Amortissement de résonances non-linéaires par couplage piézoélectrique

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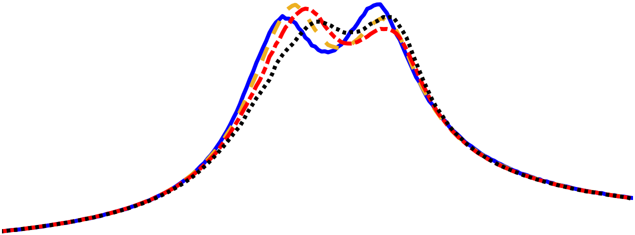
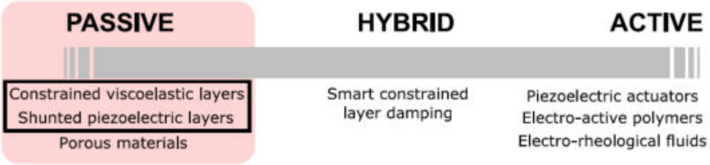


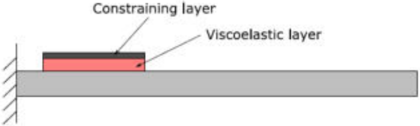
GdR EX-MODELI

9 novembre 2023

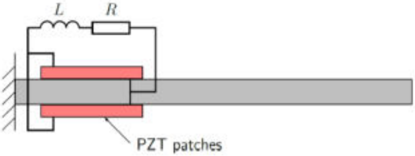
Passive vibration mitigation



- Constrained viscoelastic patches



- Piezoelectric patches connected to an electrical network



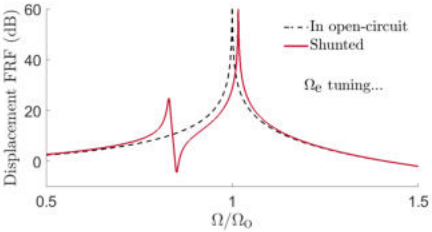
✍ *B. Lossouarn, L. Rouleau, R. Darleux, J.-F. Deü, Journal of Structural Dynamics, 2021.*

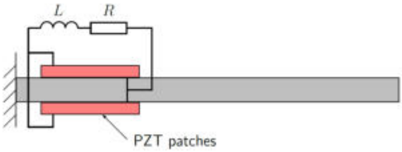
Tuning of the resonant piezoelectric shunt

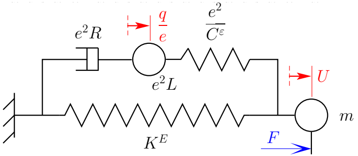
Solution for $\mathcal{H}_\infty$ **optimal tuning**:

$$L = \frac{1}{C\omega_O^2}$$

$$R = \sqrt{\frac{3}{2}}\frac{k_c}{C\omega_O}$$



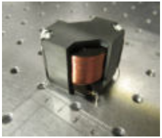


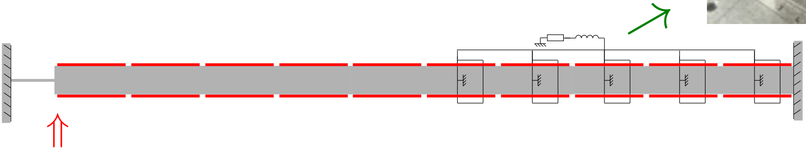


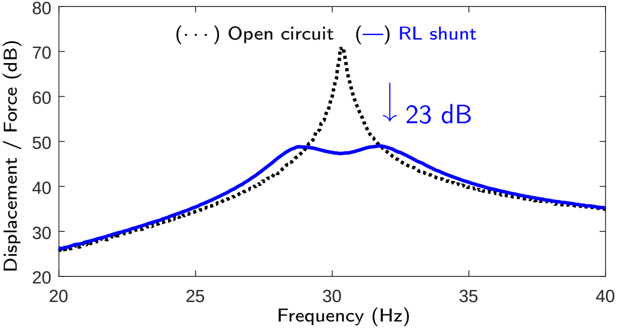
Resonant shunts can be implemented with passive inductors

Realization with a **magnetic core** in ferrite

$\rightarrow C = 250$ nF $\Rightarrow$ $L = 100$ H & $R = 3\text{k}\Omega$





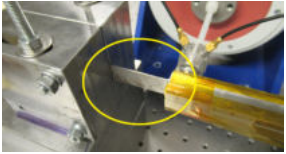


Nonlinearity may strongly affect the performance

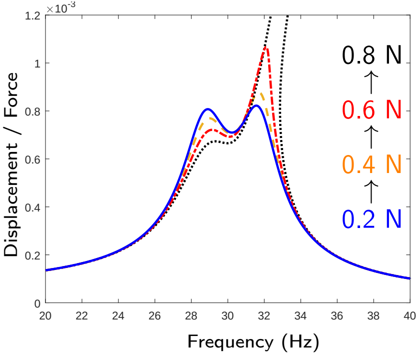
Thin lamina $\Rightarrow f \approx k^{\text{end}}u + k_{\text{NL}}u^3$

$\rightarrow$ **Hardening nonlinearity**

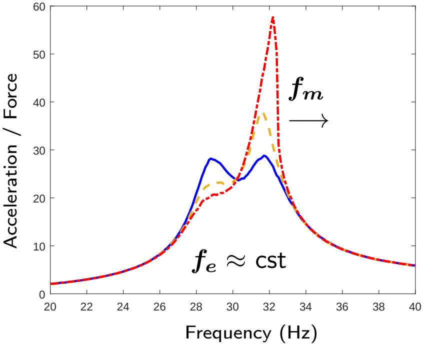
$\rightarrow$ **Detuning** of the resonant shunt + **Isola**



Simulation



Experiments



$\rightarrow$ **Nonlinear piezoelectric tuned vibration absorber required !**

Outline

1. Passive nonlinear piezoelectric tuned vibration absorber
2. Multimodal damping with an analogue twin
3. Mitigation of multiple nonlinear resonances
4. Conclusions and perspectives

Outline

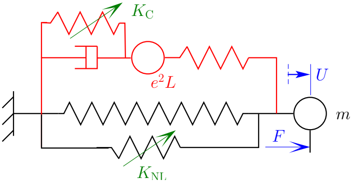
Similar nonlinearity in the absorber for global compensation

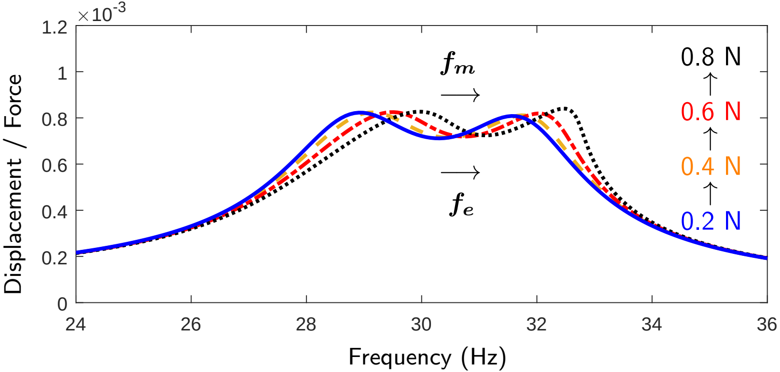
"Nonlinear + Nonlinear = Linear" (✍ Habib, 2016)

Tuning rule (✍ Soltani, 2015)

$$\rightarrow K_C \simeq 2\left(\frac{e^2 L}{m}\right)^2 K_{\mathrm{NL}}$$

for cubic nonlinearity

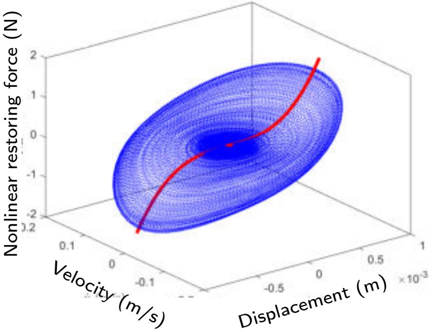


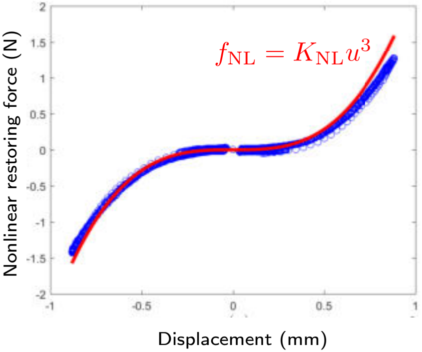


First step = characterize the mechanical nonlinearity

Sine-sweep around the resonance

Restoring force surface method → Strongly nonlinear above 0.1 mm



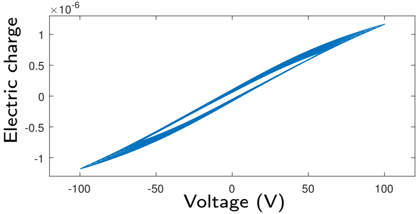


Nonlinear stiffness coefficient from the restoring force when $\dot{u} = 0$

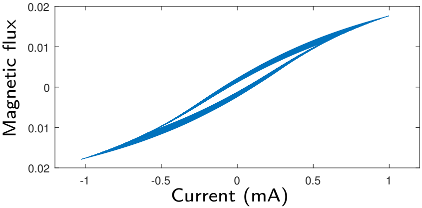
→ Lead to the **objective function** for the nonlinear electrical component

How to implement the nonlinearity in the electrical domain ?

Capacitor: $\boldsymbol{q = Cv}$



Inductor: $\boldsymbol{\Phi = L\dot{q}}$

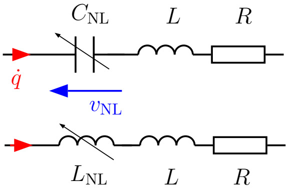


Nonlinear capacitance

$$C_{\mathrm{NL}} = C^{\star}/q^2 \quad \Rightarrow \quad v_{\mathrm{NL}} \ \alpha \ q^3$$

Nonlinear inductance

$$L_{\mathrm{NL}} = -L^{\star}\dot{q}^2 \quad \Rightarrow \quad v_{\mathrm{NL}} \ \alpha \ -\dot{q}^2\ddot{q}$$

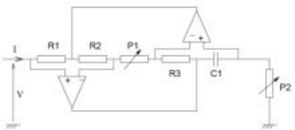


→ Same cubic voltage after one-term Harmonic Balance approximation

Several solutions to implement nonlinear impedances

Analog electronics
with AOP

✍ Silva, 2018
✍ Shami, 2022



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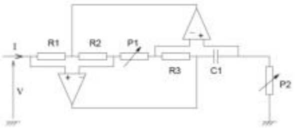
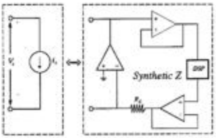
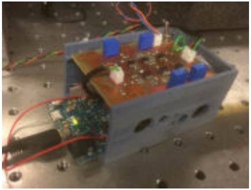


Synthetic impedance
with digital controler

✍ Raze, 2019
✍ Alfahmi, 2022





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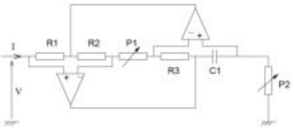

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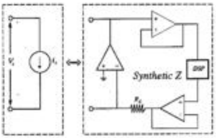
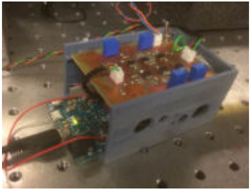
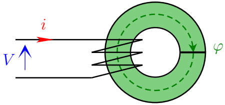




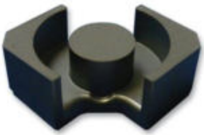
Passive magnetic components

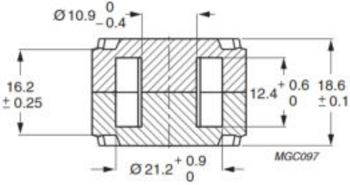
✍ Lossouarn, 2018

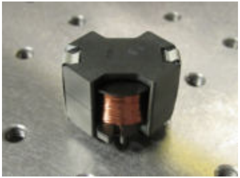


up to 1000 H !

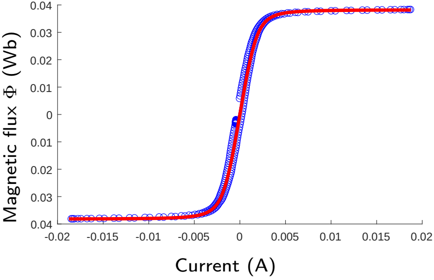
Magnetic saturation: Depends on material, geometry, number of turns...

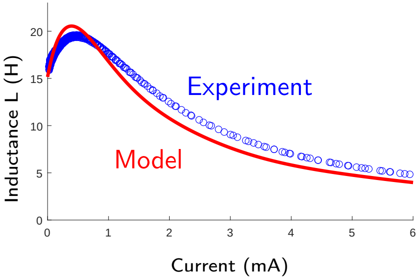






Relation between total magnetic flux and electrical current $\Rightarrow \boldsymbol{\Phi = L\dot{Q}}$

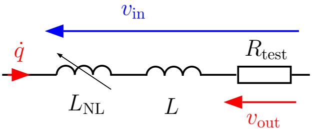


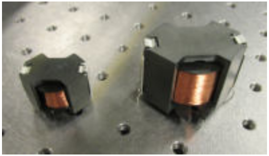


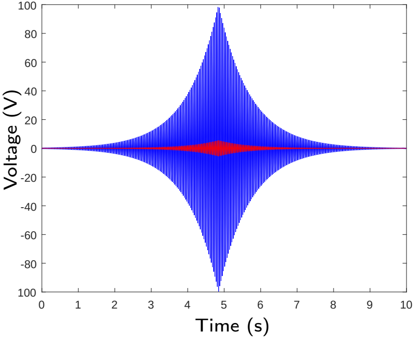
Electrical measurements validate the saturable inductor

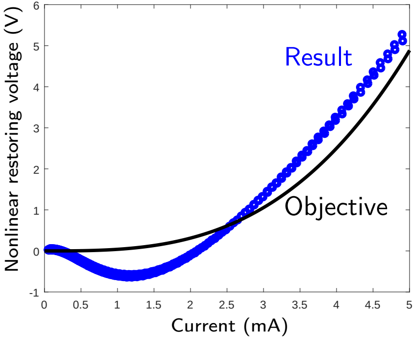
Input = Sine voltage with variable amplitude

Output = Voltage across a resistor



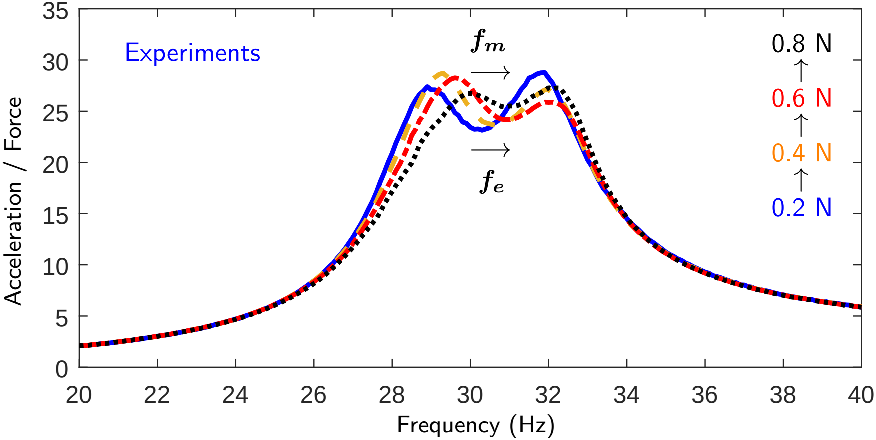






The nonlinear shunt maintains the equal peak condition

Various forcing amplitudes ⇒ **Almost no detuning**



Limitation = Merging of right peak with an **isolated resonance curve**

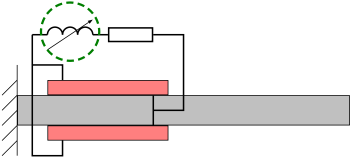
Conclusions on damping of a single nonlinear resonance

Extension of the **resonant shunt damping** to nonlinear structures

→ Principle of similarity with **passive components**

→ Experimental validation + New concept = **Saturable inductor**

→ ✍ *B. Lossouarn, J.-F. Deü, G. Kerschen, Philosophical Transactions of the Royal Society A, 2018*



Limits: Physics of magnetic circuits

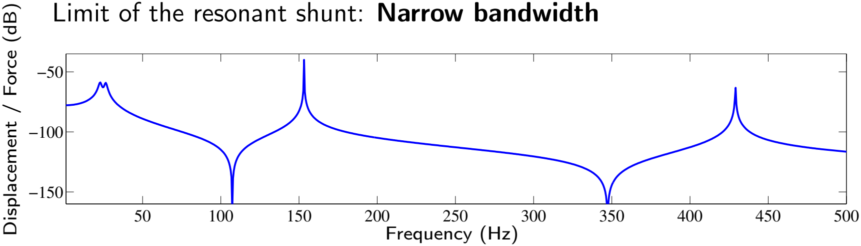
→ No pure cubic + Variable resistance

And what about the **mitigation of multiple resonances ?**

Outline

Passive technique for multimodal damping ?

Limit of the resonant shunt: **Narrow bandwidth**



Interconnected array

→ Multi-resonant network

Electrical analogue of a mechanical structure for multimodal damping

✍ *M. Porfiri, F. dell'Isola, F. M. Frattale Mascioli,*

International Journal of Circuit Theory and Applications, 2004.

Passive technique for multimodal damping ?

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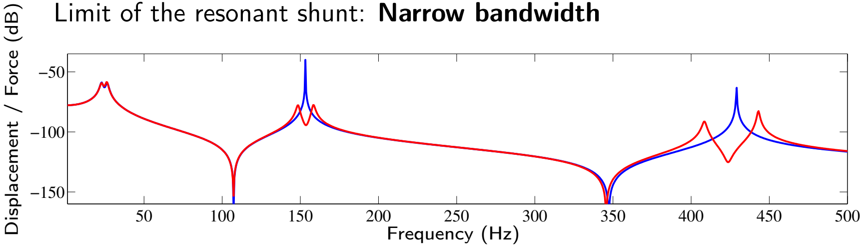

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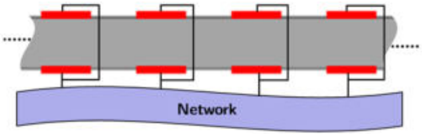

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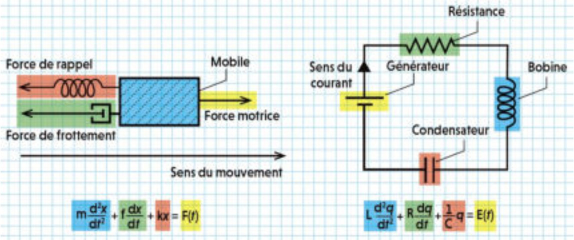
International Journal of Circuit Theory and Applications, 2004.

Analogue twin : Back to the 50's !

At that time, *Analogue electronics* refered to the *Analogy*

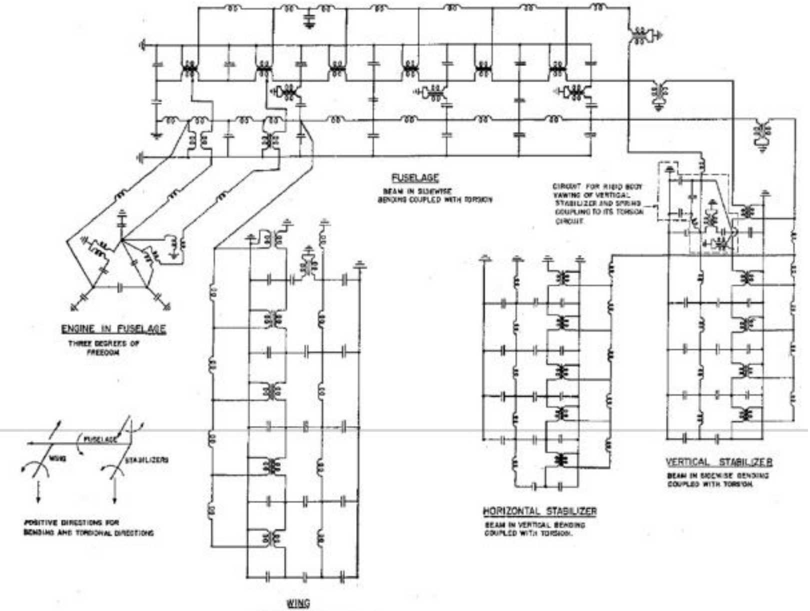
→ ✍ C. Foasso, *Quand l'informatique était analogique, Pour la Science, 2021*



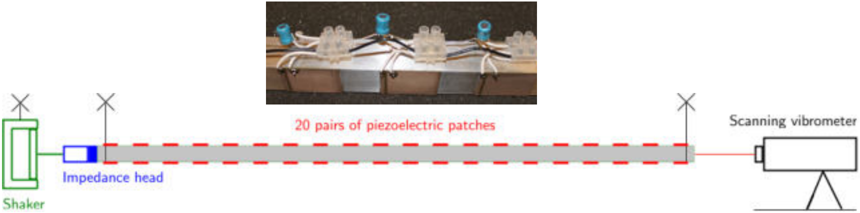
Analogue computers to simulate the dynamics of complex system

Before the development of the Finite Element Method...

Vibration analysis of an airplane → ✍ *R. MacNeal et al., J. Aeronaut. Sci, 1951.*



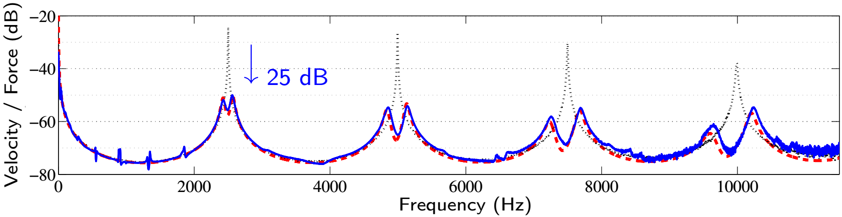
Experimenting multimodal damping on a rod



First experimental validation of the control strategy

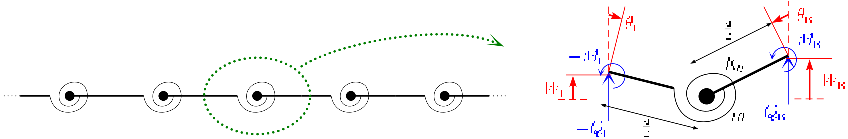
✍ *B. Lossouarn, M. Aucejo, J.-F. Deü, Smart Materials & Structures, 2015*

(...) Exp. without network, (—) Exp. with analogous network, (−−) Transfer matrix model

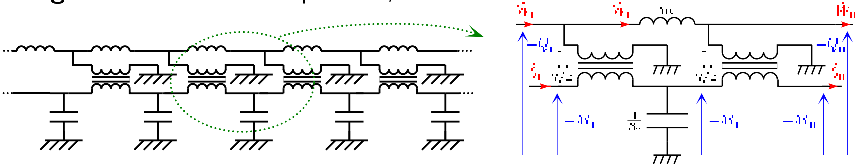


Analogue twin for a beam

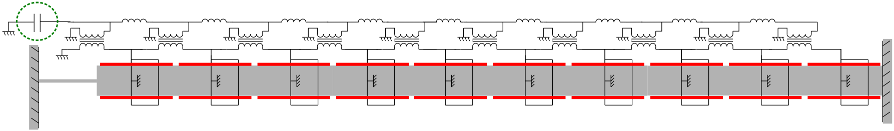
Discrete model from the beam dispersion relation $Y I \, \frac{\partial^4 q_w}{\partial x^4} + \rho S \, \ddot{q_w} = 0$



Analogous network with capacitors, inductors and transformers



+ **Analogous end capacitor** : $C^{\mathrm{end}} = \frac{k^\theta}{k^{\mathrm{end}}} \frac{\hat{a}^2}{a^2} C$



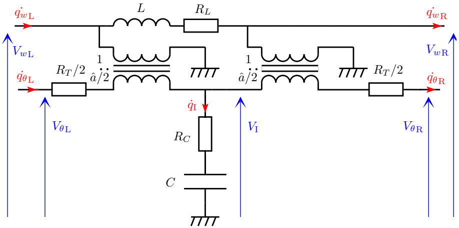
Not an exact twin because damping is required

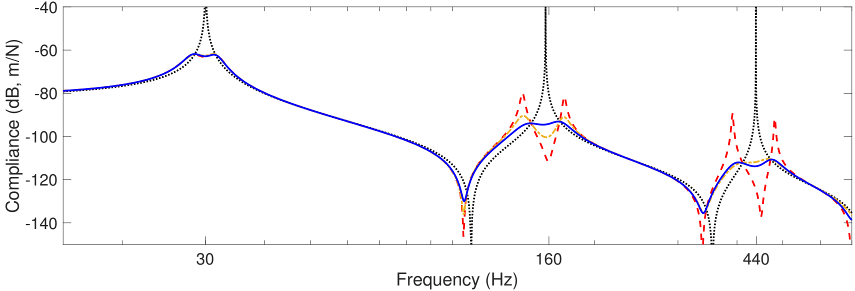
Optimal resistors for broadband damping ?

Lowest mode ⇒ $Z_L = \mathrm{j}\omega L + R_L$

Highest mode ⇒ $Z_C = \dfrac{1}{\mathrm{j}\omega C} + R_C$

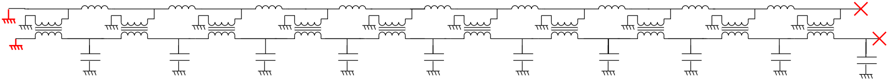
Broadband ⇒ R_T in the transformers





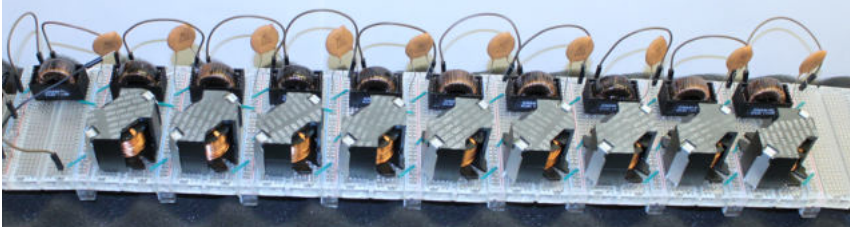
Implementation of the beam electrical analogue

Analogous boundary conditions: Free = Short circuit, Clamped = Open circuit



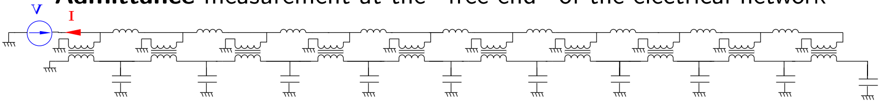
Design of **inductors** and **transformers** + Capacitors from standard series

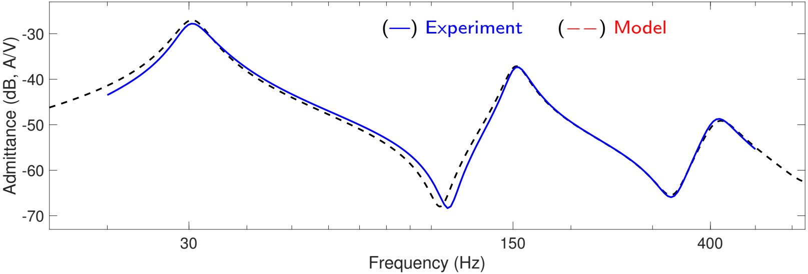
⇒ 9 ×  + 10 ×  + 10 × 

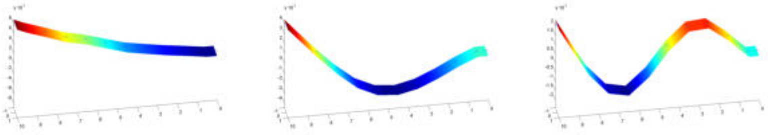


Validation of the beam electrical analogue

Admittance measurement at the "free end" of the electrical network

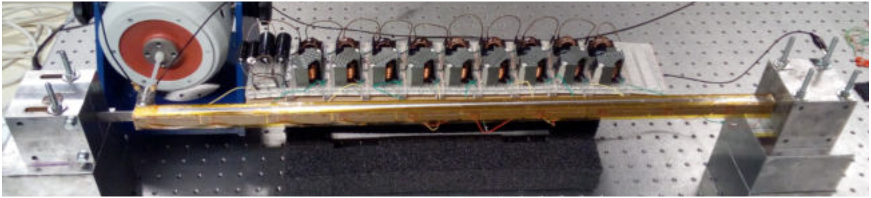




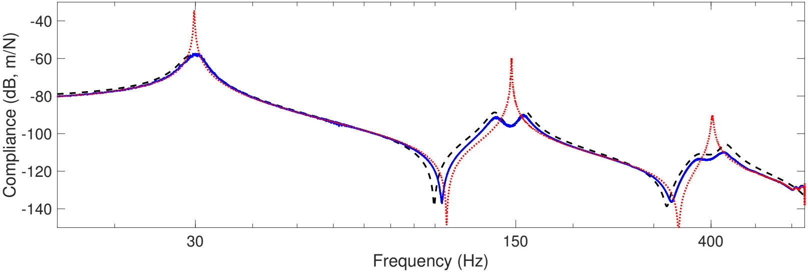


→ **Experimental modal analysis** of the analogue twin

Electromechanical coupling through piezoelectric patches

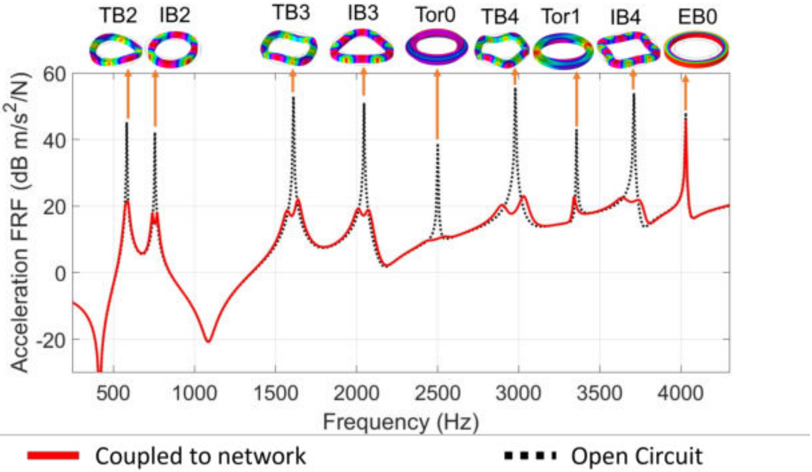


(...) Exp. without network, (—) Exp. with analogous network, (−−) Model



→ **Multimodal damping** observed on the mechanical response

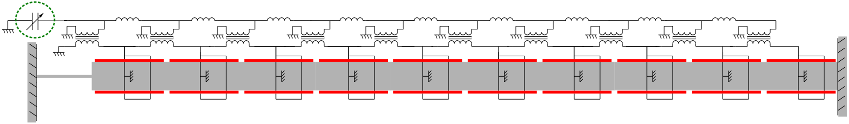
In-plane and out-of-plane motions of a thick ring : 2 independent networks



✍ *A. Luo, B. Lossouarn, A. Erturk, Journal of Sound and Vibration (under review)*

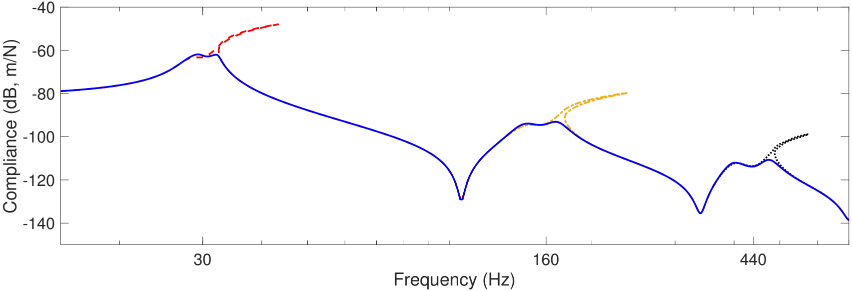
Outline

Mitigation of multiple nonlinear resonances



Nonlinear capacitor required to maintain equal-peak : $v = \frac{1}{C^{\text{end}}} q + \frac{1}{C_{\text{NL}}} q^3$

$\rightarrow$ Theoretical tuning : $\frac{1}{C_{\text{NL}}} = 2 \left(\frac{L}{m}\right)^2 k_{\text{NL}}$

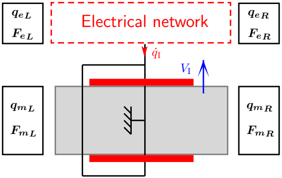


$\rightarrow$ ✍ *B. Lossouarn, G. Kerschen, J.-F. Deü, Jounal of Sound and Vibration, 2021*

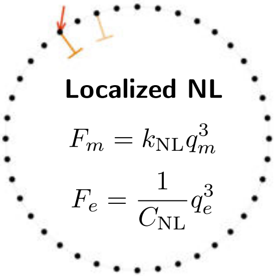
Nonlinear model for the electromecanical system

Mass and stiffness matrices

$\rightarrow$ Combination of mechanical and electrical DOFs



$$\begin{bmatrix} \boldsymbol{M_m} & 0 \\ 0 & \boldsymbol{M_e} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}_{\boldsymbol{m}} \\ \ddot{\boldsymbol{q}}_{\boldsymbol{e}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{C_m} & 0 \\ 0 & \boldsymbol{C_e} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{q}}_{\boldsymbol{m}} \\ \dot{\boldsymbol{q}}_{\boldsymbol{e}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K_m} + \boldsymbol{K_c}\boldsymbol{K_c}^{\boldsymbol{T}} & \tilde{\boldsymbol{K}}_{\boldsymbol{c}} \\ \tilde{\boldsymbol{K}}_{\boldsymbol{c}}^{\boldsymbol{T}} & \boldsymbol{K_e} \end{bmatrix} \begin{bmatrix} \boldsymbol{q_m} \\ \boldsymbol{q_e} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F_m} \\ \boldsymbol{F_e} \end{bmatrix}$$

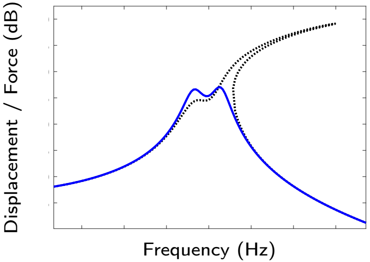


Localized NL

$$F_m = k_{\mathrm{NL}} q_m^3$$

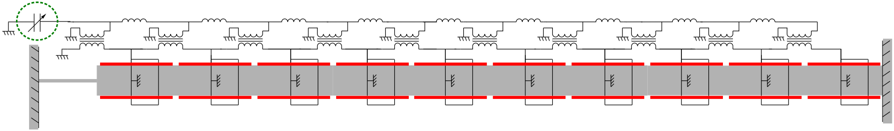
$$F_e = \frac{1}{C_{\mathrm{NL}}} q_e^3$$

Harmonic balance method



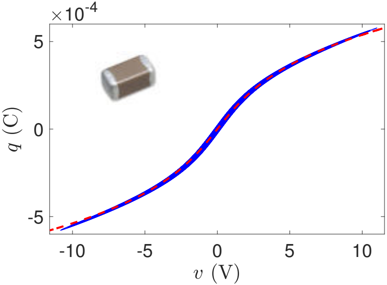
Nonlinear capacitance from a passive electrical component

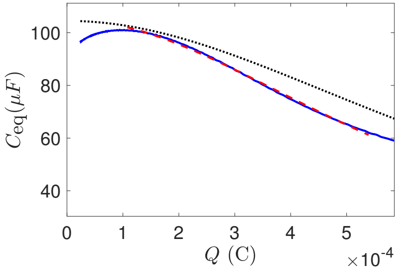
Variable electrical resonance due to variable capacitance



Nonlinear capacitor : $v = \frac{1}{C^{\text{end}}} q + \frac{1}{C_{\text{NL}}} q^3 \quad \Rightarrow \quad C(Q) \approx \frac{1}{\frac{1}{C^{\text{end}}} + \frac{3Q^2}{4C_{\text{NL}}}}$

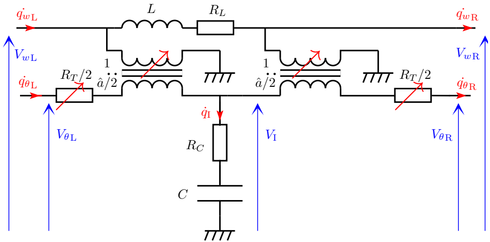
→ Solution = **Multilayer Ceramic Capacitor**





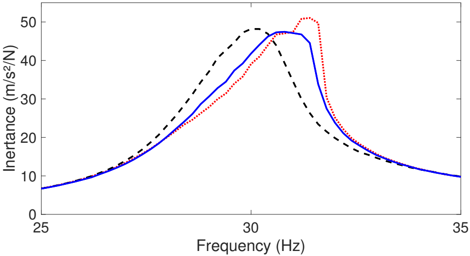
Practical limits for a full experimental validation

Nonlinearities in the transformers



Difficult compromise between nonlinearities and internal resistance

→ Nonlinear electrical network...
...even with a "linear" capacitance

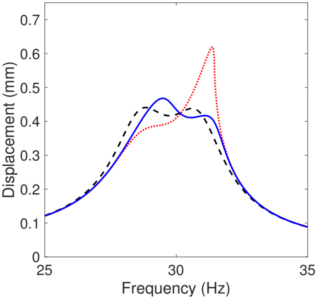


(−−) Linear (...) No NL capa, (—) With NL capa

Outline

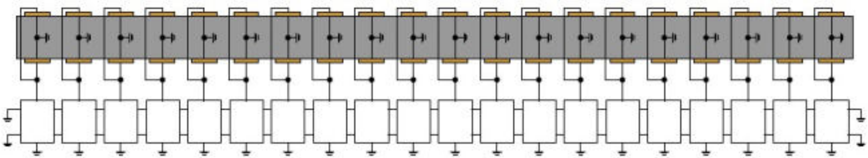
Analogue twin < Hybrid twin < Digital twin ?

Resonant shunt damping extended to nonlinear structures
→ **Principle of similarity** with passive components



Limits
→ Passive components come with their own resistance and nonlinearities

Perspectives
→ Hybrid twin for optimized tuning ?



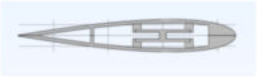
→ ✍ *G. Raze, J. Dietrich, B. Lossouarn, G. Kerschen, Mech. Systems and Signal Processing, 2022*

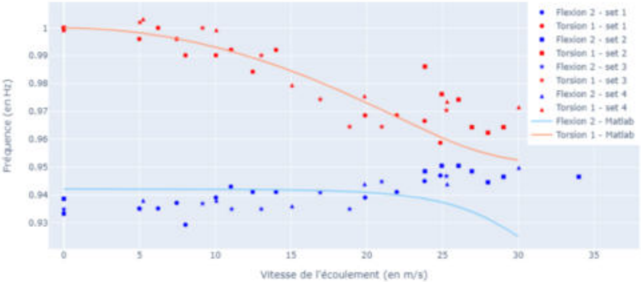
Hybrid shunts for control of flutter ? (with X. Amandolese)

1.5 m wing equipped with 26 patches (B. Prieur, Internship)

→ **Piezoelectric damping** implemented with no flow

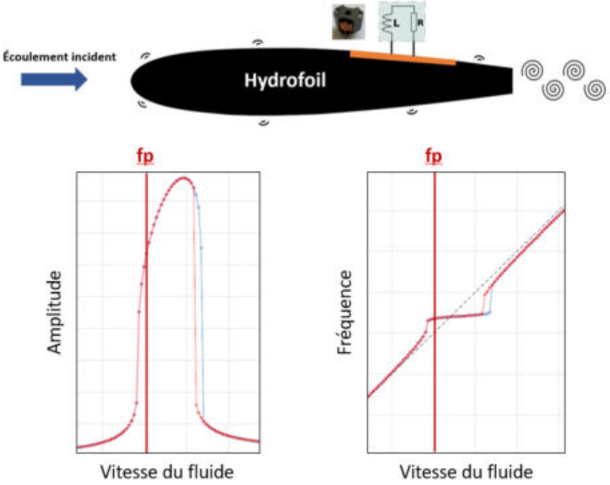
→ First experiments in **wind tunnel**





Hybrid shunts for control of VIV ? (ANR Astrid HYDRAVIB)

Hydrofoils subjected to vortex-induced vibrations : Cnam+ENSAM+Ecole Navale
→ Previous experimental results in **water tunnel** (Y. Watine)



→ First objective = explicit formula for **optimal resonant shunts** (A. Haudeville)

Merci pour votre attention !

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