Equilibrium of a non-compressible cable subjected to unilateral constraints

Applications to dynamics

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Overview

PhD Thesis - ED MEGA - C.H. Lamarque (LTDS) & V. Acary (INRIA)

Dynamics of a translating cable subjected to unilateral constraints, friction and punctual loads

Technical context

- Cable-cars were aimed to provide an alternative for public transportation
- Maintenance of existing infrastructures

Scientifical knot

- Lack of numerical tools dedicated to this particular systems
- Few objective comparisons between models

Expectations

- Depict some automatic designs considering the full dynamics of an installation
- Explain high amplitude of displacement existing in reality

Photo credit : POMA, Eiffage

Cable Mechanics

Derived from curvilinear domains : Slender structure $\longrightarrow$ Parametrized curve

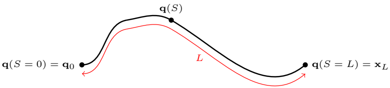
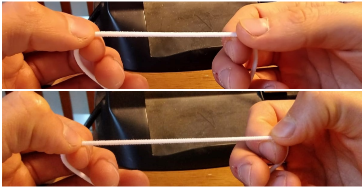
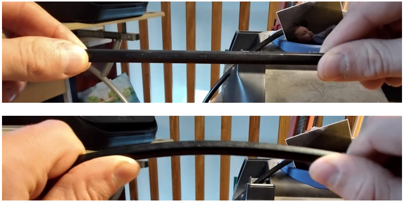


Figure: A cable and its parametrization

Two key mechanisms :





$$\text{Strain : } \quad \varepsilon(S) = \left\|\mathbf{q}'(S)\right\| - 1 \qquad (1)$$

$$\text{Bending : } \quad \kappa(S) = \omega'(S) = \alpha'(S) - \alpha_0'(S) \qquad (2)$$

Cable Mechanics

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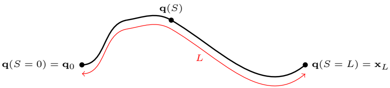

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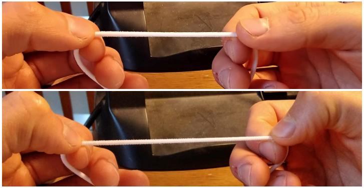

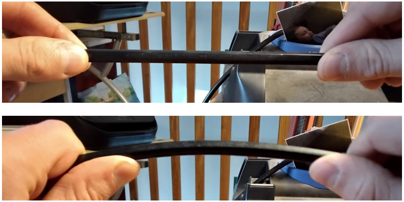

$$\text{Strain}: \quad \varepsilon(S) = \left\|\mathbf{q}'(S)\right\| - 1 \longrightarrow \text{Positive since cables only support traction} \quad (1)$$

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Cable Mechanics

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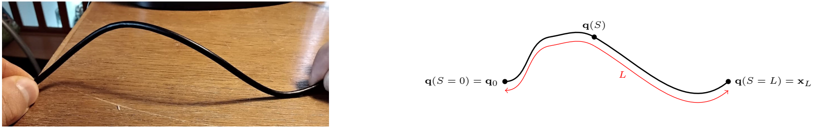

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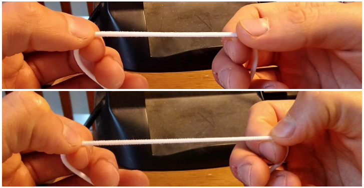

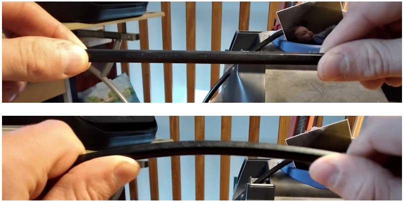

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$$\text{Bending : } \quad \kappa(S) = \omega'(S) = \alpha'(S) - \alpha_0'(S) \longrightarrow \text{Useless for the cable case} \qquad (2)$$

Equations of motion

Derived from Lagrangian mechanics

- Kinetic energy
- Elastic energy
- Work of external forces
- Non-compression condition

$$\mathcal{L}^* \left(\dot{\mathbf{q}}, \mathbf{q}', \mathbf{q}, \lambda\right) = \frac{\rho}{2} \dot{\mathbf{q}} \cdot \dot{\mathbf{q}} \tag{3}$$

$$\begin{cases} \mathbf{0} = \dfrac{\partial \mathcal{L}^*}{\partial \mathbf{q}} - \dfrac{\mathrm{d}}{\mathrm{d}S} \dfrac{\partial \mathcal{L}^*}{\partial \mathbf{q}'} - \dfrac{\mathrm{d}}{\mathrm{d}t} \dfrac{\partial \mathcal{L}^*}{\partial \dot{\mathbf{q}}} - \dfrac{\partial \mathbf{a}}{\partial \mathbf{q}}^\top \lambda + \dfrac{\mathrm{d}}{\mathrm{d}S} \left[\dfrac{\partial \mathbf{a}}{\partial \mathbf{q}'}^\top \lambda \right] + \dfrac{\mathrm{d}}{\mathrm{d}t} \left[\dfrac{\partial \mathbf{a}}{\partial \dot{\mathbf{q}}}^\top \lambda \right] \\ \mathbf{0} \leqslant \mathbf{a}(\mathbf{q}, \mathbf{q}', \dot{\mathbf{q}}) \perp \lambda \geqslant 0 \end{cases} \tag{4}$$

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t} \left(\rho \dot{\mathbf{q}}\right) = \dfrac{\mathrm{d}}{\mathrm{d}S} \left(\left[EA \left(\left\| \mathbf{q}' \right\| - 1 \right) + \lambda \right] \dfrac{\mathbf{q}'}{\left\| \mathbf{q}' \right\|} \right) + \mathbf{f}_e \\ 0 \leqslant \left\| \mathbf{q}' \right\| - 1 \perp \lambda \geqslant 0 \end{cases} \tag{5}$$

What if $\mathbf{a}$ were to embed more constraints ?..

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$$\begin{cases}\mathbf{0}=\dfrac{\partial\mathcal{L}^*}{\partial\mathbf{q}}-\dfrac{\mathrm{d}}{\mathrm{d}S}\dfrac{\partial\mathcal{L}^*}{\partial\mathbf{q}'}-\dfrac{\mathrm{d}}{\mathrm{d}t}\dfrac{\partial\mathcal{L}^*}{\partial\dot{\mathbf{q}}}-\dfrac{\partial\mathbf{a}}{\partial\mathbf{q}}^\top\lambda+\dfrac{\mathrm{d}}{\mathrm{d}S}\left[\dfrac{\partial\mathbf{a}}{\partial\mathbf{q}'}^\top\lambda\right]+\dfrac{\mathrm{d}}{\mathrm{d}t}\left[\dfrac{\partial\mathbf{a}}{\partial\dot{\mathbf{q}}}^\top\lambda\right]\\ \mathbf{0}\leqslant\mathbf{a}(\mathbf{q},\mathbf{q}',\dot{\mathbf{q}})\perp\lambda\geqslant 0\end{cases} \tag{4}$$

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What if $\mathbf{a}$ were to embed more constraints ?..

Weak form of the equilibrium

Here we present a finite element for a cable system that can solve 5. Dropping the constraints for a while, let us have a look to

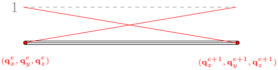
$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{v} = \frac{\mathrm{d}}{\mathrm{d}S}\left[EA\left(\left\|\mathbf{q}'\right\| - 1\right)\right] + \mathbf{f}_e \tag{6}$$

Considering a suitable weight function φ, the equilibrium of an arbitrary cable segment of length L_e reads

$$\int_0^{L_e} \rho\dot{\mathbf{v}} \cdot \varphi \,\mathrm{d}S - \int_0^{L_e} \left(\left[EA\left(\left\|\mathbf{q}'\right\| - 1\right)\right] + \mathbf{f}_e\right) \cdot \varphi \,\mathrm{d}S = 0 \tag{7}$$

Taking an interpolation as

$$\varphi = \mathbf{N}\Phi_e \;;\; \mathbf{v} = \mathbf{N}\mathbf{v}_e \;;\; \mathbf{q} = \mathbf{N}\mathbf{q}_e \tag{8}$$



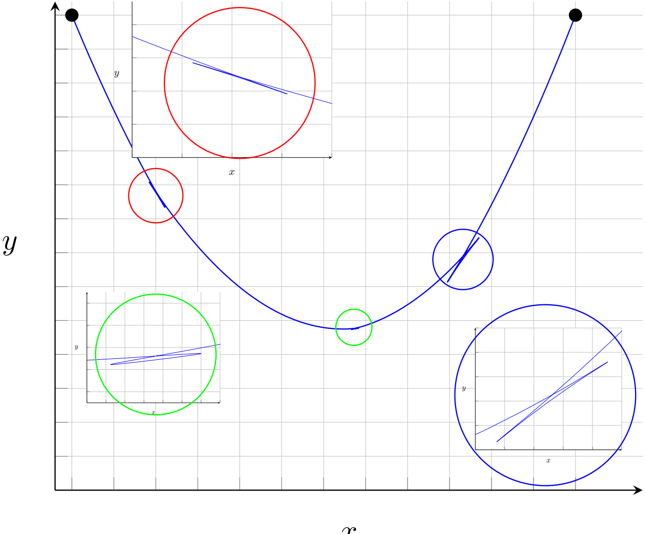
We obtain after simplification

$$\begin{cases} \mathbf{M}_e\dot{\mathbf{v}_e} + \mathbf{K}(\mathbf{q}_e)\mathbf{q}_e - \mathbf{f}_e = \mathbf{0} \\ \dot{\mathbf{q}_e} = \mathbf{v}_e \end{cases} \tag{9}$$

which can be assembled with regards to boundary conditions and mesh considered.

Compression issue with adding constraints

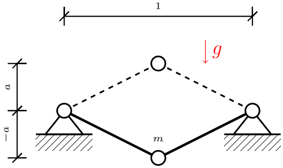
Even in statics, compressed equilibrium can occur without further considerations...
Why ?

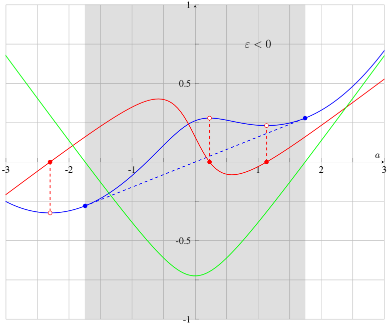


Compression issue with adding constraints

The global energy of the latter is given by

$$\begin{cases} 0 = \dfrac{4a}{L^2\sqrt{1+4a^2}}\left(\sqrt{1+4a^2}-L\right)-G \\ \end{cases} \tag{10}$$



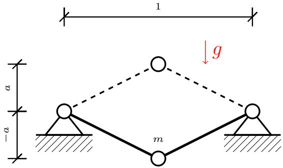


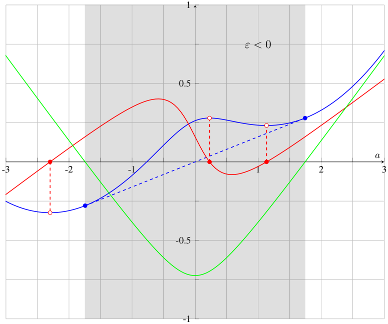
Potential energy of the weighing cable

Compression issue with adding constraints

The global energy of the latter is given by

$$\begin{cases} 0 = \dfrac{4a}{L^2\sqrt{1+4a^2}}\left(\sqrt{1+4a^2}-L\right) - G - \lambda\dfrac{4a}{L\sqrt{1+4a^2}} \\ 0 \leqslant \lambda \perp \left(\dfrac{\sqrt{1+4a^2}}{L}-1\right) \geqslant 0 \end{cases} \tag{10}$$





Potential energy of the weighing cable

One strategy

One way to cope with this issue numerically is the combined use of iterative solver with a conditional expression of stiffness matrices

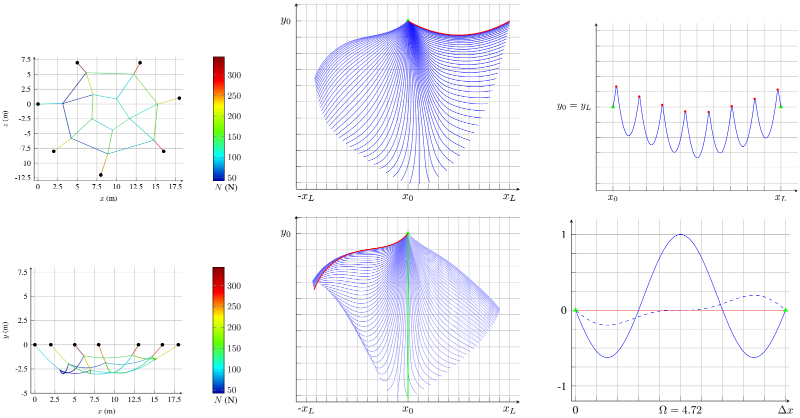
$$\varepsilon_e(S) = \left\|\mathbf{N}'(S)\mathbf{q}_e\right\| - 1 \tag{11}$$

$$\mathbf{K}_e = \begin{cases} EA\displaystyle\int_0^{Ł_e} \frac{\mathbf{N}'(S)^\top \mathbf{N}'(S)}{1+|\varepsilon_e(S)|^{-1}} \mathrm{d}S & ; \quad \varepsilon_e > 0 \\ 0 & ; \quad \varepsilon_e \leqslant 0 \end{cases} \tag{12}$$

$$\Delta\mathbf{K}_e = \begin{cases} \mathbf{K}_e + EA\displaystyle\int_0^{Ł_e} \frac{\mathbf{N}'(S)^\top \mathbf{N}'(S)\mathbf{q}_e {\mathbf{q}_e}^\top \mathbf{N}'(S)^\top \mathbf{N}'(S)}{(1+|\varepsilon_e(S)|)^3} \mathrm{d}S & ; \quad \varepsilon_e > 0 \\ EA\displaystyle\int_0^{Ł_e} \frac{\mathbf{N}'(S)^\top \mathbf{N}'(S)}{1+|\varepsilon_e(S)|^{-1}} \mathrm{d}S & ; \quad \varepsilon_e \leqslant 0 \end{cases} \tag{13}$$

The latter allows tension only computation and has various applications.

Examples


z (m)
x (m)
N (N)
y (m)
x (m)
N (N)
y_0
$-x_L$
x_0
x_L
$y_0 = y_L$
x_0
x_L
y_0
$-x_L$
x_0
x_L
1
0
-1
0
$\Omega = 4.72$
Δx

Local kinematics

The relative velocity between a node M and an obstacle point M' at the contact may be written as

$$\mathbf{u}(M, M') = \mathbf{v}(M) - \mathbf{v}_{\text{obs}}(M') \tag{14}$$

which is decomposed along a local basis as

$$\mathbf{u}(M, M') \rightarrow \begin{cases} \mathbf{n} & : \mathbf{u}_N(M, M') = \mathbf{H}_N(M, M')\mathbf{u}(M, M') \\ \mathbf{t} & : \mathbf{u}_T(M, M') = \mathbf{H}_T(M, M')\mathbf{u}(M, M') \end{cases} \tag{15}$$

where $\mathbf{t} = [\mathbf{t}_1, \mathbf{t}_2]^\top$ and $\mathbf{n} \perp \mathbf{t}_1 \perp \mathbf{t}_2 \perp \mathbf{n}$.
The dynamics can be considered both in the local or in the global frame via

$$\mathbf{M}\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{f} + \mathbf{p} \leftrightarrow \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = \tilde{\mathbf{f}} + \widehat{\mathbf{W}}\mathbf{r} \tag{16}$$

where $\widehat{\mathbf{W}} = \begin{bmatrix} \widehat{\mathbf{W}}_{NN} & \widehat{\mathbf{W}}_{NT} \\ \widehat{\mathbf{W}}_{TN} & \widehat{\mathbf{W}}_{TT} \end{bmatrix}$ is the Delassus operator given by

$$\widehat{\mathbf{W}}_{NN} = \mathbf{H}_N\hat{\mathbf{M}}\mathbf{H}_N \;;\; \widehat{\mathbf{W}}_{NT} = \mathbf{H}_N\hat{\mathbf{M}}\mathbf{H}_T \;;\; \widehat{\mathbf{W}}_{TN} = \mathbf{H}_T\hat{\mathbf{M}}\mathbf{H}_N \;;\; \widehat{\mathbf{W}}_{TT} = \mathbf{H}_T\hat{\mathbf{M}}\mathbf{H}_T \tag{17}$$

Coulomb friction

Let us introduce the following sets

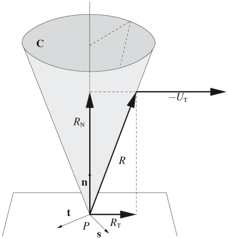
$$\mathbf{K} = \left\{\mathbf{r} \in \mathbb{R}^3 \ , \ \|\mathbf{r}_T\| \leqslant \mu \mathbf{r}_N\right\} \tag{18}$$

$$\mathbf{K}^* = \left\{\mathbf{u} \in \mathbb{R}^3 \ , \ \forall \mathbf{r} \in \mathbf{K} \ , \ \mathbf{u} \cdot \mathbf{r} \geqslant 0\right\} \tag{19}$$

(20)

The contact is embedded into the following possibilities

- No contact: $\mathbf{r} = 0$ and $\mathbf{u}_N \geqslant 0$
- Sticking: $\mathbf{r} \in \mathbf{K}$ and $\mathbf{u} = 0$
- Sliding: $\mathbf{r} \in \partial\mathbf{K} - 0$ and $\mathbf{r}_T = -\alpha \mathbf{u}_T$



Credits: NSCD - Acary, V. & Brogliato B (must read)

A couple $(\mathbf{u}, \mathbf{r})$ is said to belong to $\mathcal{C}$ if it satisfies all above conditions i.e.

$$\mathbf{K}^* \ni \mathbf{u} + \mu \|\mathbf{u}_T\| \mathbf{n} \perp \mathbf{r} \in \mathbf{K} \tag{21}$$

Stepping scheme

The dynamics obtained via FEM, coupled to a vector inequality are discretized in time via the θ-method

$$\begin{cases} \mathbf{0} = \mathbf{M}\dfrac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} + \mathbf{C}(\mathbf{q},\mathbf{v})\mathbf{v} + \mathbf{K}(\mathbf{q})\mathbf{q} - \mathbf{f} \\ \quad \text{such that } \mathbf{g}(\mathbf{q},t) \geqslant 0 \end{cases} \underset{\theta\text{-method}}{\rightsquigarrow} \begin{cases} \hat{\mathbf{M}}_k\left(\mathbf{v}_{k+1} - \mathbf{v}_k\right) - \hat{\mathbf{f}} = \mathbf{p}_{k+1} \\ \mathbf{q}_{k+1} = \mathbf{q}_k + h\theta\mathbf{v}_k + h(1-\theta)\mathbf{v}_{k+1} \end{cases} \tag{22}$$

where we have set

$$\begin{aligned} \hat{\mathbf{M}}_k &= \mathbf{M} + h\theta\mathbf{C} + h^2\theta^2\Delta\mathbf{K}_k \ , \ \mathbf{p}_{k+1} = \int_{t_k}^{t_{k+1}} \mathrm{d}\mathbf{p} \\ \hat{\mathbf{f}} &= h\theta\mathbf{f}_{k+1} + h(1-\theta)\mathbf{f}_k - h\mathbf{C}\mathbf{v}_k - h\mathbf{K}_k\mathbf{q}_k - h^2\theta\Delta\mathbf{K}_k\mathbf{v}_k \end{aligned} \tag{23}$$

The key idea is to split the evolution into a smooth and a non-smooth part (according to Lebesgue decomposition theorem)

$$\begin{aligned} \hat{\mathbf{M}}\left(\mathbf{v}_f - \mathbf{v}_k\right) &= \hat{\mathbf{f}}_k \\ \hat{\mathbf{M}}\left(\mathbf{v}_{k+1} - \mathbf{v}_f\right) &= \mathbf{p}_{k+1} \rightsquigarrow \mathbf{u}_{k+1} = \mathbf{u}_f + \widehat{\mathbf{W}}\mathbf{r}_{k+1} \end{aligned} \tag{24}$$

The cone complementarity formulation can be used as

$$\mathbf{K}^* \ni \mathbf{u} + \mu\left\|\mathbf{u}_T\right\|\mathbf{n} \perp \mathbf{r} \in \mathbf{K} \tag{25}$$

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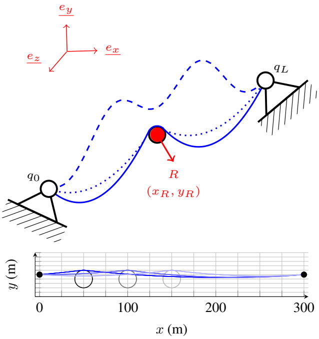
Solving the 2D-contact problem

In every iteration, contact and friction are solved via OSNSP[1]:

$$\begin{cases} \mathbf{v}_f = \mathbf{v}_k + \widehat{\mathbf{M}}_k^{-1}\widehat{\mathbf{f}}_k \\ \mathbf{u}_{Nf}^{\alpha} = \mathbf{H}_N(t_k)^{\alpha}\mathbf{v}_f \\ \mathbf{u}_{Nk}^{\alpha} = \mathbf{H}_N(t_k)^{\alpha}\mathbf{v}_k \\ \mathbf{u}_{Tf}^{\alpha} = \mathbf{H}_T(t_k)^{\alpha}\mathbf{v}_f \\ \\ \forall\alpha \in \mathcal{A} \quad , \quad \text{Solve LCP} \\ \begin{cases} \mathbf{0} \leqslant \widehat{\mathbf{W}}_{NN}\bar{\boldsymbol{\lambda}}_{N,k+1}^{\alpha} + \widehat{\mathbf{W}}_{NT}\left(\mu\bar{\boldsymbol{\lambda}}_{N,k+1}^{\alpha} - \bar{\boldsymbol{\lambda}}_1^{\alpha}\right) + \mathbf{u}_{Nf}^{\alpha} + e\mathbf{u}_{Nk}^{\alpha} \perp \mathbf{r}_{N,k+1}^{\alpha} \geqslant \mathbf{0} \\ \mathbf{0} \leqslant -\widehat{\mathbf{W}}_{TN}\mathbf{r}_{N,k+1}^{\alpha} - \widehat{\mathbf{W}}_{TT}\left(\mu\mathbf{r}_{N,k+1}^{\alpha} - \bar{\boldsymbol{\lambda}}_1^{\alpha}\right) + \mathbf{u}_{T,k+1}^{+,\alpha} - \mathbf{u}_{Tf}^{\alpha} \perp \bar{\boldsymbol{\lambda}}_1^{\alpha} \geqslant \mathbf{0} \\ \mathbf{0} \leqslant 2\mu\mathbf{r}_{N,k+1}^{\alpha} - \bar{\boldsymbol{\lambda}}_1^{\alpha} \perp \mathbf{u}_{T,k+1}^{+,\alpha} \geqslant \mathbf{0} \end{cases} \\ \\ \mathbf{p}_{k+1} = \begin{cases} \mathbf{r}_{N,k+1}^{\alpha}{\mathbf{H}_N}^{\top} + \left(\mu\mathbf{r}_{N,k+1}^{\alpha} - \bar{\boldsymbol{\lambda}}_1^{\alpha}\right){\mathbf{H}_T}^{\top} & , \quad \forall\alpha \in \mathcal{A} \\ \mathbf{0} & , \quad \forall\alpha \notin \mathcal{A} \end{cases} \\ \widehat{\mathbf{M}}_k\left(\mathbf{v}_{k+1} - \mathbf{v}_f\right) = \mathbf{p}_{k+1} \end{cases} . \tag{26}$$

[1] One Step Non-Smooth Problem

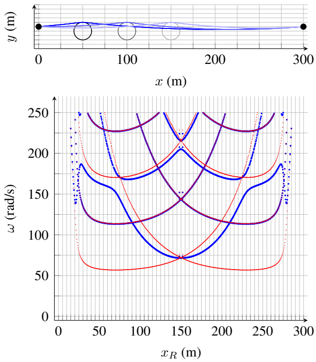
Applications



Hanging cable subjected to a cylinder obstacle

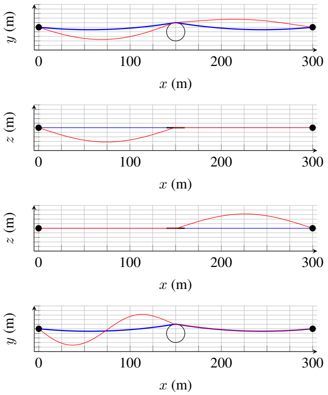
- Restitution coefficient < 1
- Obstacle modeled as a cylinder (moving or not)

Applications



Variations of frequencies WRT obstacle positions

- Restitution coefficient < 1
- Obstacle modeled as a cylinder (moving or not)



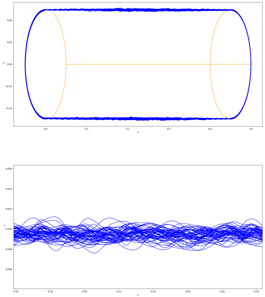
Examples of modes obtained

Applications

Belt-pulley system

- Low friction coefficient for the driven pulley
- High friction coefficient for the driving pulley
- Assembly that preserves conveyor connections
- Local velocity of the driving pulley used as input
- Pulleys are modeled as circles

Applications



Trajectory of one material point

- Low friction coefficient for the driven pulley
- High friction coefficient for the driving pulley
- Assembly that preserves conveyor connections
- Local velocity of the driving pulley used as input
- Pulleys are modeled as circles

Conclusion

Presented today

- Mechanics of a constrained cable via calculus of variations
- Derivation of a cable element and its usage for friction and contact
- Naive applications to various system and possible use

Perspective

- Applications to more complex structures and geometry
- Investigate other constitutive law and other governing equations
- Develop periodic solutions tracking

Open discussion