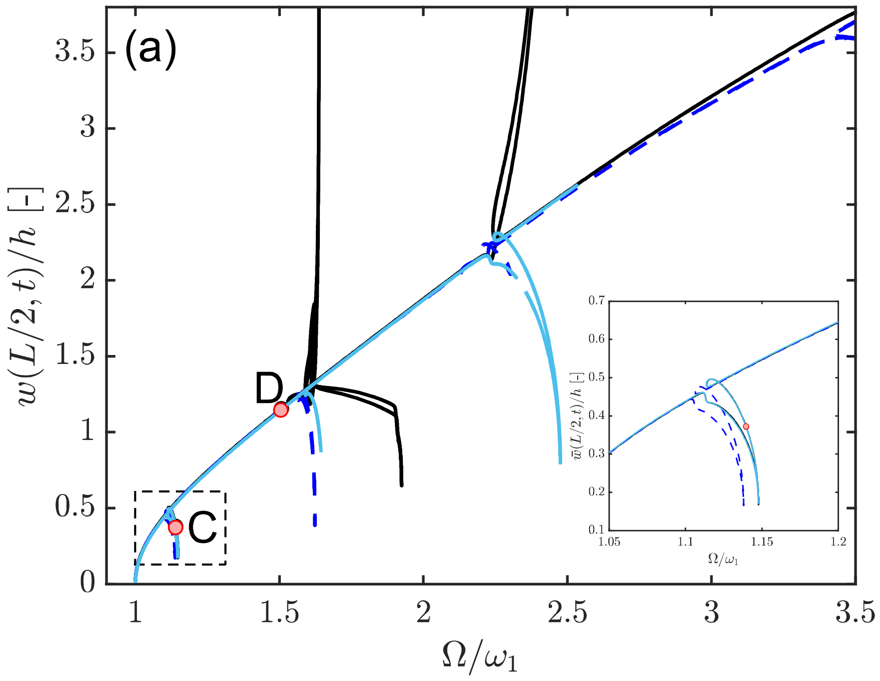
Clamped-clamped beam – axial-bending coupling

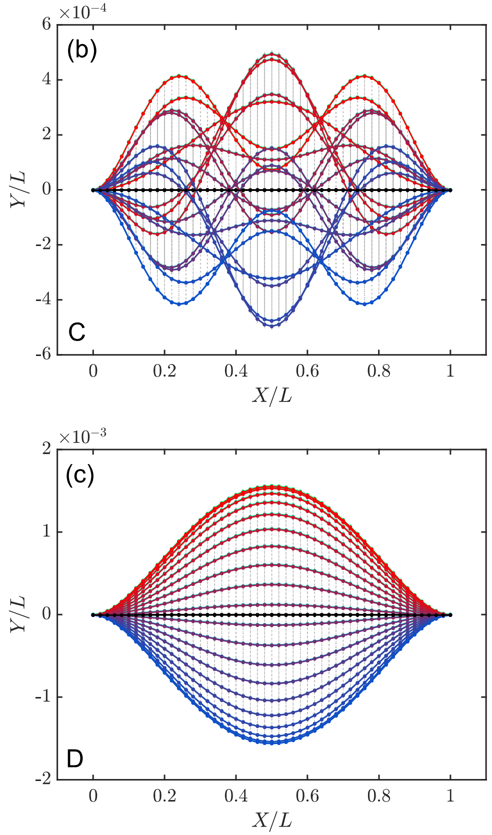
- Comparison between formulations: 2D rotation matrix-based (50 elements) vs. 2D quaternion-based (50 elements) vs nonlinear Von Kármán [4]
- More computationally efficient formulation?

Observations:

- Converged with 50 (linear) elements
  → amplitude
- Main backbone curve
- Deformed shapes
- Internal resonance branches







[4] Givois, A., Grolet, A., Thomas, O., Deü, J.-F.: On the frequency response computation of geometrically nonlinear flat structures using reduced-order finite element models. *Nonlinear Dynamics* **97**(2), p. 1747-1781 (2019).

Conclusion

Remarks and future work

In addition to our presentations…

- Stability adapted to large FE systems with many degrees of freedom [5]

Future work

- Extension of 3D model to initially curved structures
- Nonlinear damping considerations
- Full analysis of computational efficiency

[5] Bayer, F., Leine, R.: Sorting-free Hill-based stability analysis of periodic solutions through Koopman analysis. *Nonlinear Dynamics* **111**, p. 8439-8466 (2023).

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In memory of
Simon Benacchio (1988-2023)

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Thank you for your attention!

Questions?

In memory of
Simon Benacchio (1988-2023)

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Nonlinear dynamics of highly flexible beam structures: frequency domain-based finite element computation of the nonlinear modes

Marielle Debeurre
Supervisor: Olivier Thomas | Co-supervisor: Aurélien Grolet

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