

A Nonlinear Piezoelectric Shunt Absorber with 2:1 Internal Resonance

GDR EX-MODELI – Nov. 9-10, 2023, Besançon

Olivier THOMAS

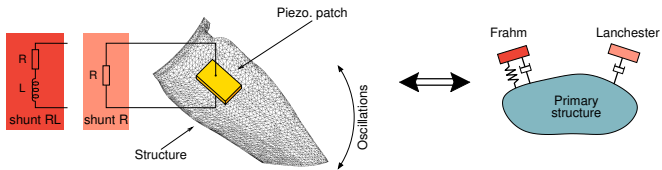
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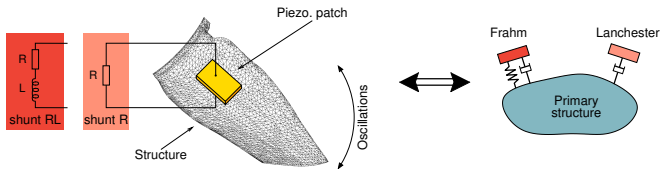
Piezoelectric shunts



▷ Principle

- Piezoelectric patches: mechanical / electrical energy conversion
- electric circuits:
 - ↔ mechanical energy dissipation,
 - ↔ counter vibrations.

Piezoelectric shunts



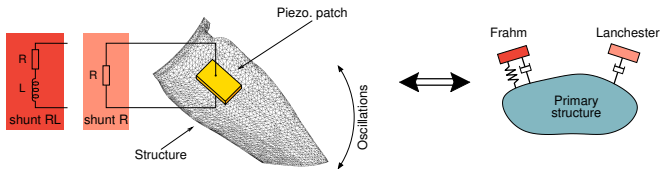
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▷ Basic shunts *electric analogs of mechanical dampers* [Hagood & von Flotow, JSV 1991]

- Resistive shunt: the energy is dissipated in the electrical resistance
- Resonant shunt: electric equivalent of a tuned mass damper

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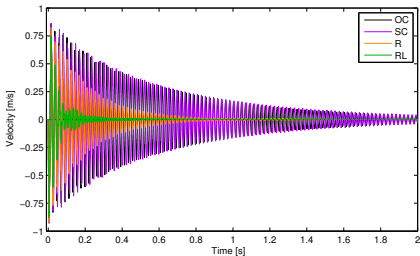
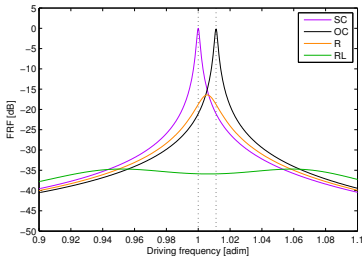
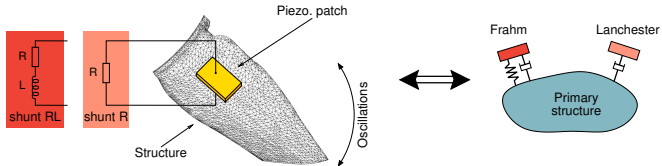
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- Resistive shunt: the energy is dissipated in the electrical resistance
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▷ Passive devices

- no external energy injected into the system (always stable)
- powered electronic components (semi-passive)

Resistive and resonant shunt responses

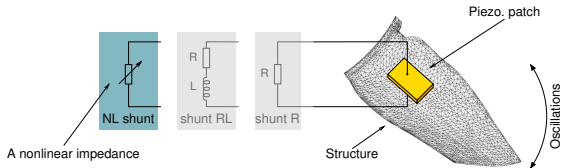


▷ Resonant / resistive shunts [Thomas et al SMS 2012]

Good performance, but fully linear. . .

↪ why not including nonlinearities in the shunt ?

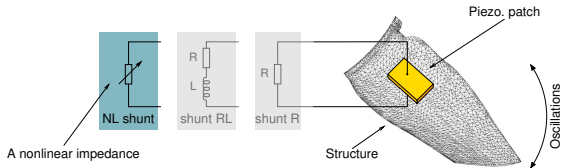
Today's talk and state of the art



▷ Our work:

- including nonlinearities in the shunt
- using 1:2 internal resonance for a saturation effect

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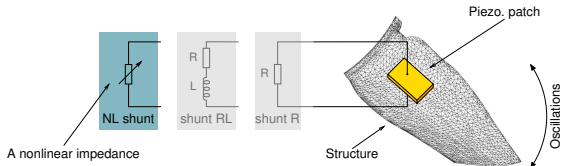
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- Some works using 1:2 internal resonance exist in active control [Nayfeh & Pai teams, 1996-]
- Our solution is a **semi-passive shunt**

Today's talk and state of the art



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▷ Other families of works on piezoelectric nonlinear dampers:

- Nonlinear Energy Sinks (NES) [Erturk team, SMS 2018], [Dekemele, Smart 2023]
- Nonlinear tuned vibration absorbers: a mirror of a primary nonlinear structure [Lossouarn, Raze, Kerschen, Deü 2018–]
- Synchronized Switch Damping [Guyomar team 2000–], . . .
- A non-smooth shunt (using a diode) [Shami, Thomas, Giraud-Audine, ND, 2023]

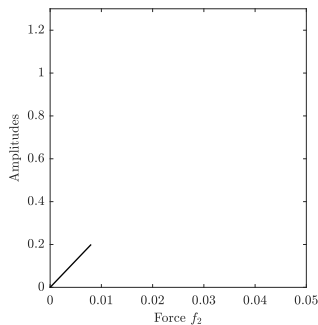
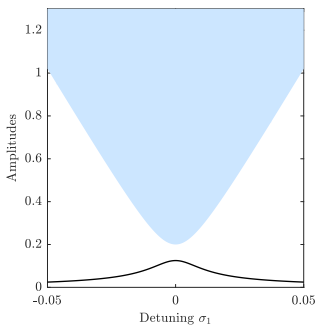
Two oscillators coupled in 1:2 internal resonance

$$\begin{cases} \ddot{x}_1 + 2\mu_1\dot{x}_1 + \omega_1^2 x_1 + \alpha_1 x_1 x_2 = 0 & \leftarrow \text{secondary oscillator} \\ \ddot{x}_2 + 2\mu_2\dot{x}_2 + \omega_2^2 x_2 + \alpha_2 x_1^2 = f_2 \cos \Omega t & \leftarrow \text{primary oscillator} \end{cases}$$

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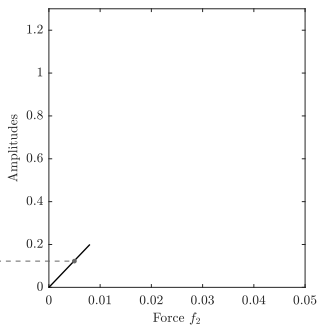
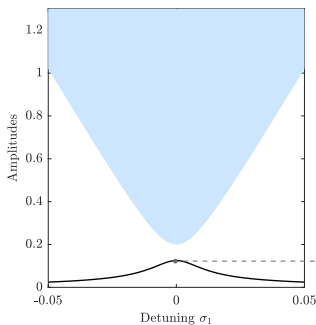
$$\boxed{\omega_1 \simeq \omega_2/2, \Omega \simeq \omega_2} \Rightarrow \begin{cases} x_1(t) = 0 \\ x_2(t) = a_2 \cos(\Omega t + \varphi_2) \end{cases}$$



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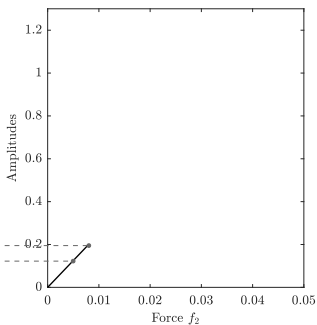
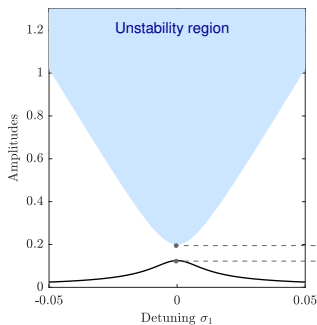
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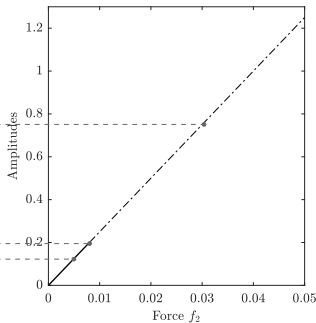
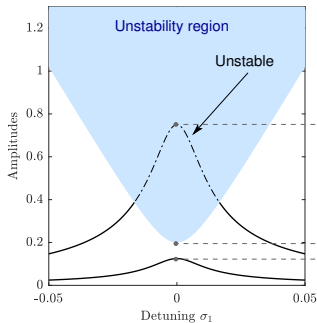
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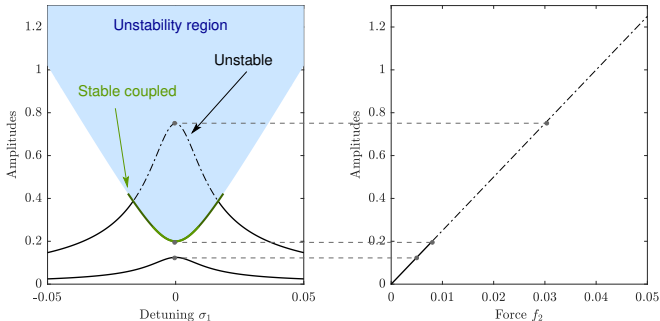
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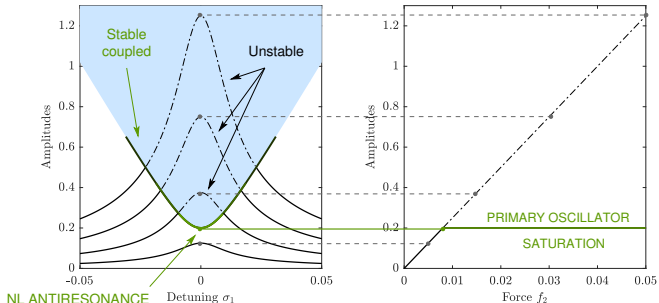
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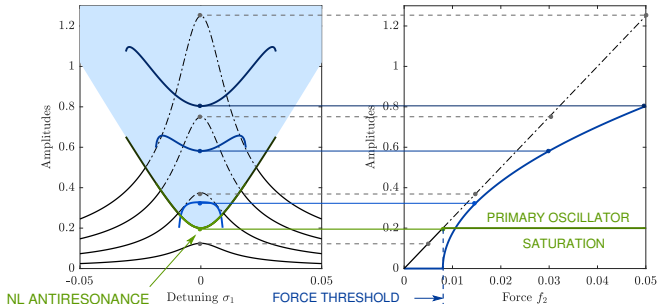
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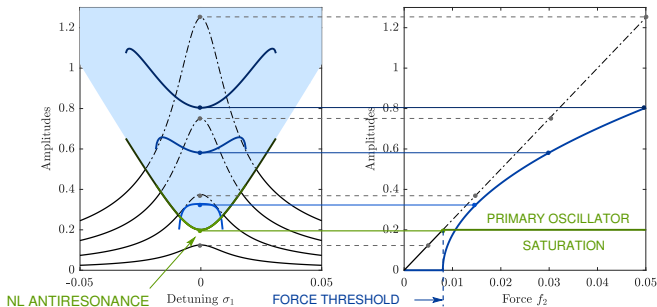
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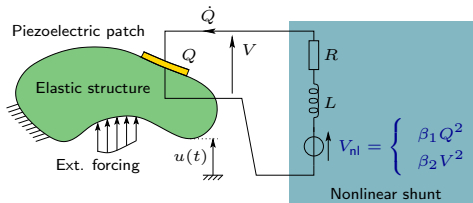
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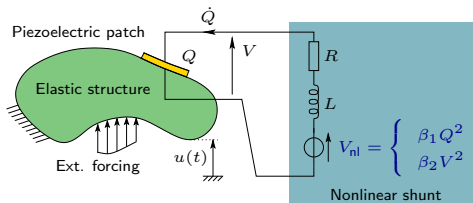


- Below the threshold $f_2 < f_2^*$: linear uncontrolled response;
- Above the threshold $f_2 > f_2^*$: nonlinear antiresonance + saturation of the amplitude thanks to a subharmonic transfer of energy

Coupling a mechanical mode with an electrical circuit



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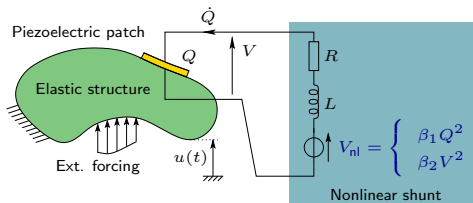


- One eigenmode expansion + piezoelectric coupling + resonant circuit:

$$\mathbf{u}(t) = \Phi_i q_i(t) \Rightarrow \begin{cases} \ddot{q}_i + 2\xi_i \hat{\omega}_i \dot{q}_i + \hat{\omega}_i^2 q_i + \frac{\theta_i}{m_i C_{pi}} Q = \frac{F_i}{m_i} \cos \Omega t \\ \ddot{Q} + 2\xi_e \omega_e \dot{Q} + \omega_e^2 Q + \frac{\theta_i}{LC_{pi}} q_i + \frac{V_{nl}}{L} = 0 \end{cases}$$

$q_i(t)$ mechanical modal coordinate; $Q(t)$ electric charge; $(\hat{\omega}_i, \omega_e)$ mechanical and electrical natural frequencies: $\omega_e = 1/\sqrt{LC} \simeq \hat{\omega}_i/2$

Coupling a mechanical mode with an electrical circuit



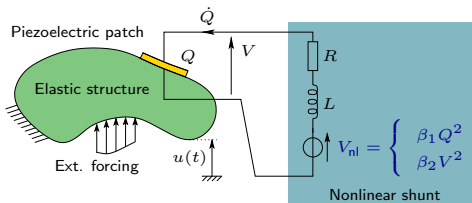
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- **Linear piezoelectric coupling**
- **Nonlinear voltage source**
 - $\rightsquigarrow V_{nl} = \beta_1 Q^2 \Leftrightarrow$ a nonlinear capacitor (no simple passive component)
 - $\rightsquigarrow V_{nl} = \beta_2 V^2$: easier in practice (see after).

Back to the canonical oscillators

- Expansion onto electromechanical modes (ψ_1, ψ_2):

$$\begin{pmatrix} q_i \\ Q \end{pmatrix} = \Psi \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\Rightarrow \begin{cases} \ddot{x}_1 + 2\mu_1\dot{x}_1 + \omega_1^2 x_1 + \Lambda_1 x_1^2 + \Lambda_2 x_1 x_2 + \Lambda_3 x_2^2 = 0 \\ \ddot{x}_2 + 2\mu_2\dot{x}_2 + \omega_2^2 x_2 + \Lambda_4 x_1^2 + \Lambda_5 x_1 x_2 + \Lambda_6 x_2^2 = f_2 \cos \Omega t \end{cases}$$

with $q_i \simeq x_2$ (\simeq mechanical mode) & $Q \simeq x_1$ (\simeq electrical mode) because the piezoelectric coupling is often low.

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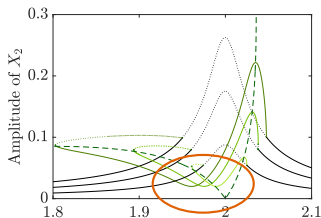
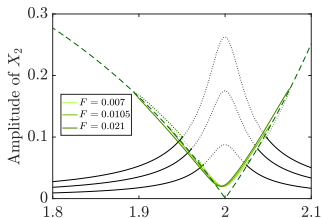
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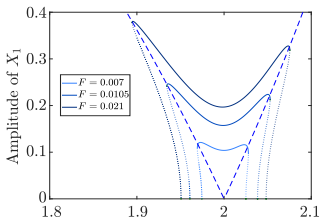
↪ but also non-resonant terms of large value (Λ_1 in particular).

the **non-resonant terms** are responsible of a **detuning of the nonlinear antiresonance** as a function of the amplitude of forcing

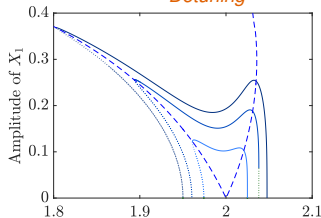
Effect of quadratic non resonant terms



Detuning



Resonant terms only



All quadratic NL terms

detuning of the nonlinear antiresonance \Rightarrow loss of saturation of $x_2(t)$

Normal form transform & cubic terms

- From basic normal form theory, non-resonant quadratic terms are equivalent to cubic terms in the normal form:

$$\begin{cases} \ddot{x}_1 + \omega_1^2 x_1 + g_{11}^1 x_1^2 + g_{12}^1 x_1 x_2 + g_{22}^1 x_2^2 + \sum_{i,j,k} h_{ijk}^1 x_i x_j x_k = 0 \\ \ddot{x}_2 + \omega_2^2 x_2 + g_{11}^2 x_1^2 + g_{12}^2 x_1 x_2 + g_{22}^2 x_2^2 + \sum_{i,j,k} h_{ijk}^2 x_i x_j x_k = f_2 \cos \Omega t \end{cases}$$

$$\begin{pmatrix} x_p \\ \dot{x}_p \end{pmatrix} = \begin{pmatrix} r_p \\ \dot{r}_p \end{pmatrix} + \begin{pmatrix} \mathcal{P}_p^{(3)}(r_i, \dot{r}_i) \\ \mathcal{Q}_p^{(3)}(r_i, \dot{r}_i) \end{pmatrix}$$

$$\Rightarrow \begin{cases} \ddot{r}_1 + \omega_1^2 r_1 + g_{12}^1 x_1 x_2 + \underbrace{(h_{111}^1 + A_{111}^1) r_1^3 + B_{111}^1 r_1 \dot{r}_1^2 + \dots}_{\text{cubic terms}} = 0 \\ \ddot{r}_2 + \omega_2^2 r_2 + g_{11}^2 x_1^2 + \underbrace{(h_{112}^2 + A_{111}^1 - D_{112}^2) r_1^2 r_2 + \dots}_{\text{cubic terms}} = f_2 \cos \Omega t \end{cases}$$

with $A_{ijl}^k, B_{ijl}^k, D_{ijl}^k$ functions of quadratic **non-resonant** coefficients.

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$$\begin{cases} \ddot{x}_1 + \omega_1^2 x_1 + g_{11}^1 x_1^2 + g_{12}^1 x_1 x_2 + g_{22}^1 x_2^2 + \sum_{i,j,k} h_{ijk}^1 x_i x_j x_k = 0 \\ \ddot{x}_2 + \omega_2^2 x_2 + g_{11}^2 x_1^2 + g_{12}^2 x_1 x_2 + g_{22}^2 x_2^2 + \sum_{i,j,k} h_{ijk}^1 x_i x_j x_k = f_2 \cos \Omega t \end{cases}$$

$$\begin{pmatrix} x_p \\ \dot{x}_p \end{pmatrix} = \begin{pmatrix} r_p \\ \dot{r}_p \end{pmatrix} + \begin{pmatrix} \mathcal{P}_p^{(3)}(r_i, \dot{r}_i) \\ \mathcal{Q}_p^{(3)}(r_i, \dot{r}_i) \end{pmatrix}$$

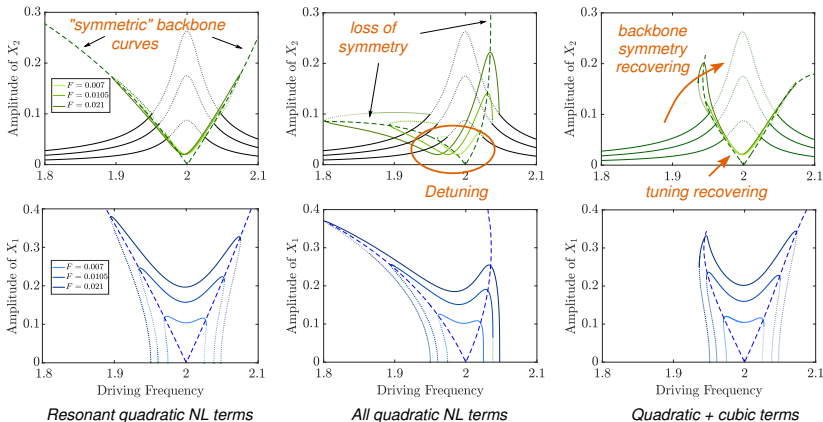
$$\Rightarrow \begin{cases} \ddot{r}_1 + \omega_1^2 r_1 + g_{12}^1 x_1 x_2 + \underbrace{(h_{111}^1 + A_{111}^1) r_1^3 + B_{111}^1 r_1 \dot{r}_1^2 + \dots}_{\text{cubic terms}} = 0 \\ \ddot{r}_2 + \omega_2^2 r_2 + g_{11}^2 x_1^2 + \underbrace{(h_{112}^2 + A_{111}^1 - D_{112}^2) r_1^2 r_2 + \dots}_{\text{cubic terms}} = f_2 \cos \Omega t \end{cases}$$

with $A_{ijl}^k, B_{ijl}^k, D_{ijl}^k$ functions of quadratic **non-resonant** coefficients.

- with **cubic terms** in the initial oscillators

Adding a **cubic term** in the shunt may help
cancelling the effect of **quadratic non resonant** terms

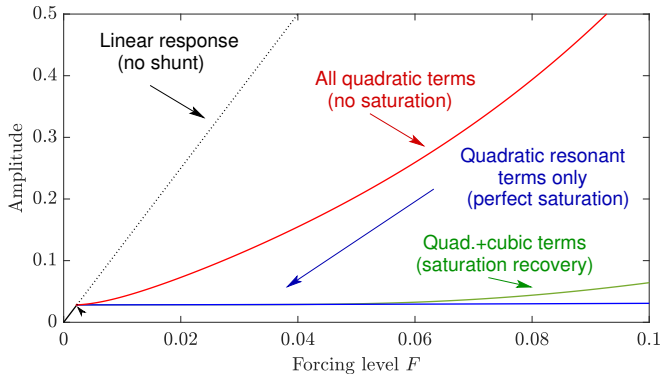
Annihilation of the effect of non-resonant terms



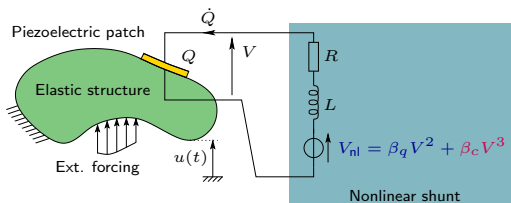
We tune the cubic h_{ijl}^k terms to cancel the effect of the quadratic non-resonant terms on the **backbone curves** of the coupled 1:2 nonlinear modes:

$$\text{explicit formula } h_{ijl}^k = f(g_{ij}^k).$$

Saturation recovery



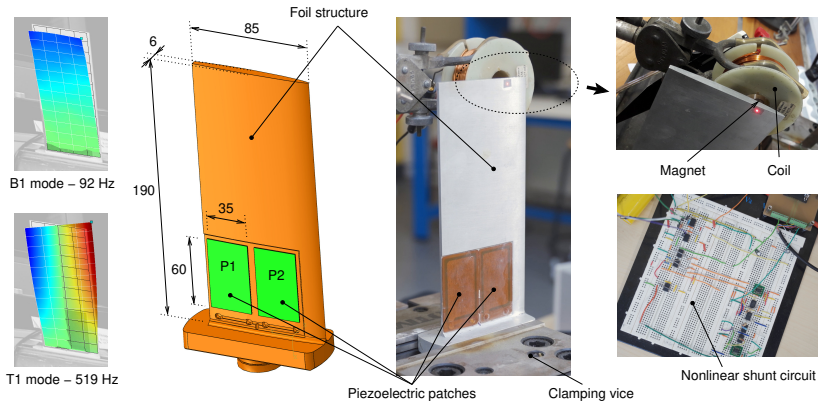
Quadratic & cubic shunt



- We add both quadratic and cubic NL terms in the shunt
- With only one degree of freedom (the value of coef. β_c), we are able to cancel only **the largest quadratic non resonant term** (g_{11}^1), leading to:

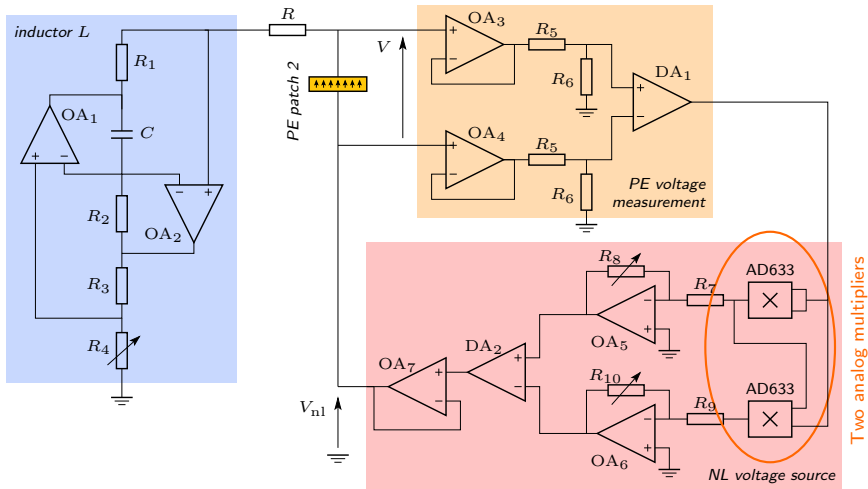
$$\beta_c = \frac{10}{9} \beta_c^2$$

Proof of concept on a hydroelastic foil model



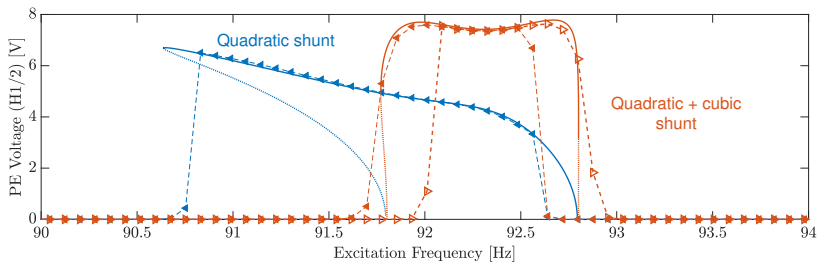
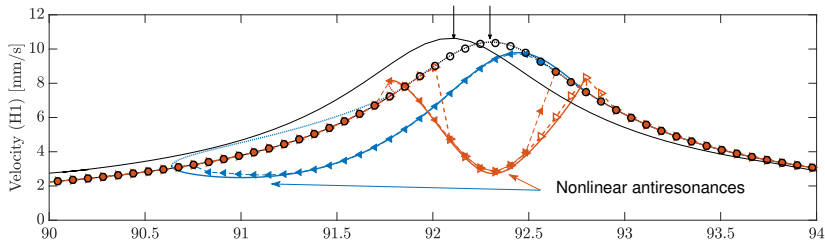
Nonlinear shunt tuned to damp the first bending mode (92 Hz)

Analog electronic circuit with multipliers



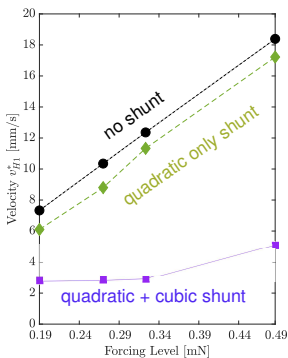
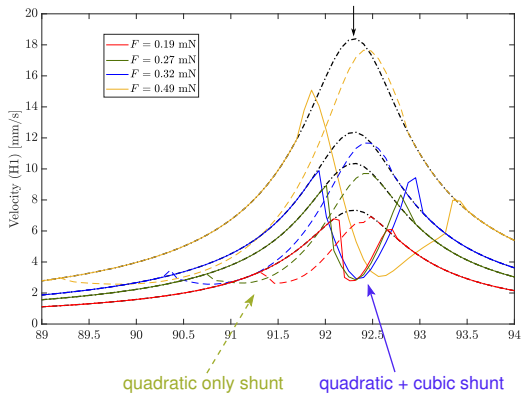
Two analog multipliers

Experiments vs. theory



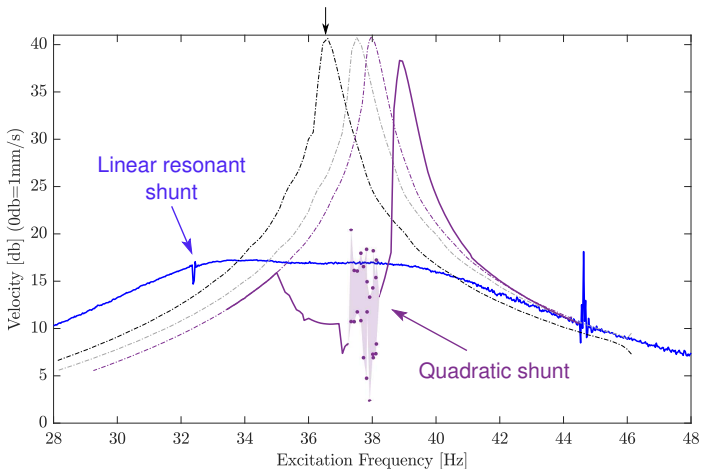
Perfect match !

Saturation effect



The cubic shunt clearly helps enhancing the saturation response range

Comparison to a linear resonant shunt



Because of the saturation, above a certain threshold (tunable) the nonlinear shunt is **better** than the linear resonant (RL) shunt.

Conclusions & perspectives

- The use of a 2:1 internal resonance with a **piezoelectric shunt** to obtain a control device based on **saturation** is fully demonstrated, theoretically & experimentally with analog electronic components;
- An unexpected effect of **non resonant** quadratic terms (*detuning* and *quasiperiodic responses*) was cancelled thanks to **cubic terms & normal form theory**;
- All optimization results are detailed in publications. Especially, the **figure of merit** of the shunt, for the saturation amplitude, is $\xi_e / (\kappa \beta_q)$ with ξ_e [-] electrical damping ratio, κ [-] piezoelectric coupling factor & β_c [V^{-1}] the quadratic voltage constant.

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- Z. A. Shami, C. Giraud-Audine, and O. Thomas. "A nonlinear piezoelectric shunt absorber with a 2:1 internal resonance : Theory" *Mechanical Systems and Signal Processing*, **170**:108768, 2022.
- Z. A. Shami, C. Giraud-Audine, and O. Thomas. "A nonlinear piezoelectric shunt absorber with 2:1 internal resonance: experimental proof of concept" *Smart Materials and Structures*, **31**:035006, 2022.
- Z. A. Shami, Y. Shen, C. Giraud-Audine, C. Touzé, and O. Thomas. "Nonlinear dynamics of coupled oscillators in 1 :2 internal resonance : effects of the non-resonant quadratic terms and recovery of the saturation effect" *Meccanica*, **57**:2701-2731, 2022.
- Z. A. Shami, C. Giraud-Audine, and O. Thomas. "Saturation correction for a piezoelectric shunt absorber based on 2:1 internal resonance using a cubic nonlinearity" *Smart Materials and Structures*, **32**:055024, 2023.
- Z. A. Shami, C. Giraud-Audine, and O. Thomas. "A nonlinear tunable piezoelectric resonant shunt using a bilinear component : theory and experiment". *Nonlinear Dynamics*, **111**:2701-273, 2023.

THANK YOU !

Une annonce



Localisation du poste :
Campus de Lille

Informations complémentaires :
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Unité d'affectation : **laboratoire LISPEN**

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