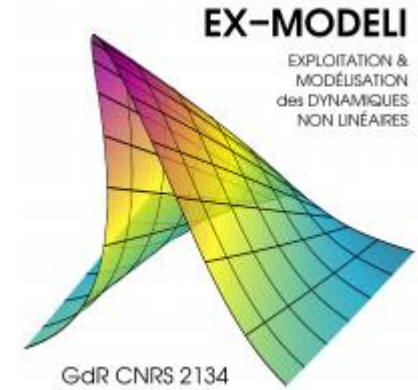


# Prise en compte de l'amortissement non linéaire dans le dimensionnement d'un NES

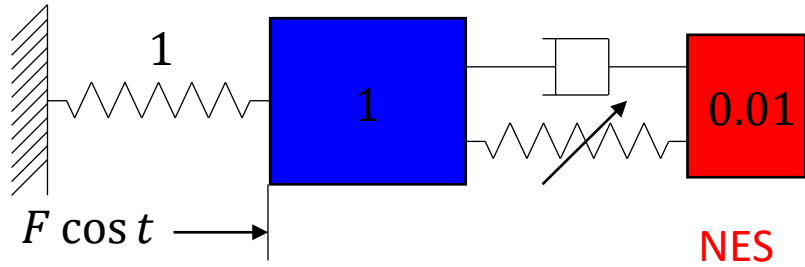


Etienne Gourc<sup>1</sup>, Pierre-Olivier Mattei<sup>1</sup>, Renaud Côte<sup>1</sup>, Mattéo Capaldo<sup>2</sup>

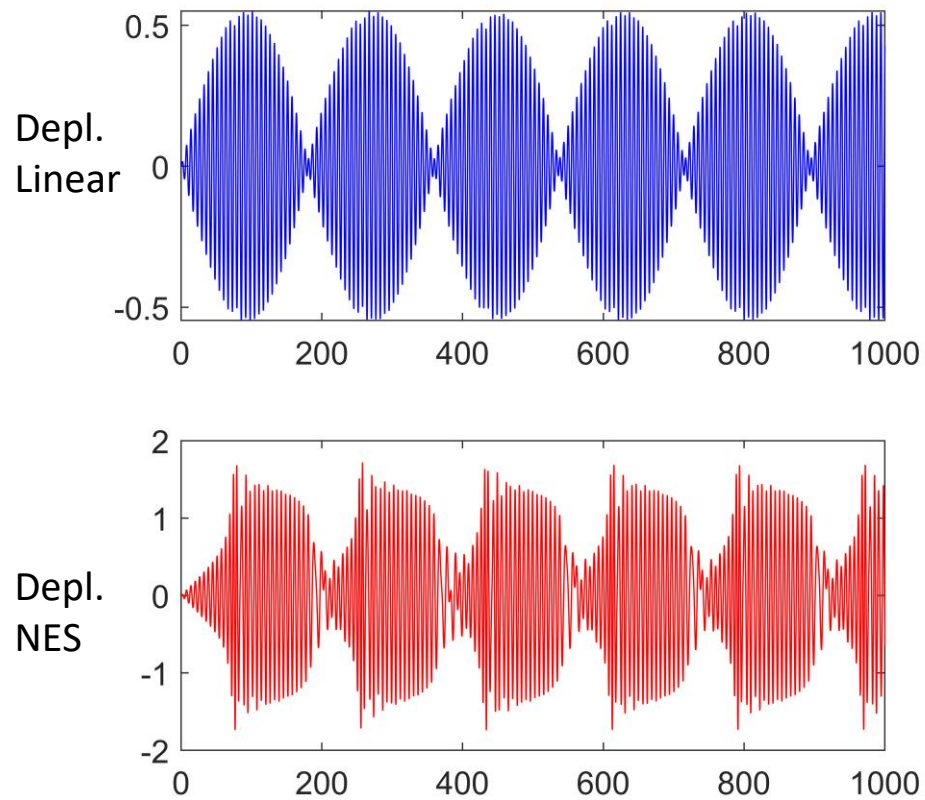
[1] Laboratoire de Mécanique et d'Acoustique, CNRS, Marseille

[2] Total Energies, Paris

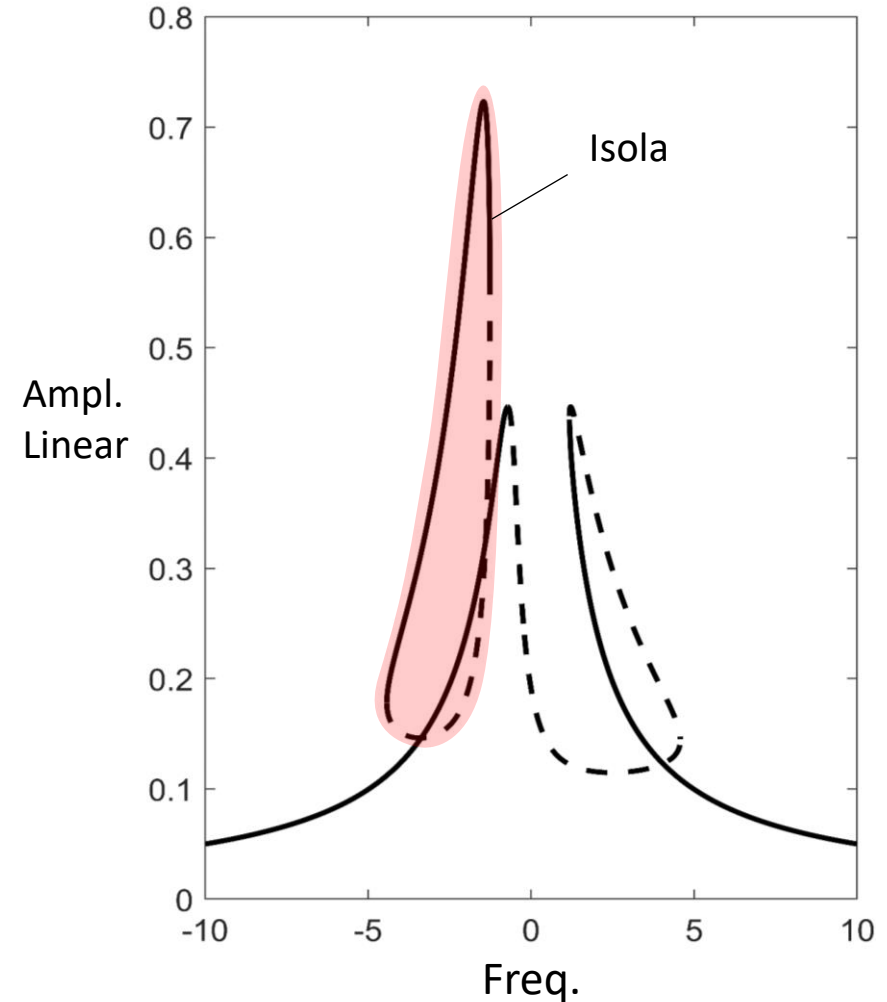
# Working principle under harmonic excitation



## Passive control through relaxation oscillations



## High amplitude Isola limits the performance of NES



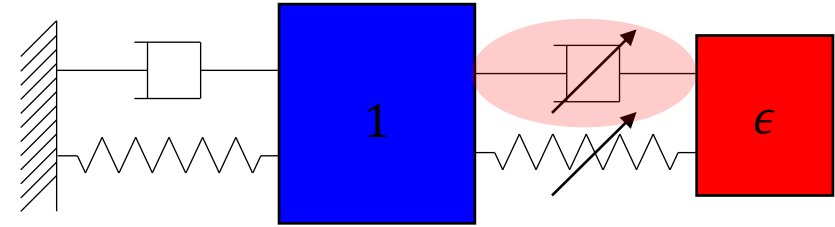
# A nonlinear energy sink with nonlinear damping

Adimensional equation of motion

$$\ddot{x} + x + 2\zeta\dot{x} + \epsilon(\mu\dot{w} + \kappa w^3 + \lambda w^2\dot{w}) = G \cos(\omega t)$$

$$\epsilon(\ddot{x} - \ddot{w} - \mu\dot{w} - \kappa w^3 - \lambda w^2\dot{w})$$

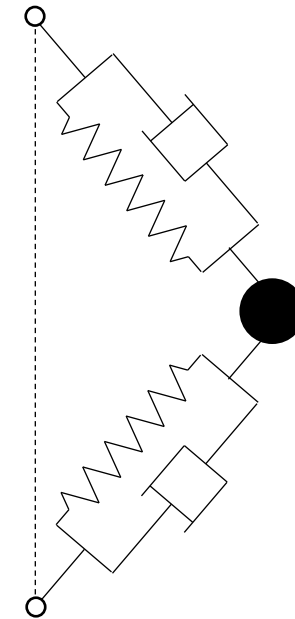
Nonlinear damping



$\epsilon$  : mass ratio  $\ll 1$

Objectives :

- Describe the dynamical behavior
- Propose a tuning methodology
- Quantify the effect of nonlinear damping

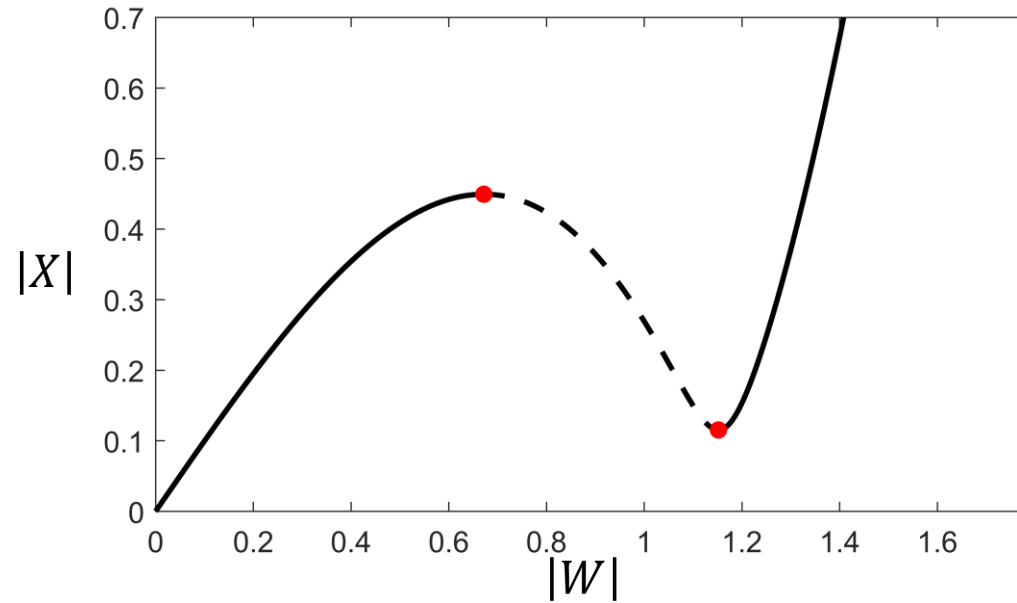


# Theoretical analysis using MS/HBM<sup>[1]</sup>

order  $\mathcal{O}(\epsilon^0)$

The Slow Invariant Manifold

$$X = (1 - i\mu)W - \frac{1}{4}(3\kappa + i\lambda)W^2\bar{W}$$



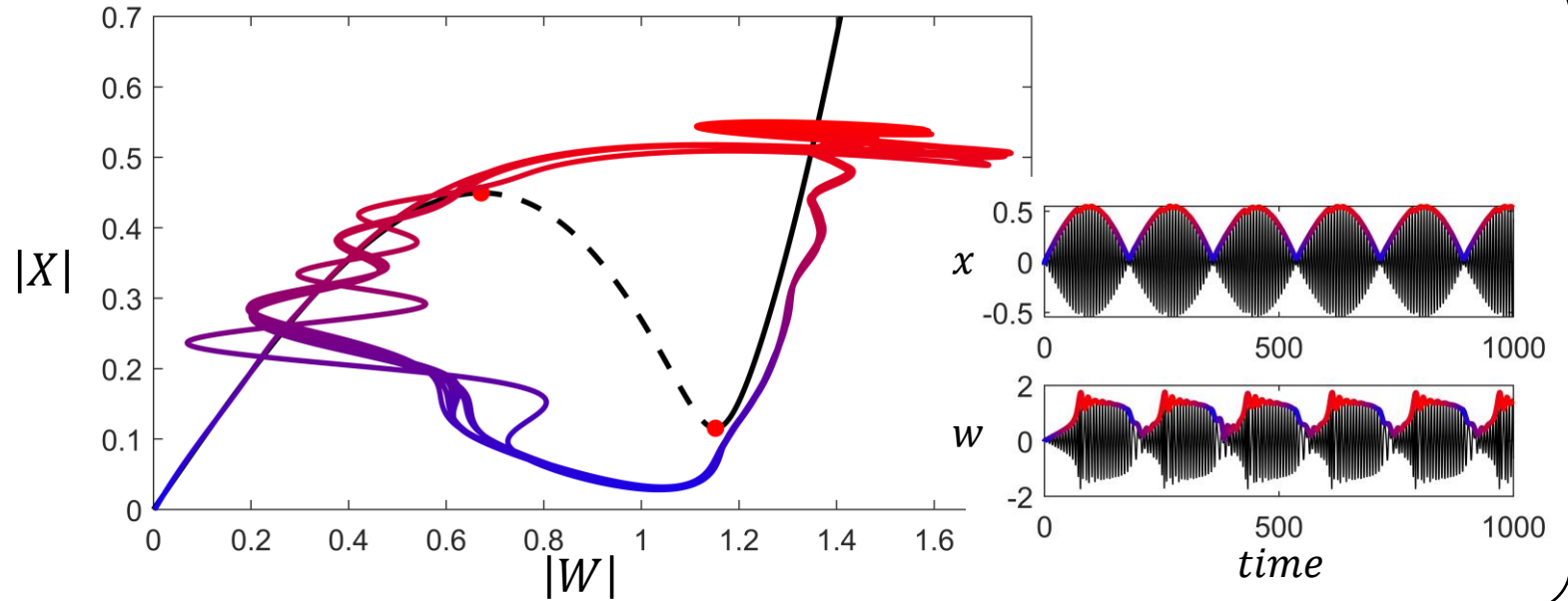
[1] Luongo, A., Zulli, D. (2012). Dynamic analysis of externally excited NES-controlled systems via a mixed Multiple Scale/Harmonic Balance algorithm. *Nonlinear Dynamics*

# Theoretical analysis using MS/HBM<sup>[1]</sup>

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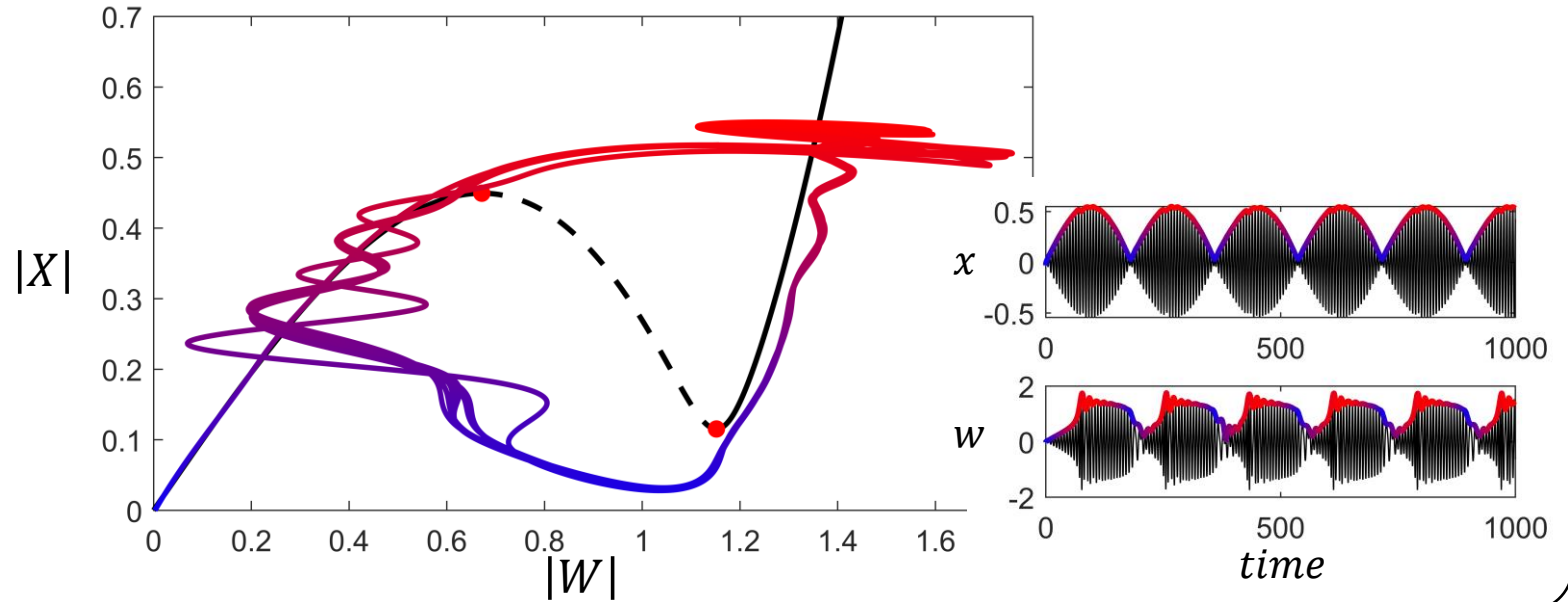


# Theoretical analysis using MS/HBM<sup>[1]</sup>

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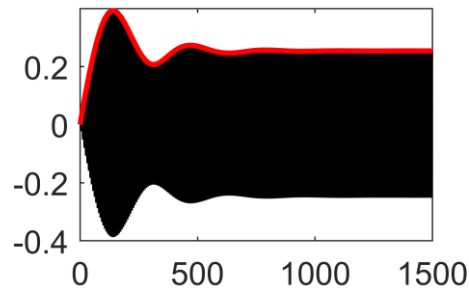


order  $\mathcal{O}(\epsilon^1)$

Modulation equation

$$|\dot{W}| = \frac{f_1(W)}{g(W)}$$

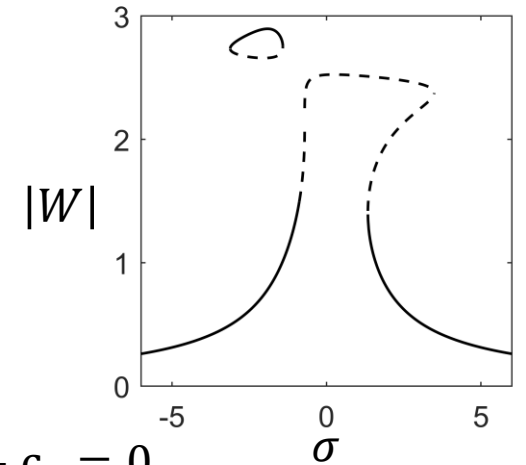
$$\arg \dot{W} = \frac{f_2(W, \sigma)}{g(W)}$$



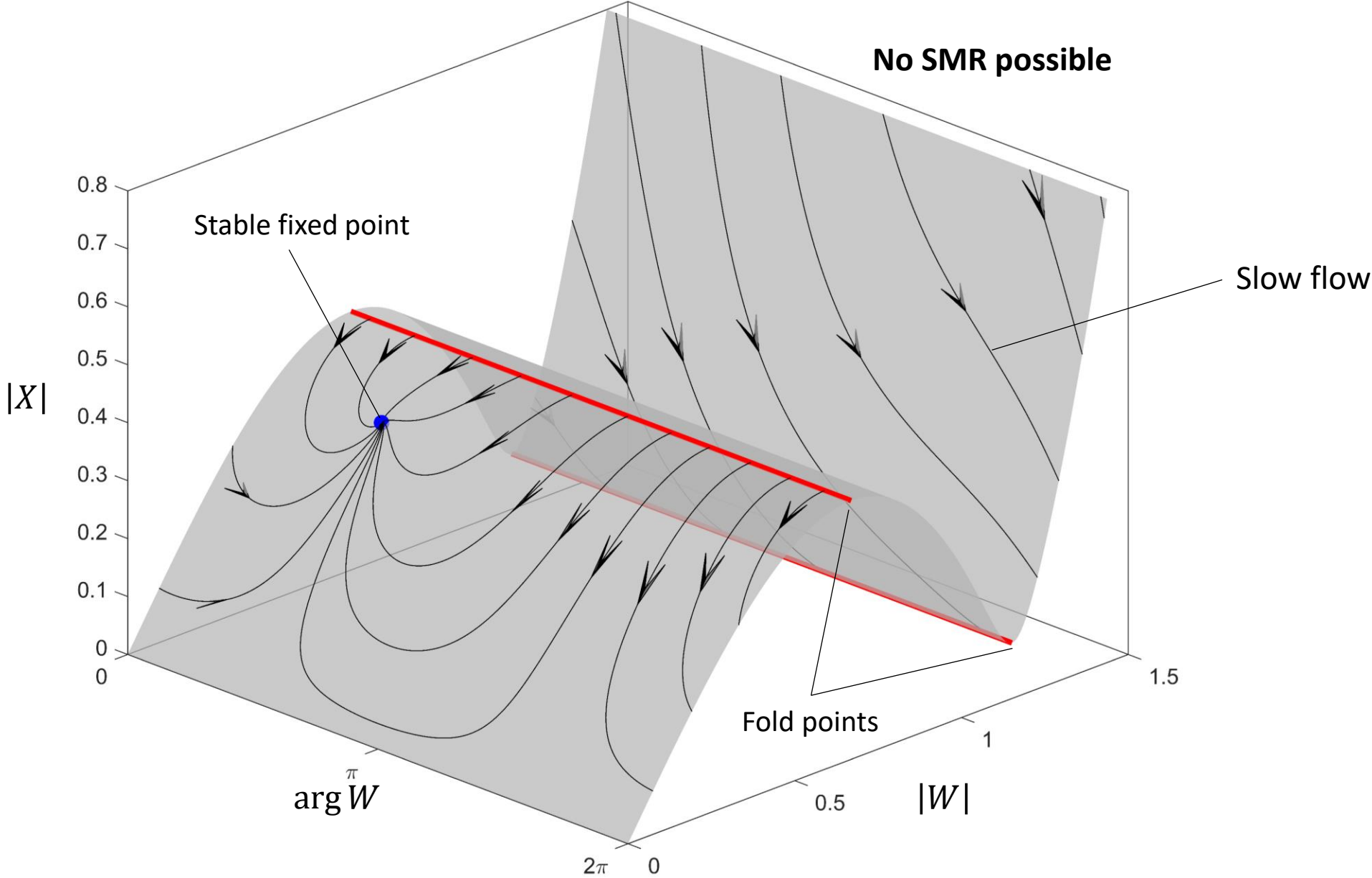
Fixed points

$$\begin{cases} f_1(W) = 0 \\ f_2(W, \sigma) = 0 \end{cases}$$

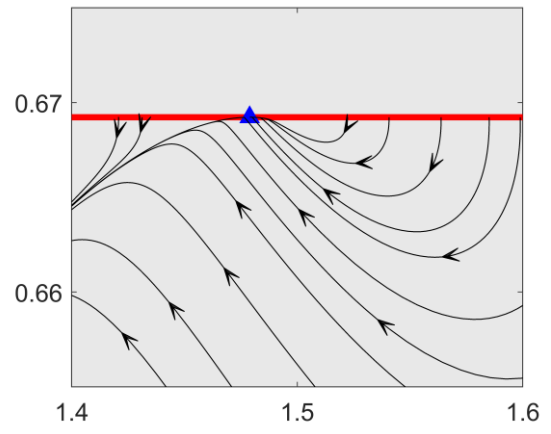
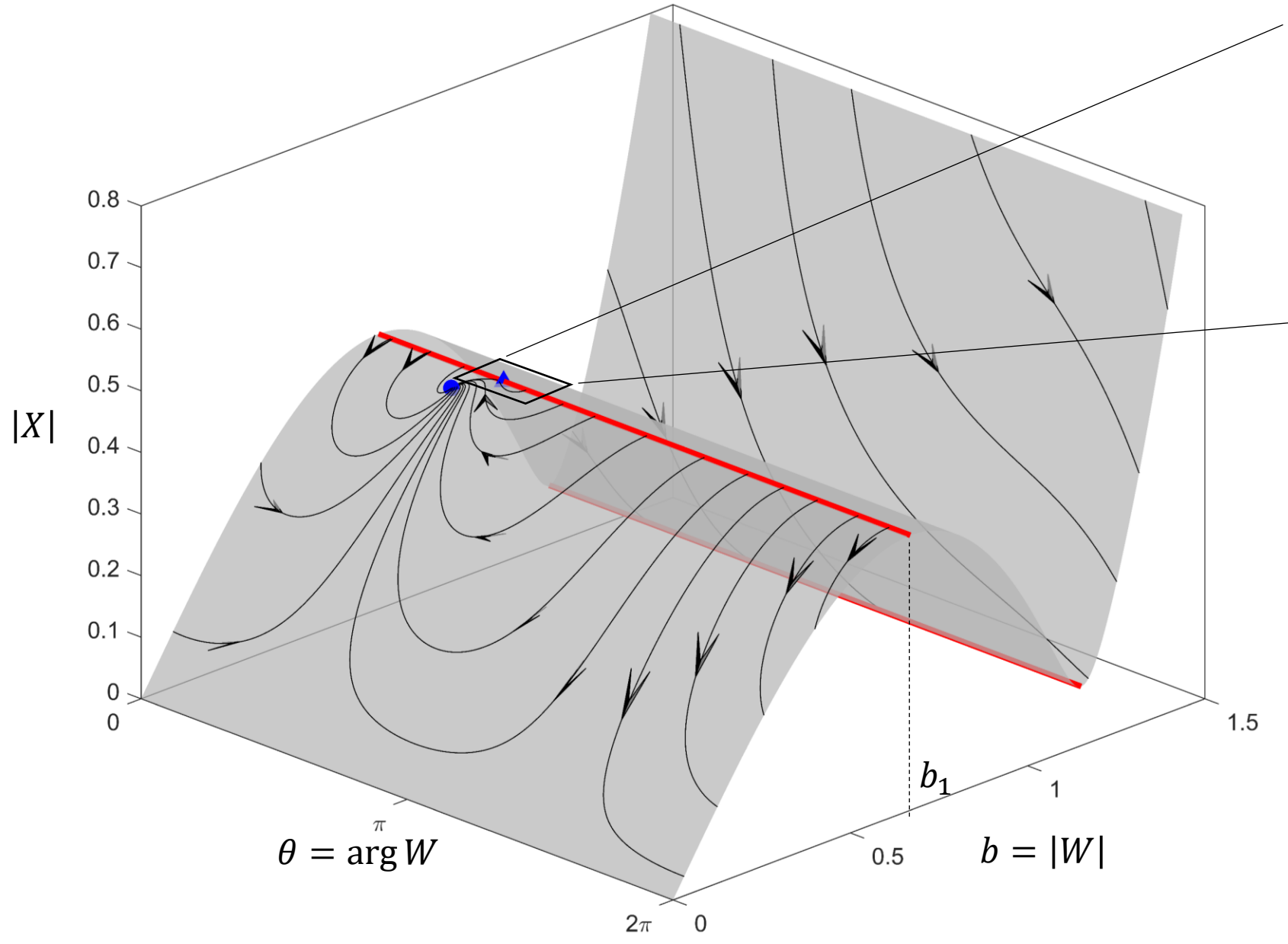
$$f_3(W, \sigma) \equiv c_3|W|^6 + c_3|W|^4 + c_3|W|^2 + c_0 = 0$$



# Activation threshold of SMR



# SMR triggered by grazing flow



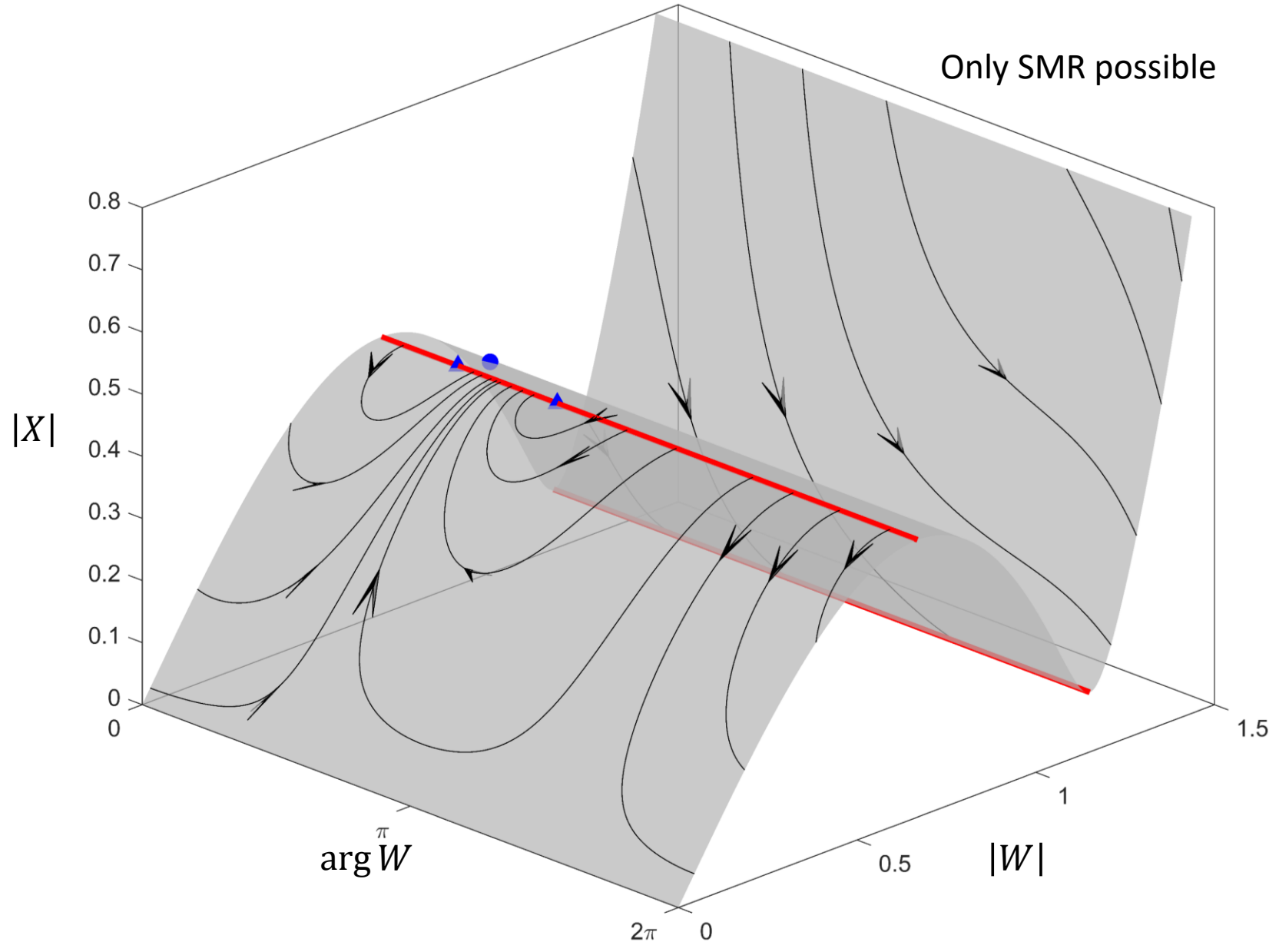
Grazing flow

$$\underbrace{\left. \frac{db}{d\theta} \right|_{b=b_1} = \left. \frac{db}{dt} \right|_{b=b_1} = \frac{f_1}{f_2} \right|_{b=b_1} = 0}_{\text{Grazing flow}}$$

Critical forcing amplitude  $G_{SMR}$

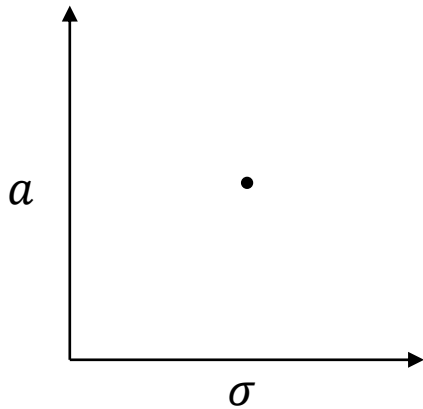


# SMR triggered by grazing flow

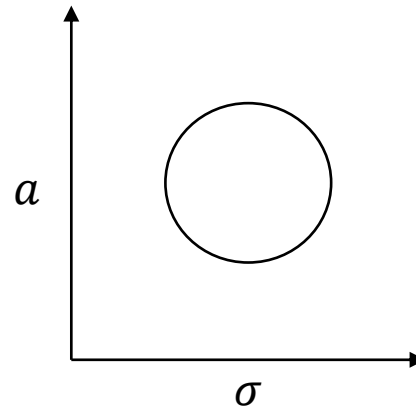
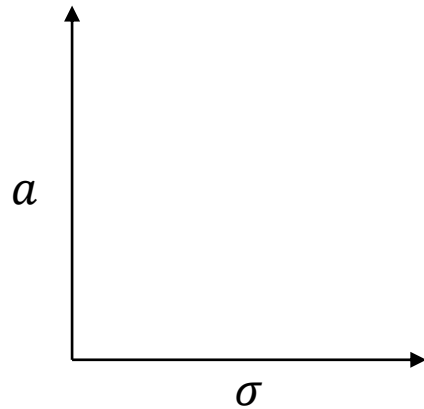


# Singularity theory<sup>[2,3]</sup> : a usefull tool to detect DRC

The isola singularity



Unperturbed diagram



Perturbed diagram

$$f_3(|W|, \sigma) = 0, \quad \frac{\partial f_3}{\partial \sigma} = \frac{\partial f_3}{\partial |W|} = 0, \quad \left. \frac{f_3}{\partial |W|^2} \neq 0, \det(d^2 f_3) > 0 \right\}$$

Fixed points  
expression

Isola singularity  
defining conditions

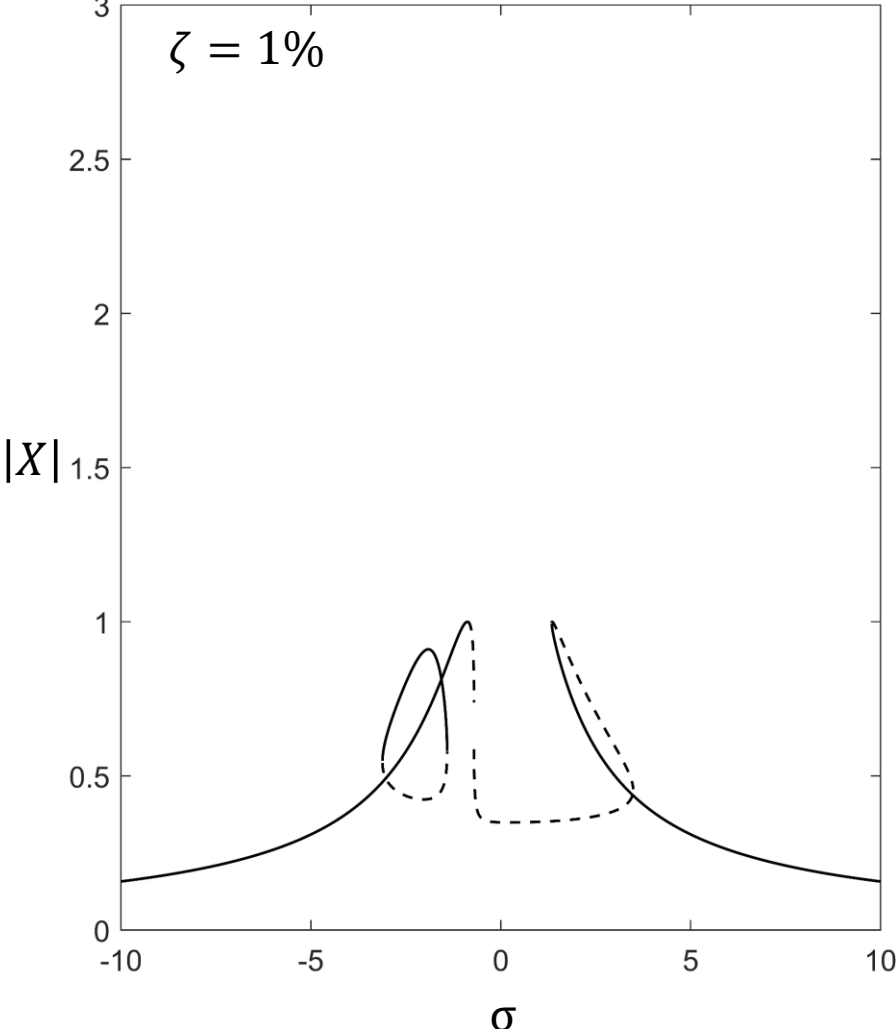
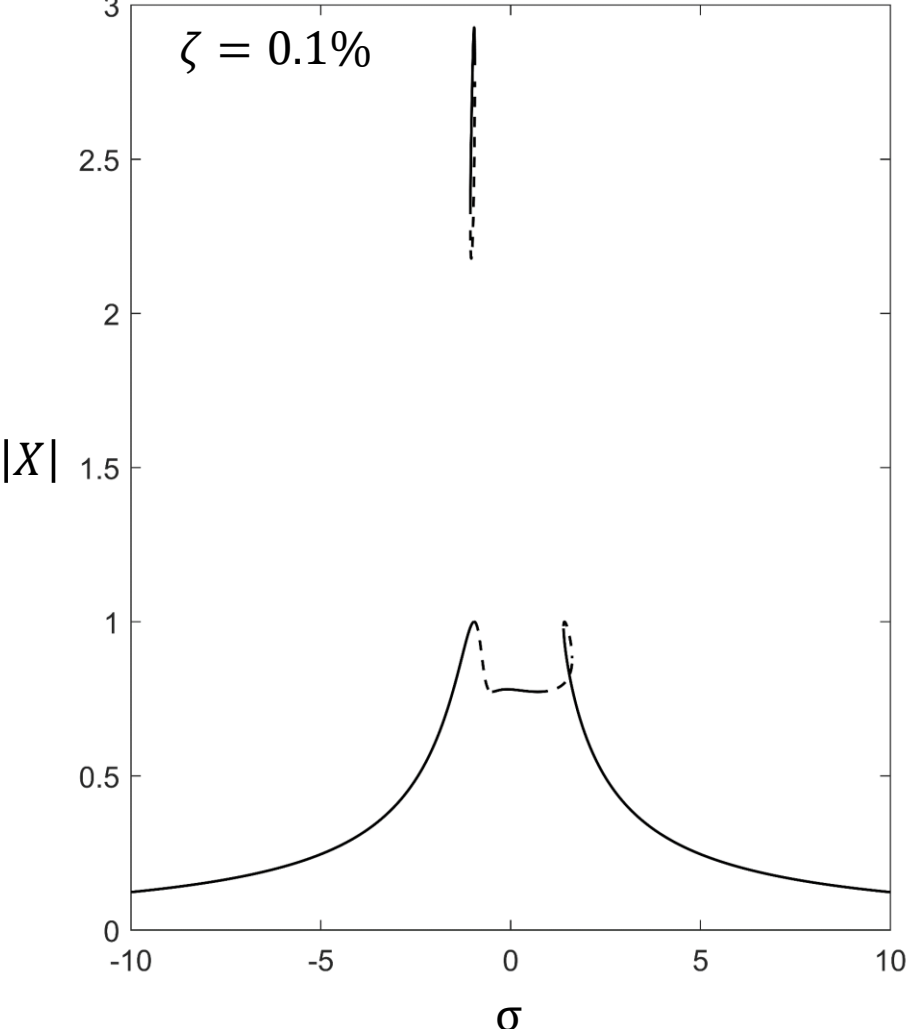
Non-degeneracy  
conditions

Critical forcing amplitude  $G_{DRC}$

[2] M. Golubitsky, D. Schaeffer, *Singularities and groups in bifurcation theory*

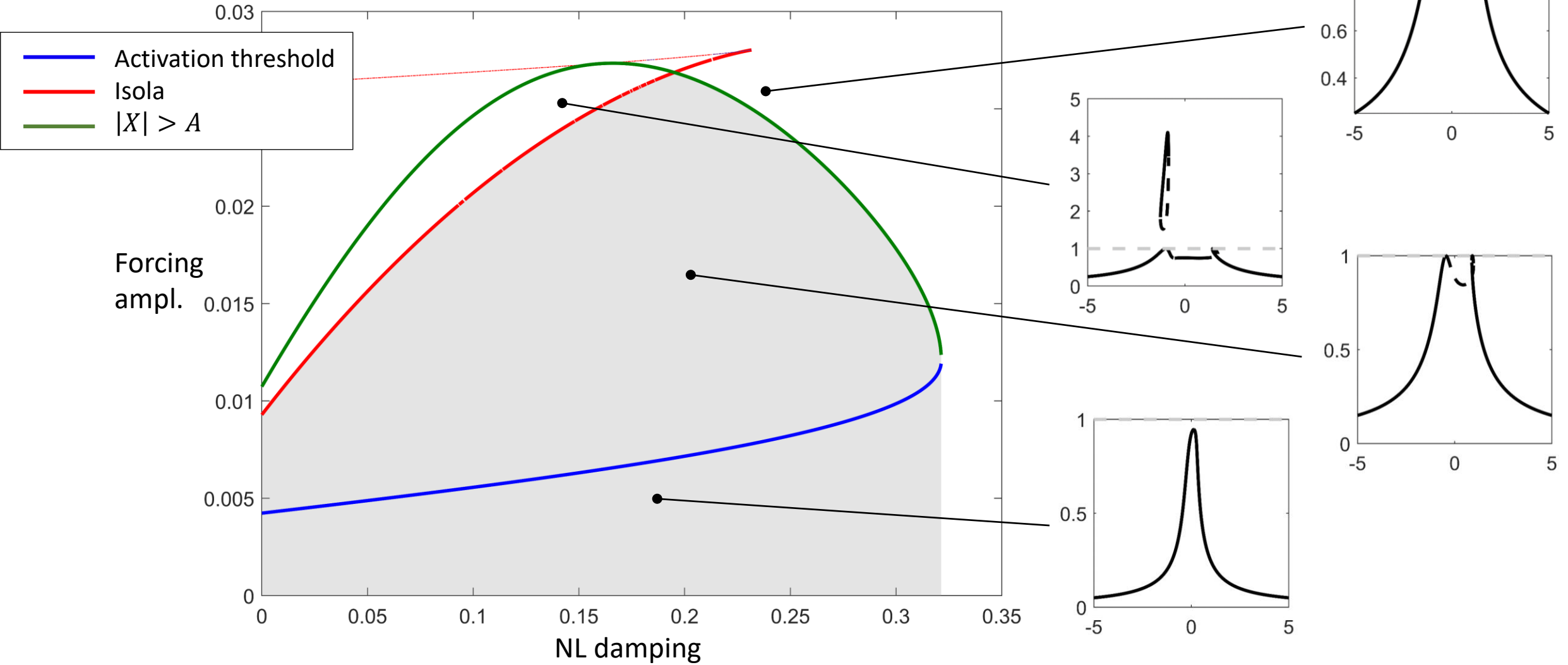
[3] I. Cirillo, G. Habib, K. Kerschen, R. Sepulchre, *Analysis and design of nonlinear resonances via singularity theory*

# All DRC are not problematic!

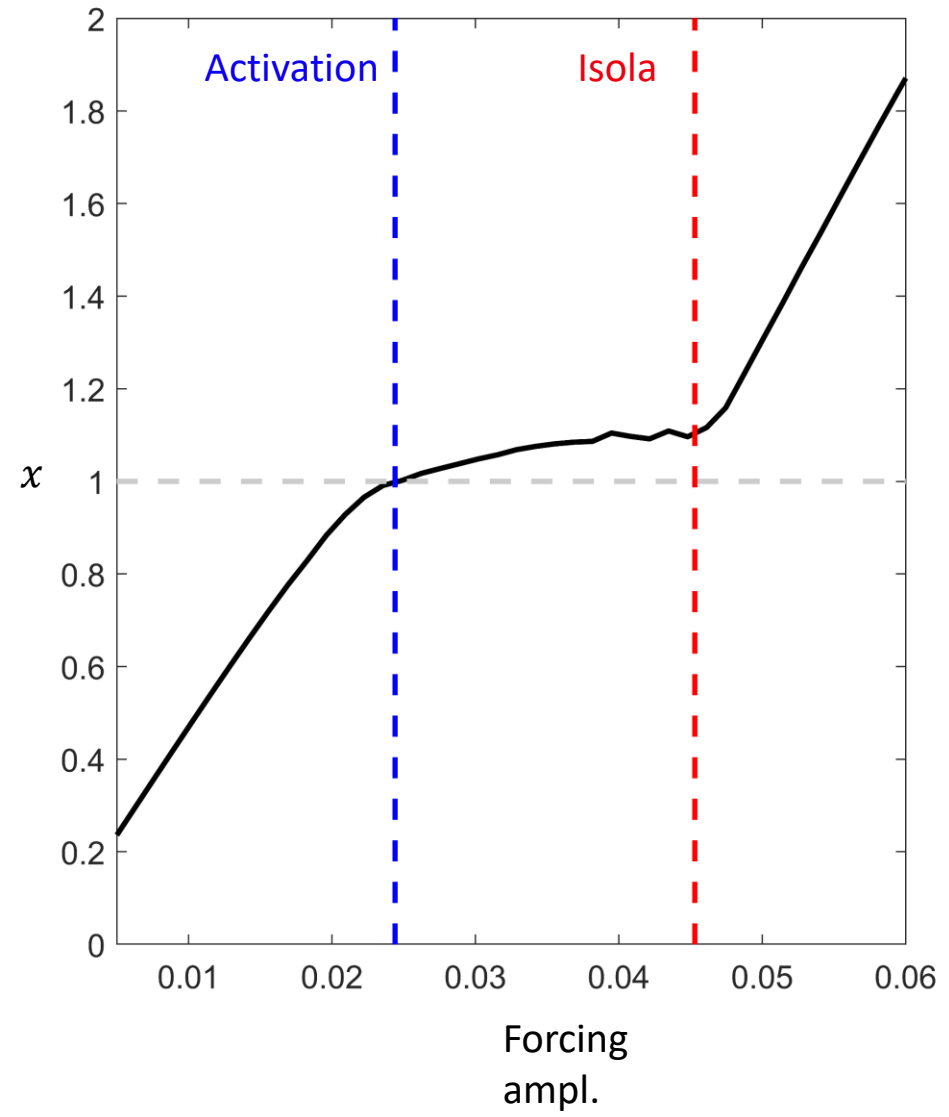
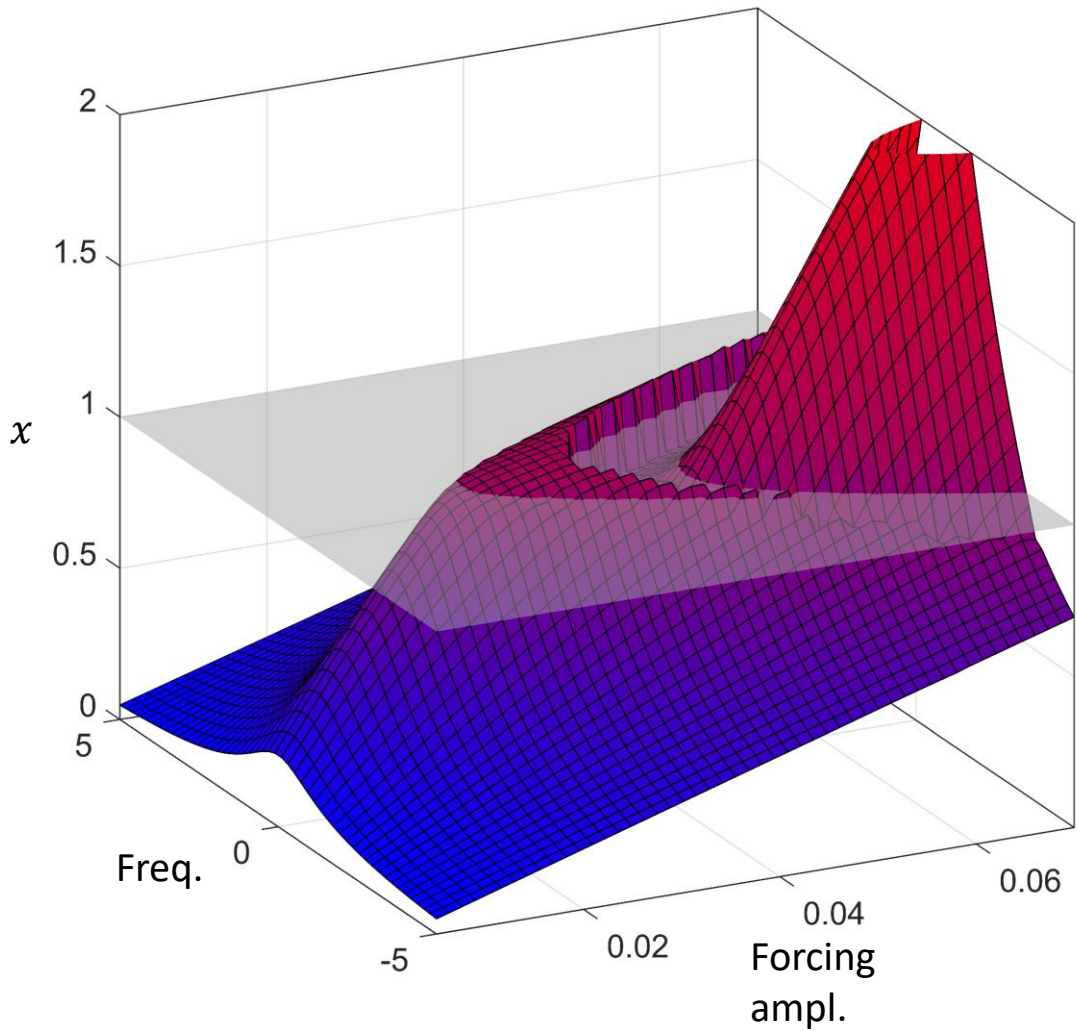


# A simple design procedure

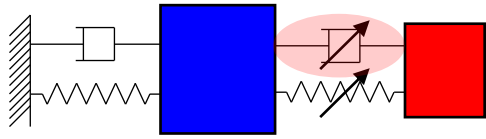
*Given a maximal allowed vibration amplitude  $A$  of the primary system, what are the parameters of the NES that maximize the dynamic range?*



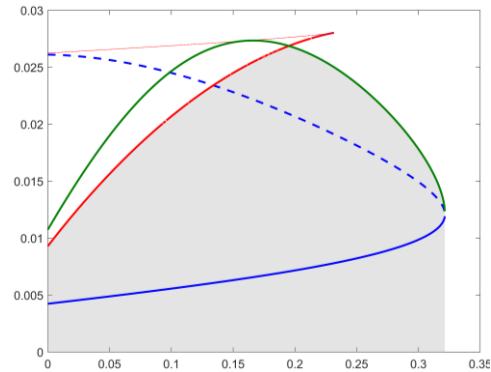
# Numerical validation of the design procedure



# Conclusion



A Nonlinear Energy Sink with nonlinear damping



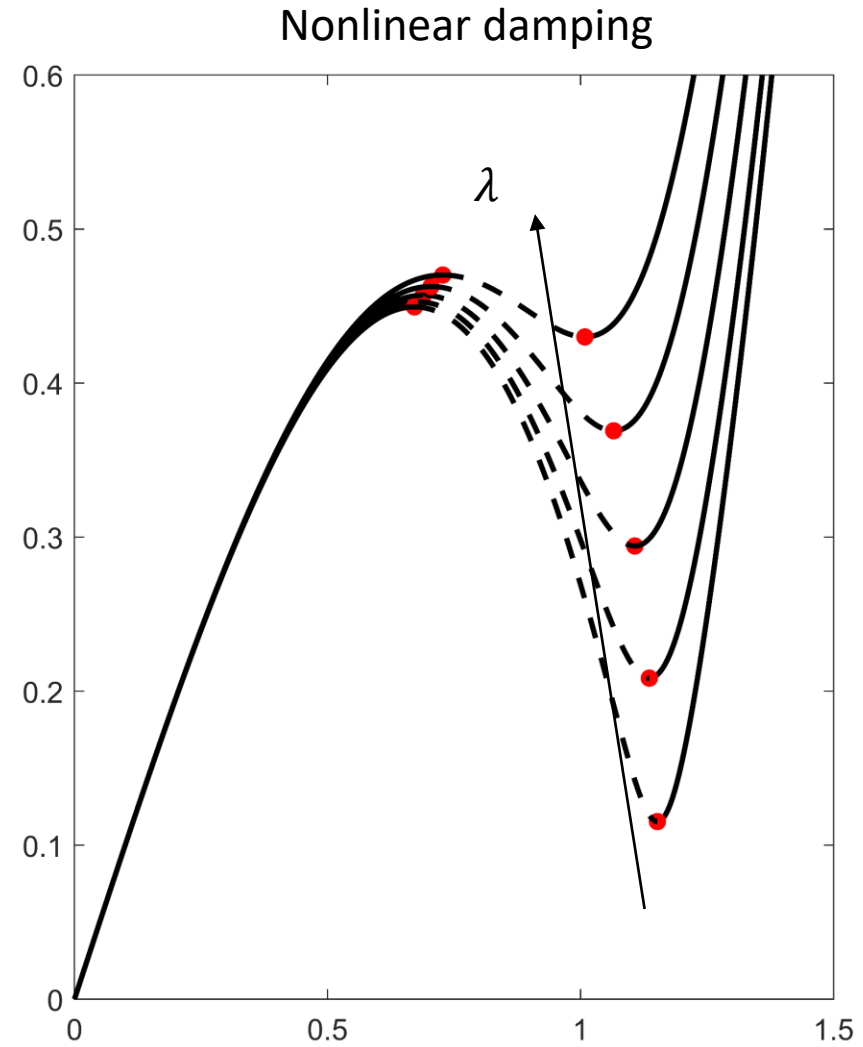
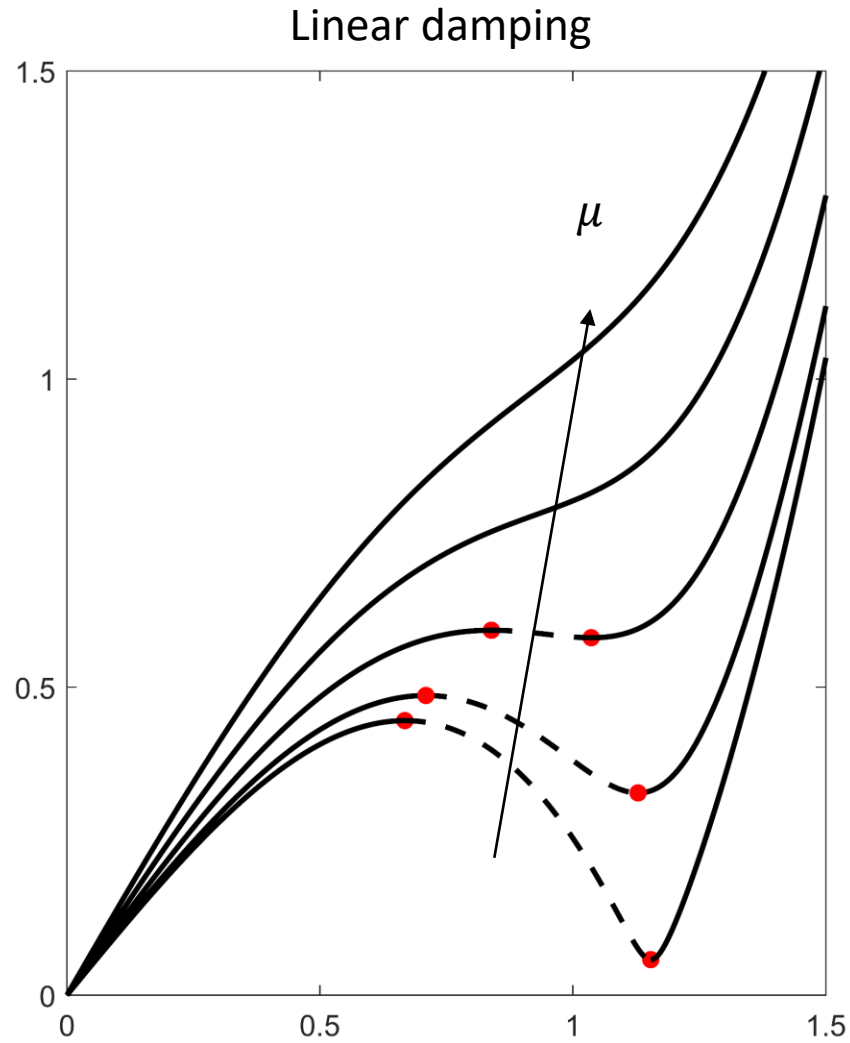
Tuning procedure based on theoretical developments

# Perspectives

Experimental validation

More realistic model of FOWT

# How does the parameters of the NES affects the SIM?



Too high damping  $\rightarrow$  NO TET