Resonant Phase Lags of Nonlinear Mechanical Systems

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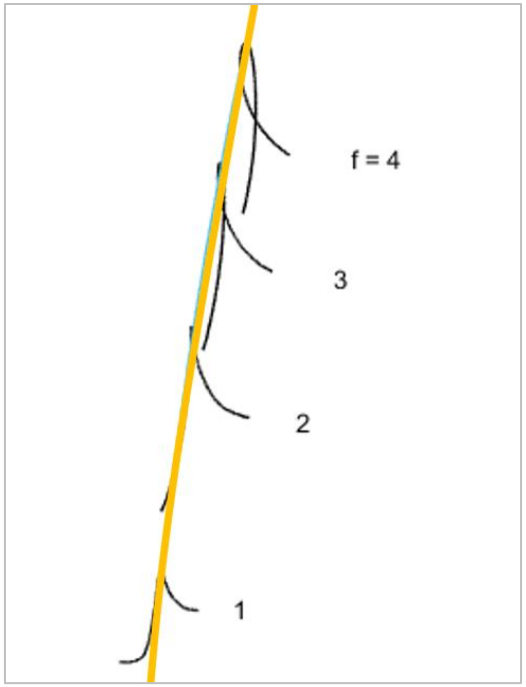

Where do we stand ?

1. Nonlinear normal modes can characterize the resonant behavior of structures, including modal interactions.

2. Nonlinear normal modes of real-life structures can be calculated numerically.

3. Where do we stand in the area of experimental identification of nonlinear normal modes ?

How to identify a linear normal mode ?

a. ***Critère de la quadrature de phase***

Lorsque l'on réalise un essai de vibration harmonique sur un système amorti, les amplitudes des forces appliquées et les amplitudes des réponses observées aux différents points vérifient la relation complexe (3.1.18)

$$\left(\mathbf{K} - \omega^2\mathbf{M} + i\omega\mathbf{C}\right)\mathbf{z} = \mathbf{f} \qquad (3.1.21)$$

Isoler un mode propre par une excitation appropriée revient à réaliser

$$\mathbf{z} = \mathbf{x}_{(k)} \qquad \text{et} \qquad \omega = \omega_{0k}$$

L'équation (3.1.21) devient alors

$$\left(\mathbf{K} - \omega_{0k}^2\mathbf{M} + i\omega_{0k}\mathbf{C}\right)\mathbf{x}_{(k)} = \mathbf{f}_{(k)} \qquad (3.1.22)$$

où $\mathbf{f}_{(k)}$ est le mode de sollicitation qui permet de réaliser l'excitation appropriée. ω_{0k}^2 et $\mathbf{x}_{(k)}$ étant solutions propres du système conservatif associé, on a

$$\left(\mathbf{K} - \omega_{0k}^2\mathbf{M}\right)\mathbf{x}_{(k)} = \mathbf{0}$$

et l'expression de la sollicitation qui permet d'exciter le mode $\mathbf{x}_{(k)}$ à sa fréquence de résonance découle de l'équation (3.1.22)

$$\mathbf{f}_{(k)} = i\omega_{0k}\mathbf{C}\mathbf{x}_{(k)} \qquad (3.1.23)$$

Elle montre que la sollicitation est alors en phase avec les forces de dissipation et présente donc un déphasage de 90° par rapport à la réponse.

F. de Veubeke

Géradin and Rixen

Linear phase lag quadrature criterion

How to identify a nonlinear normal mode ?

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Dynamic testing of nonlinear vibrating structures using nonlinear normal modes

M. Peeters *, G. Kerschen, J.C. Golinval

Nonlinear phase lag quadrature criterion (Rosenberg NNM)

$$\mathbf{x}(t) = \sum_{k=1}^{\infty} \mathbf{X}_k \cos(k\omega t)$$

$$\mathbf{p}(t) = \sum_{k=1} \mathbf{P}_k \sin(k\omega t)$$

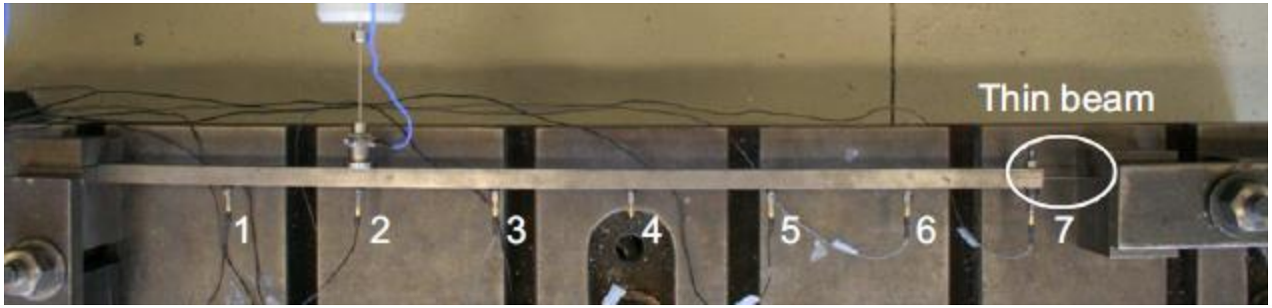
⇒

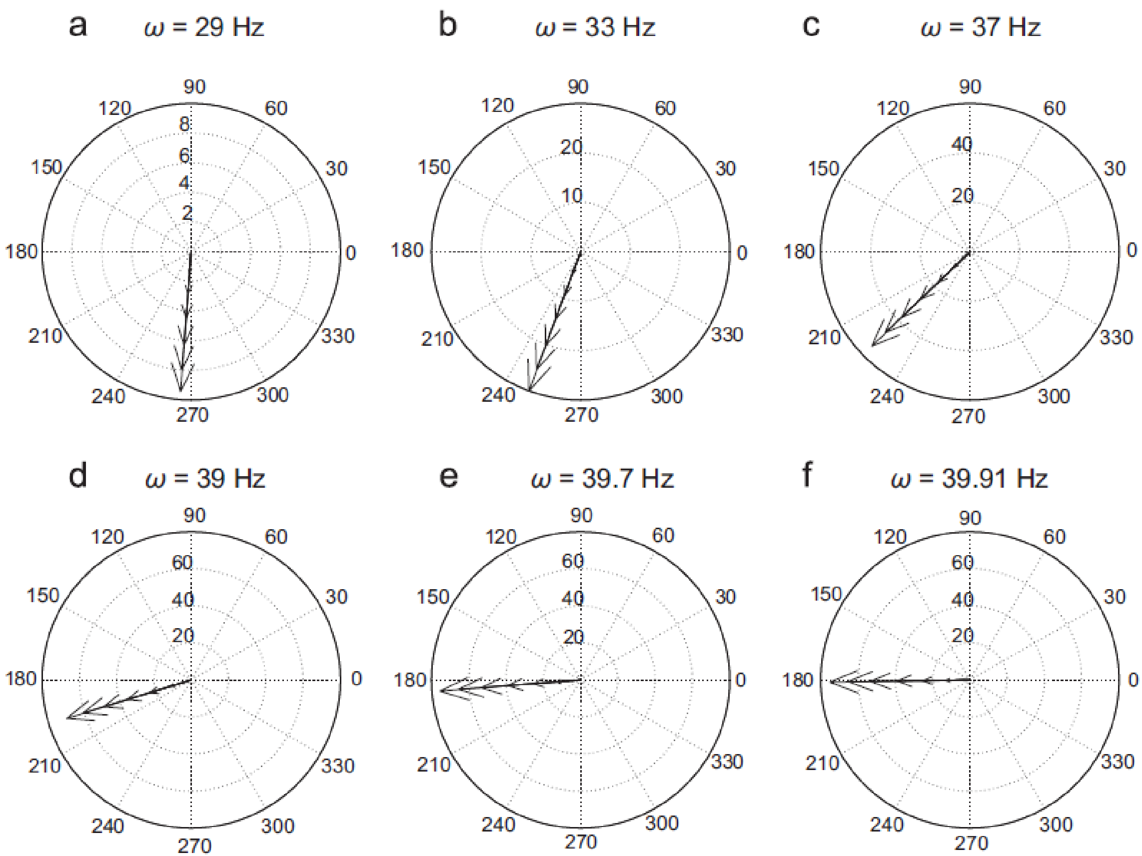
$$-k^2\omega^2\mathbf{M}\mathbf{X}_k + \mathbf{K}\mathbf{X}_k + \mathbf{F}_{\mathrm{nl},k}(\mathbf{X}_r) = \mathbf{0}$$

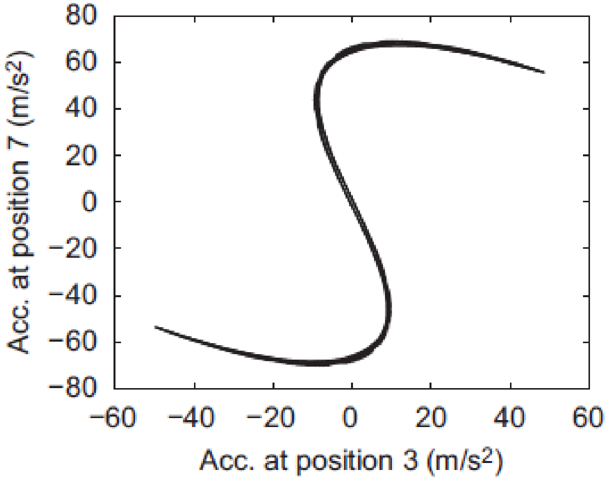
$$-k\omega\mathbf{C}\mathbf{X}_k = \mathbf{P}_k$$

⇒ Multiharmonic forcing is needed to isolate a NMM perfectly.

Experimental demonstration



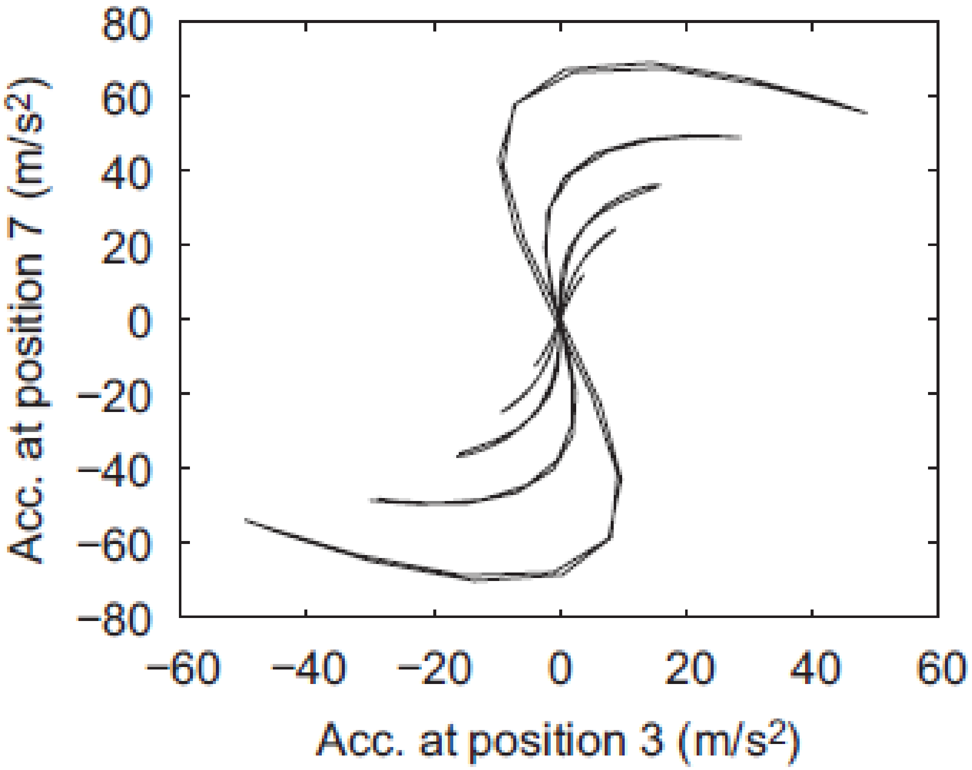




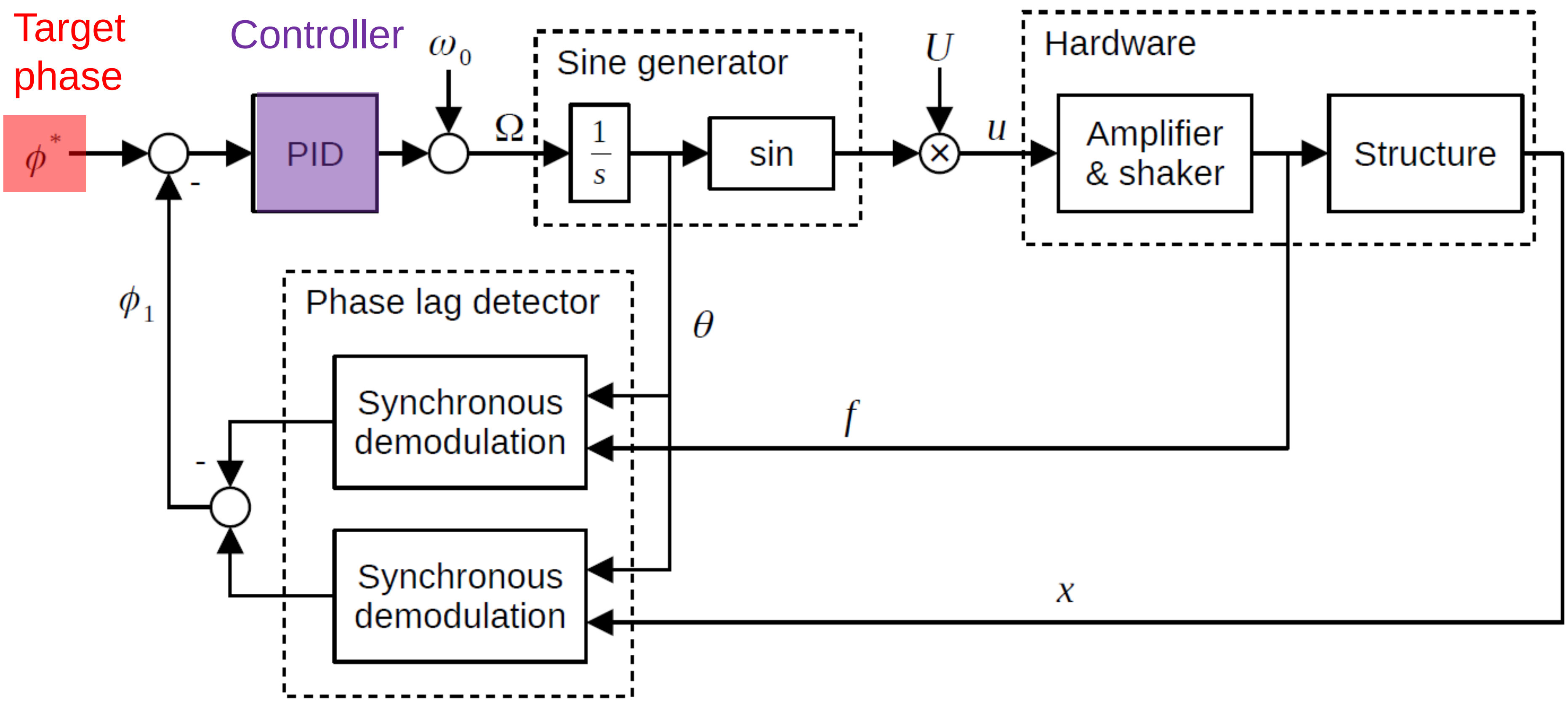
Appropriation of the first beam mode

Identification of the frequency-energy dependence

Turn off the shaker when the first mode is appropriated



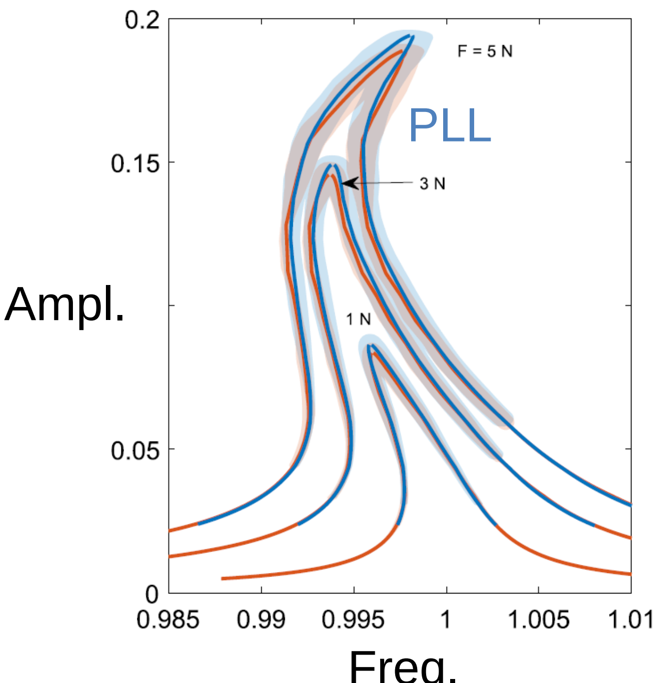
Key enabling technology: phase locked loops



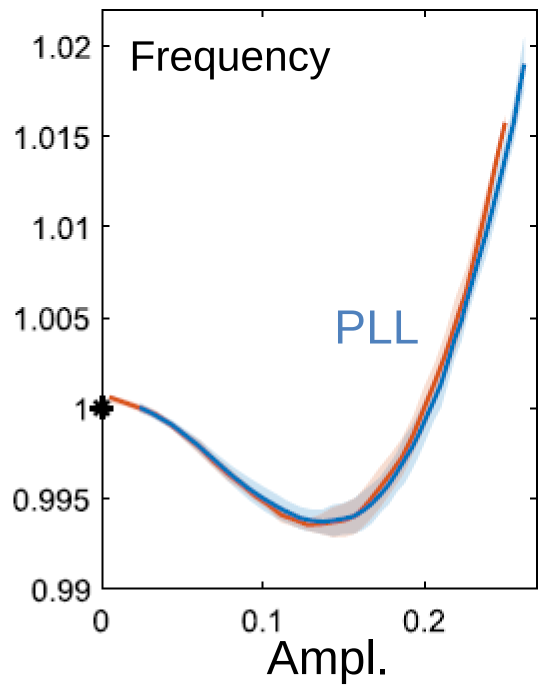
First contributions: *J. Twiefel et al., S. Peter and R. Leine, O. Thomas et al., L. Renson et al.*

Experimental demonstration

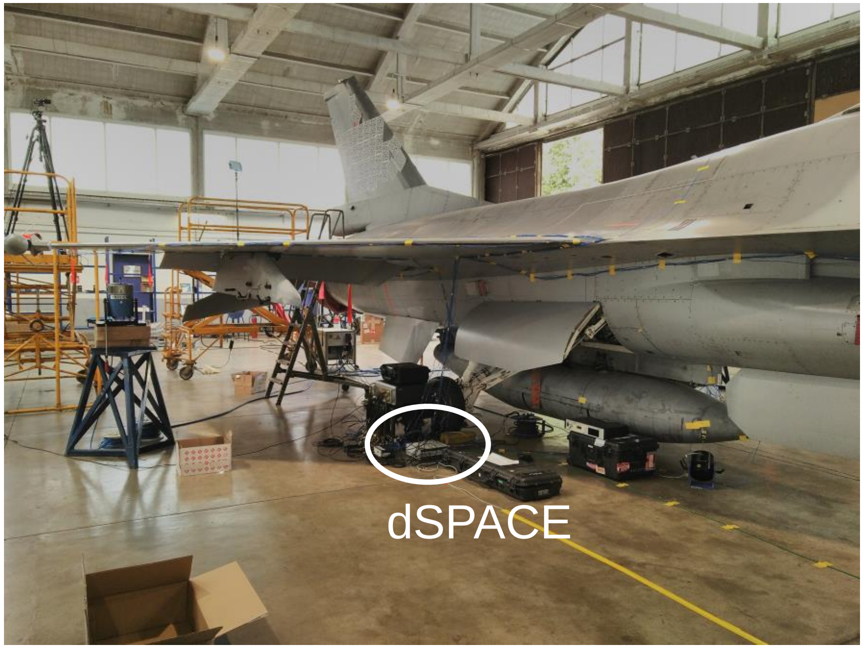
Phase from 0 to 180°

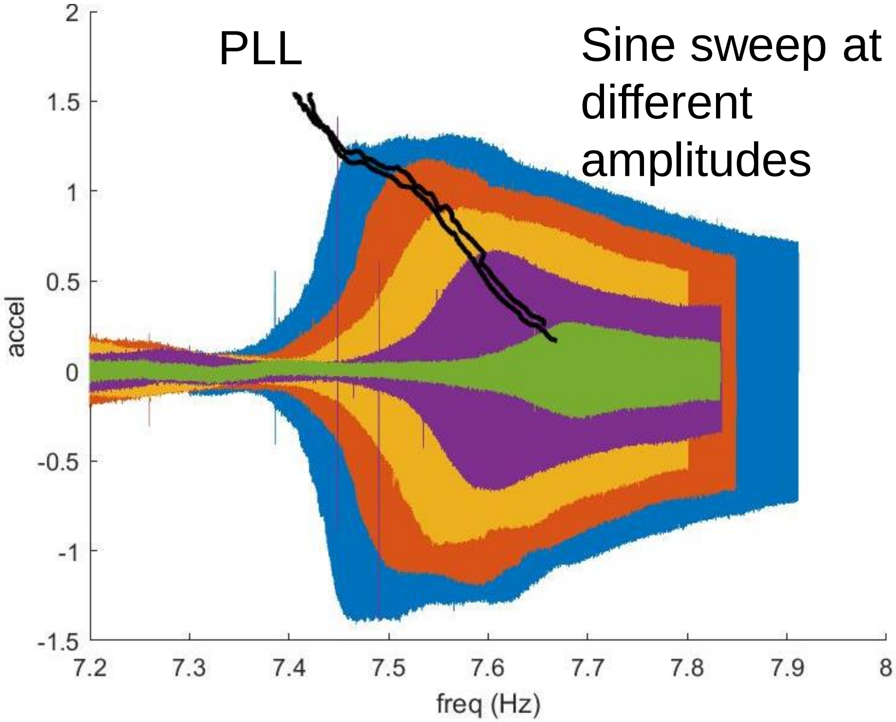


Phase locked at 90°



Application to a F-16 aircraft





Phase resonance nonlinear normal modes (PRNMs)

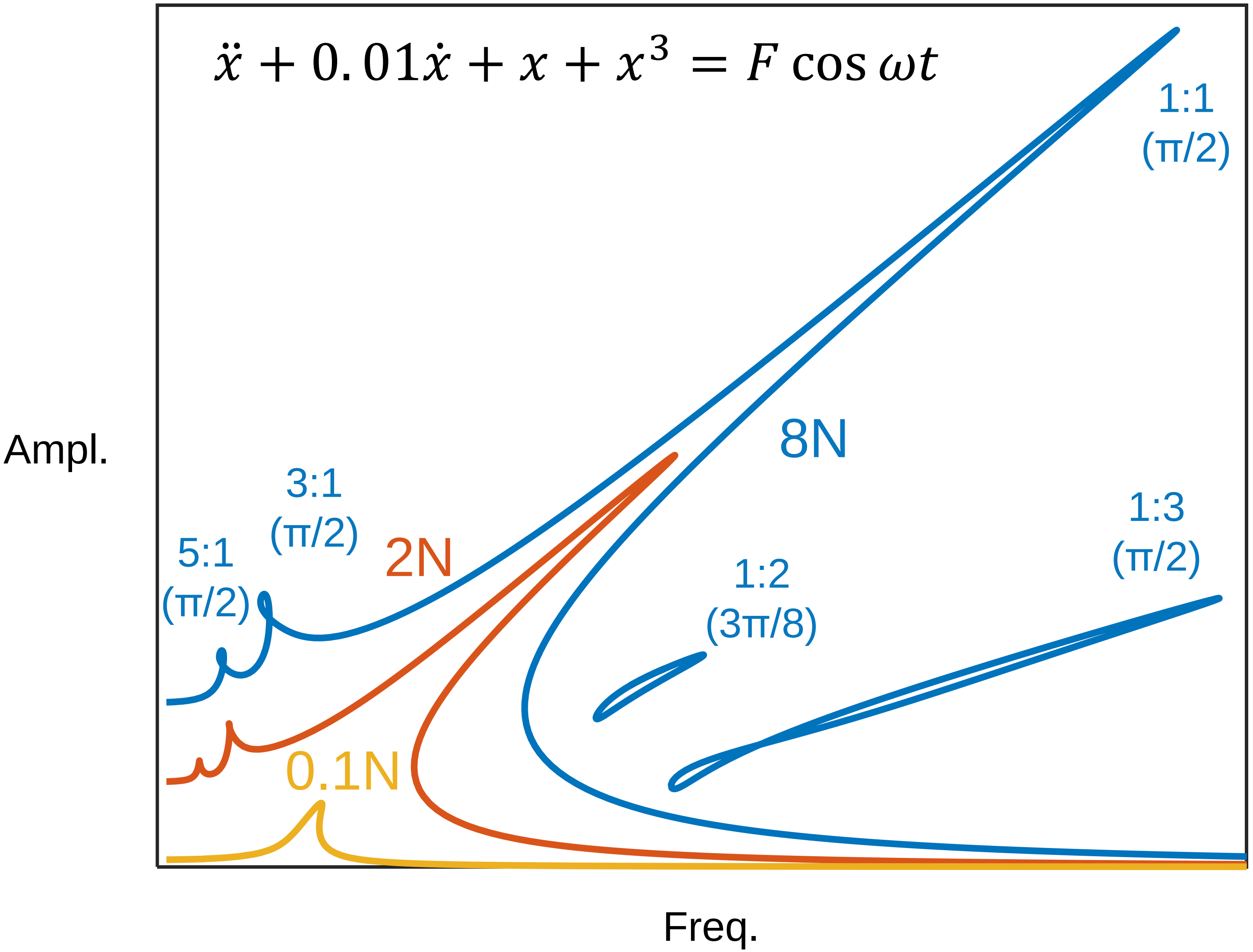
1. NNMs need multiharmonic forcing to be isolated (particularly around modal interactions).
2. NNMs are usually associated with primary resonances.

PRNMs: resonant phase lags between the *k*th harmonic of the displacement and the monoharmonic forcing.

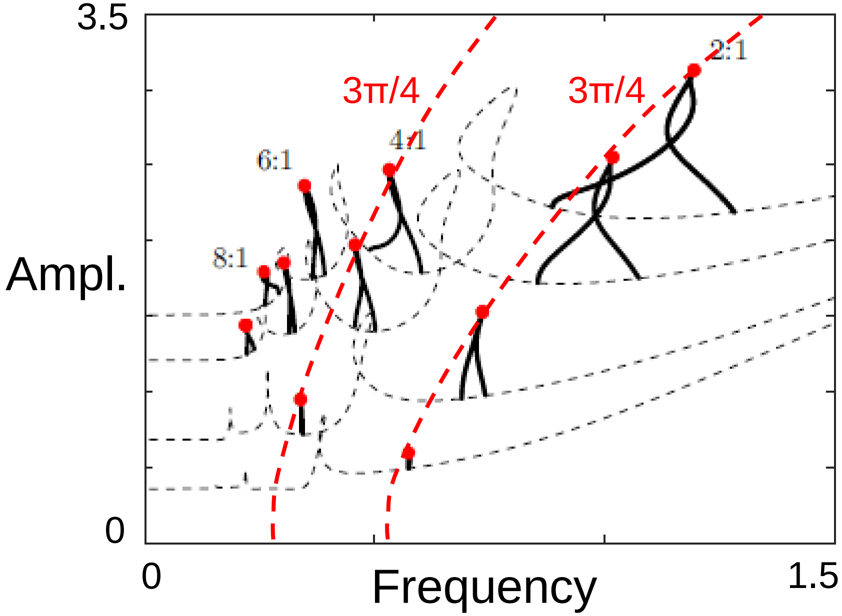
Phase resonance for primary and secondary nonlinear resonance:

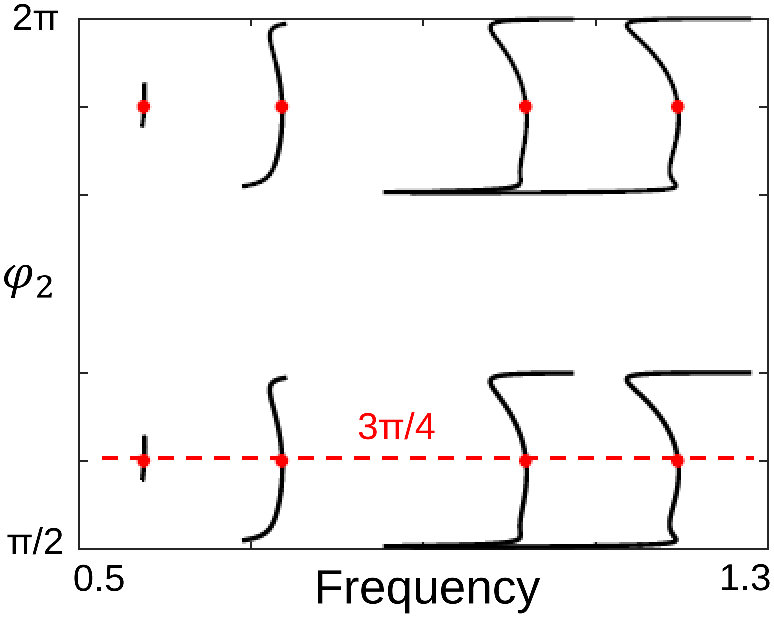
- Primary (1:1): π/2
- Secondary (k:v): determined in what follows

Resonant phase lags obtained numerically



Even-order superharmonic resonances





Phase resonance for Hk: 3π/4

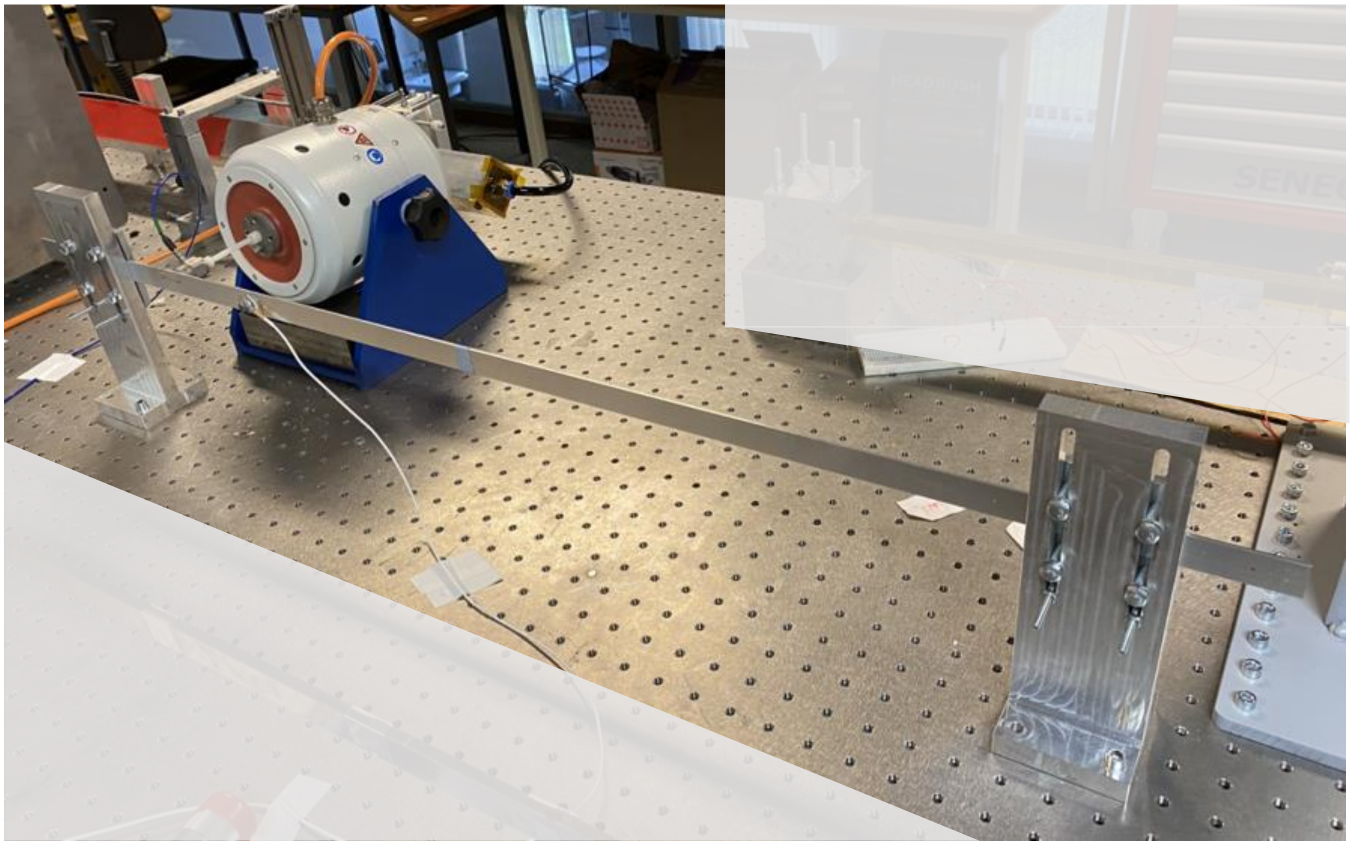
Resonant phase lags for the Duffing oscillator

PRNMs correspond to the amplitude resonance of the *k*-th harmonic:

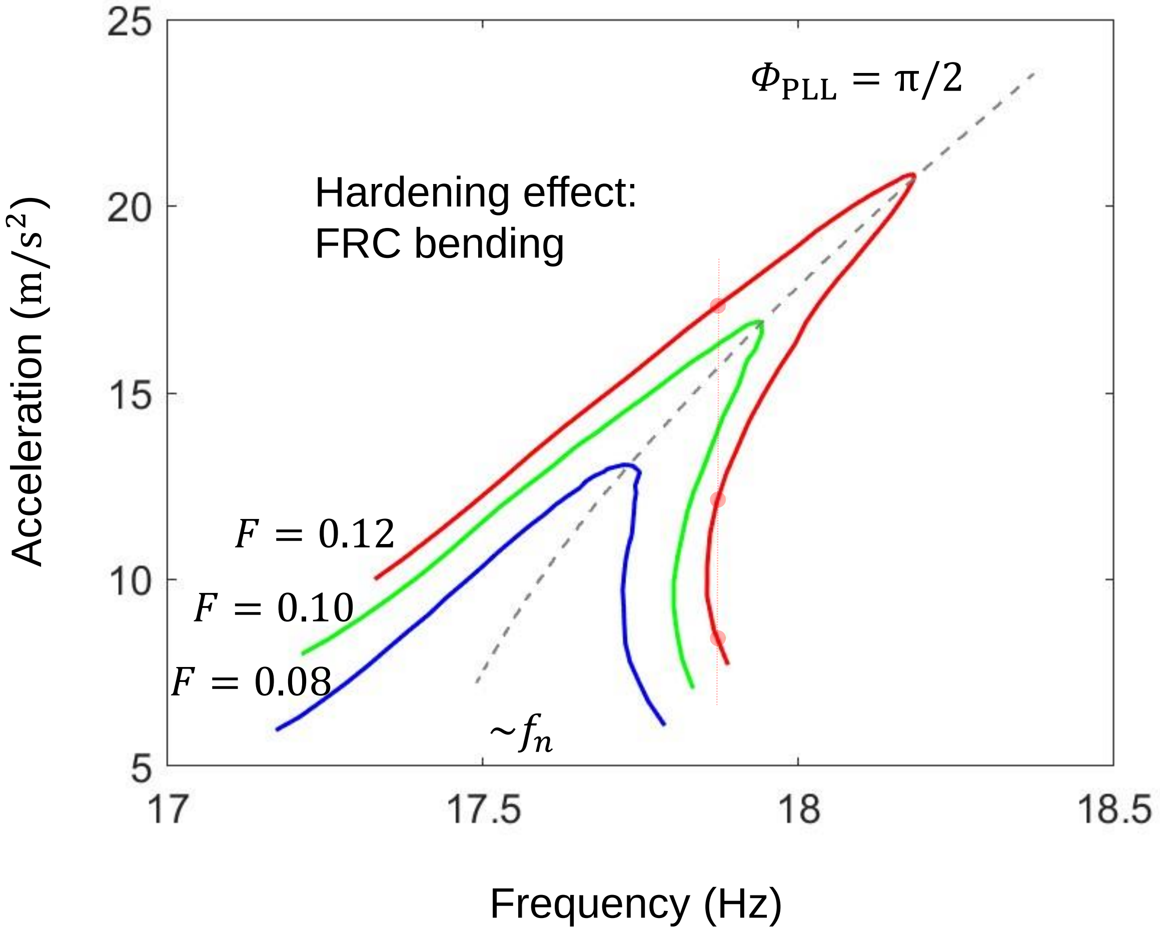
k and ν are odd:	$\pi/2$
k or ν is even:	$3\pi/4$

These results can be confirmed analytically using first-order and higher-order averaging.

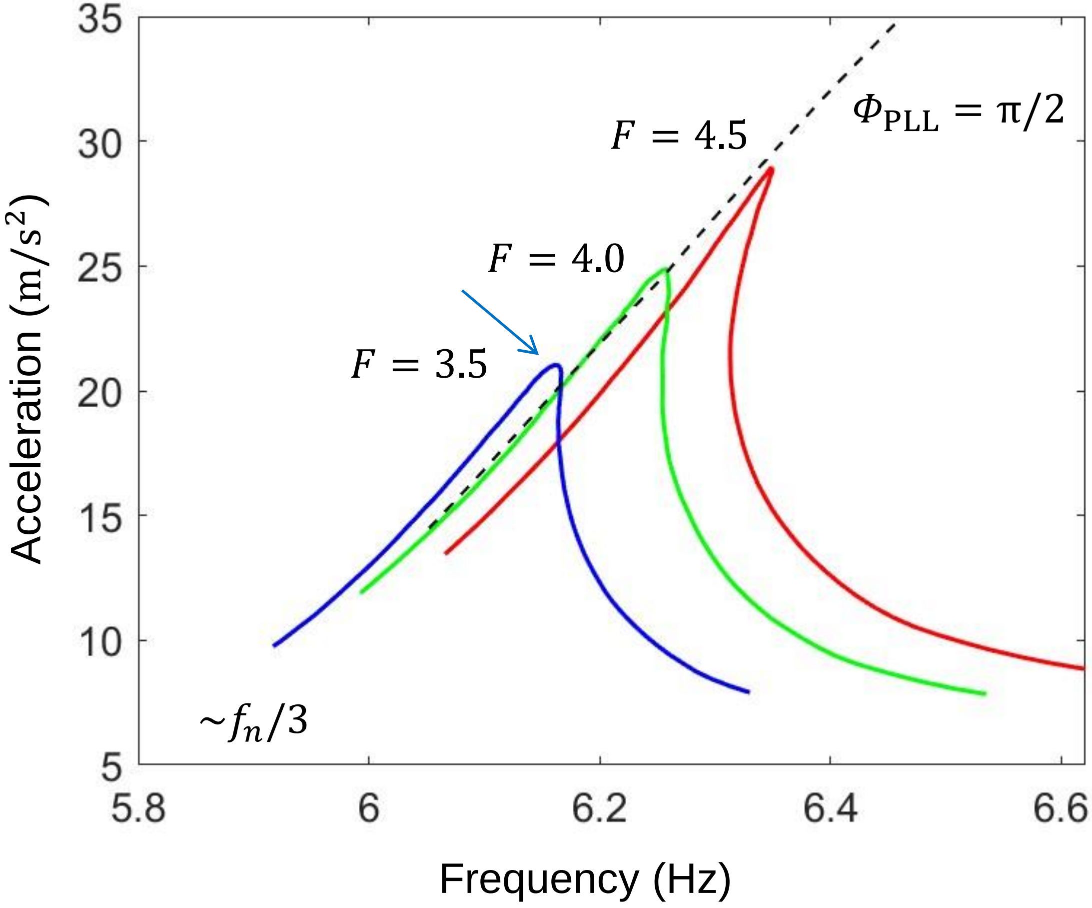
Experimental demonstration: clamped-clamped beam



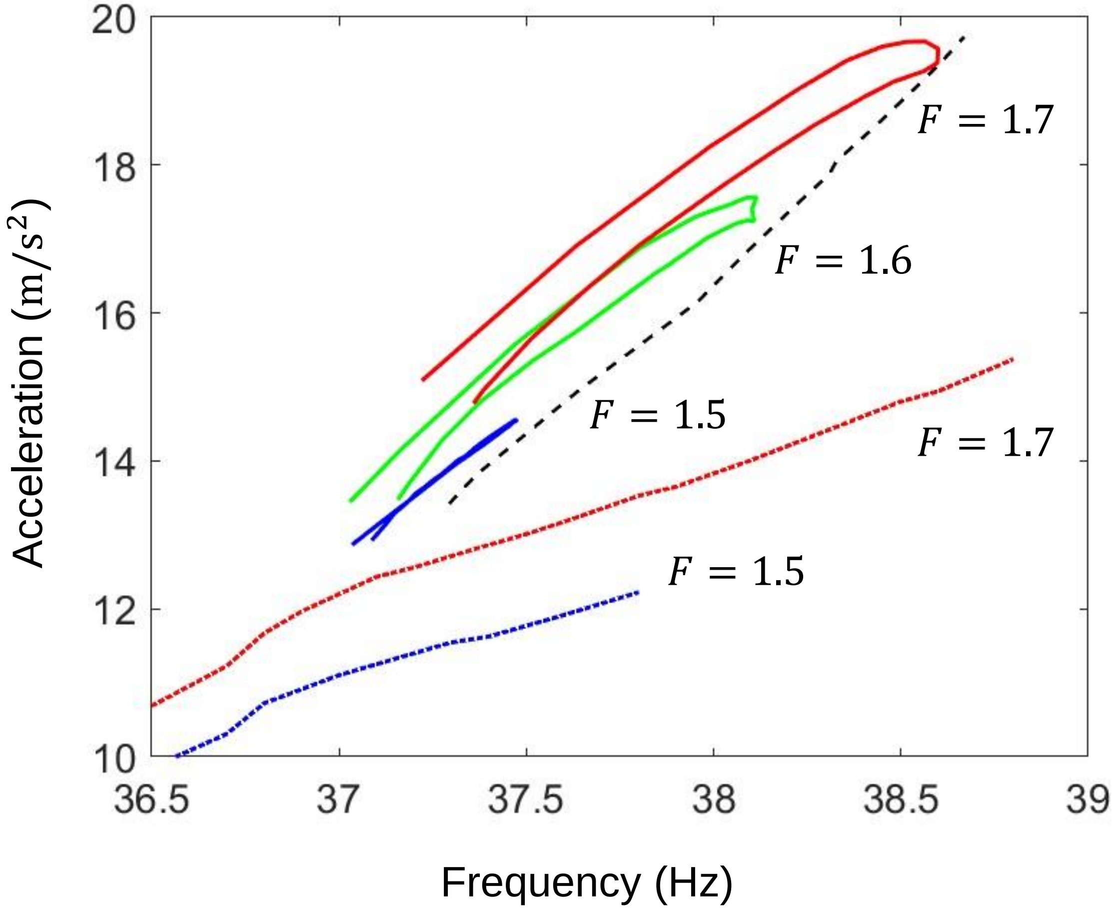
Identification of the primary resonance (mode 1)



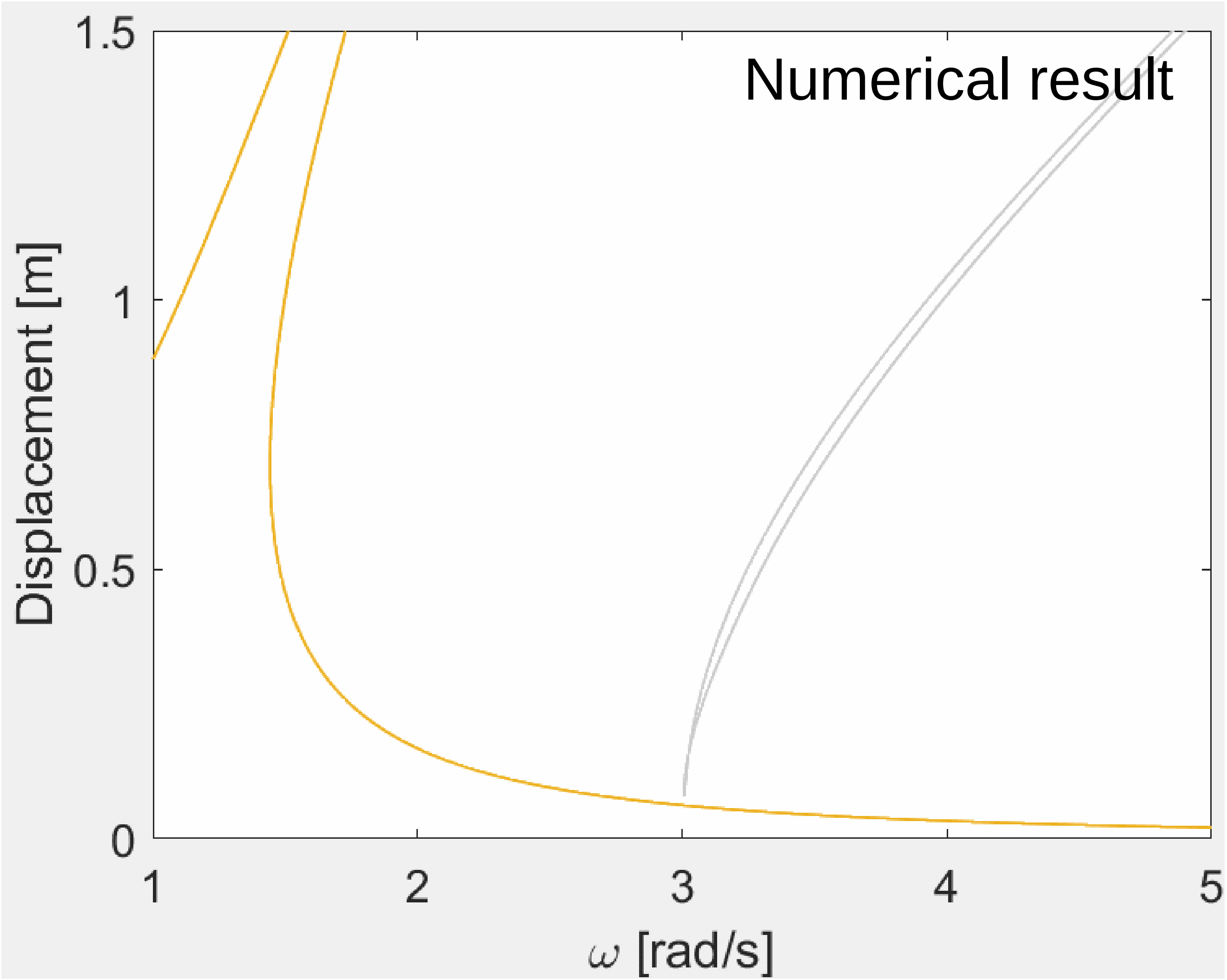
Identification of the 3:1 superharmonic resonance



Identification of the 1:2 subharmonic resonance



Automatic transition to an isola using PLL



Concluding remarks

Motivations:

- Obtain numerical and experimental modes which can be rigorously compared.
- Characterize primary and secondary resonances.

Definition of PRNMs based on resonant phase lags.

PRNMs can be easily calculated numerically using HBM and identified experimentally using PLLs.

Thank you for your attention!

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