Theoretical and experimental study of a continuously tunable NES

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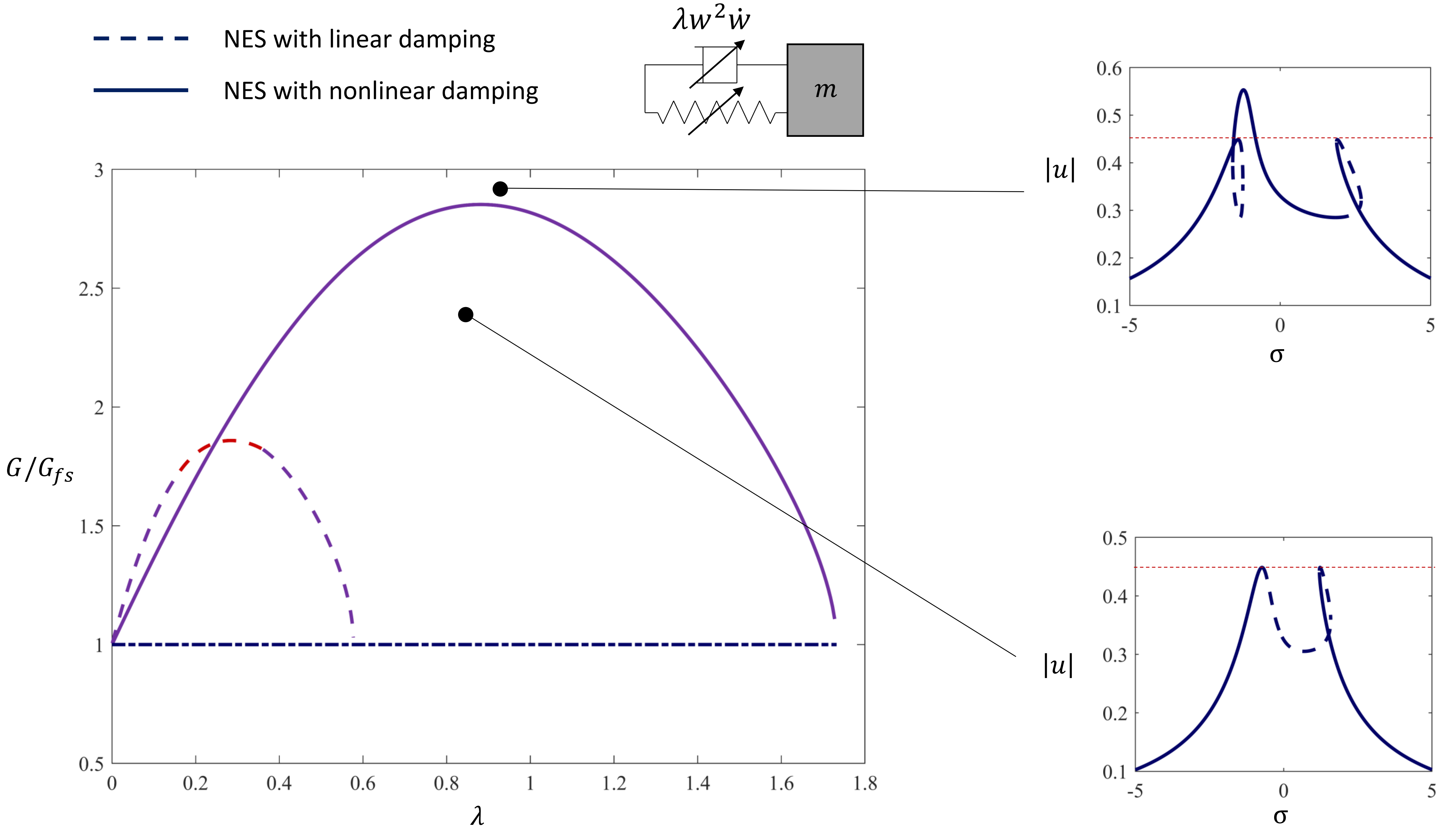
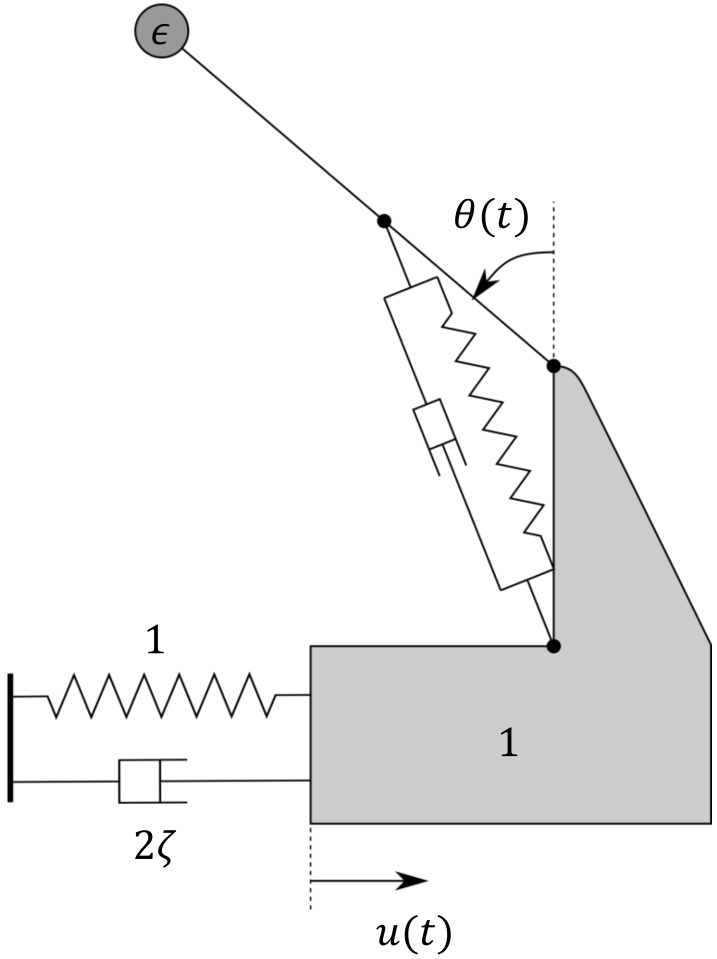
Ex-Modeli Besançon – Extending dynamic range of NES using nonlinear damping



Proposed design – adimensional equation of motion



- Approximate EoM (Series expansion)

$$(1+\epsilon)\ddot{u} + u + 2\zeta\dot{u} - \epsilon\left(\ddot{\theta} - \frac{1}{2}\theta^2\ddot{\theta} - \theta\dot{\theta}^2\right) = F\cos\Omega t$$

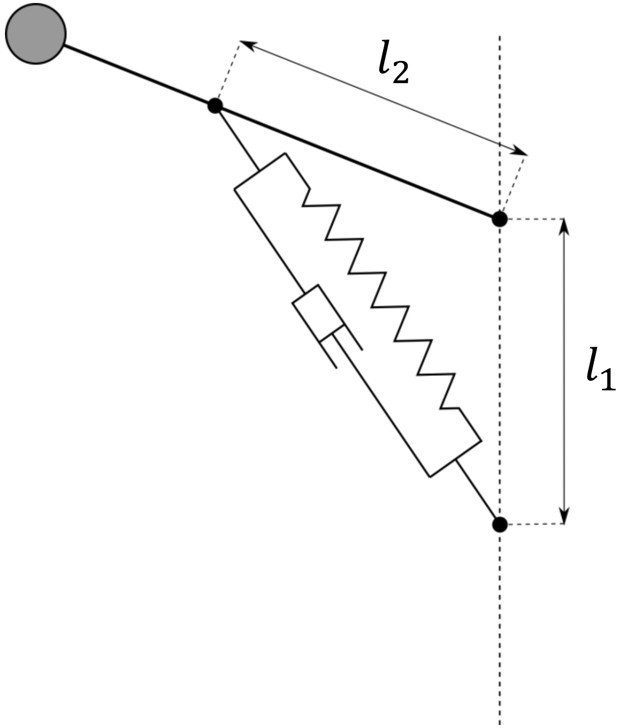
$$\ddot{\theta} - \ddot{u}\left(1 - \frac{1}{2}\theta^2\right) + \kappa\theta^3 + \lambda\theta^2\dot{\theta} = 0$$

Objectives

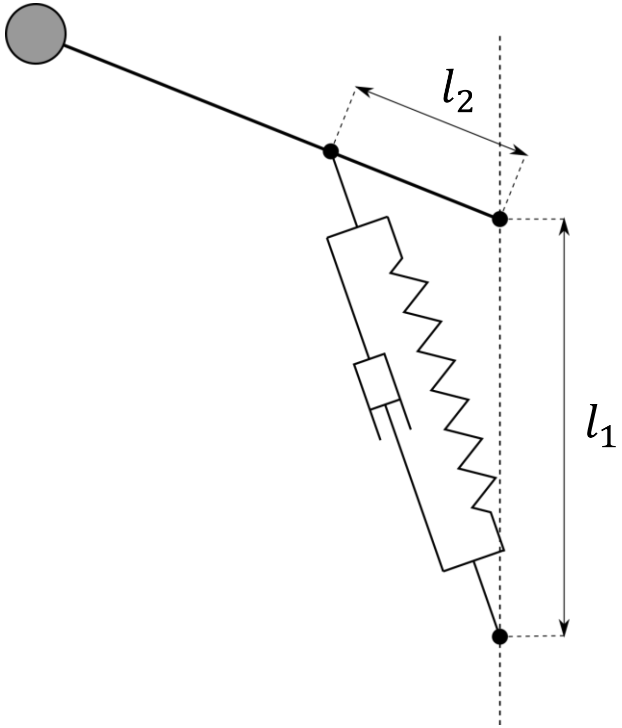
- Describe the dynamical behavior of the system
- Compare between translational and pendulum NES
- Experimental exploration

Proposed design – How to adjust the nonlinear stiffness?

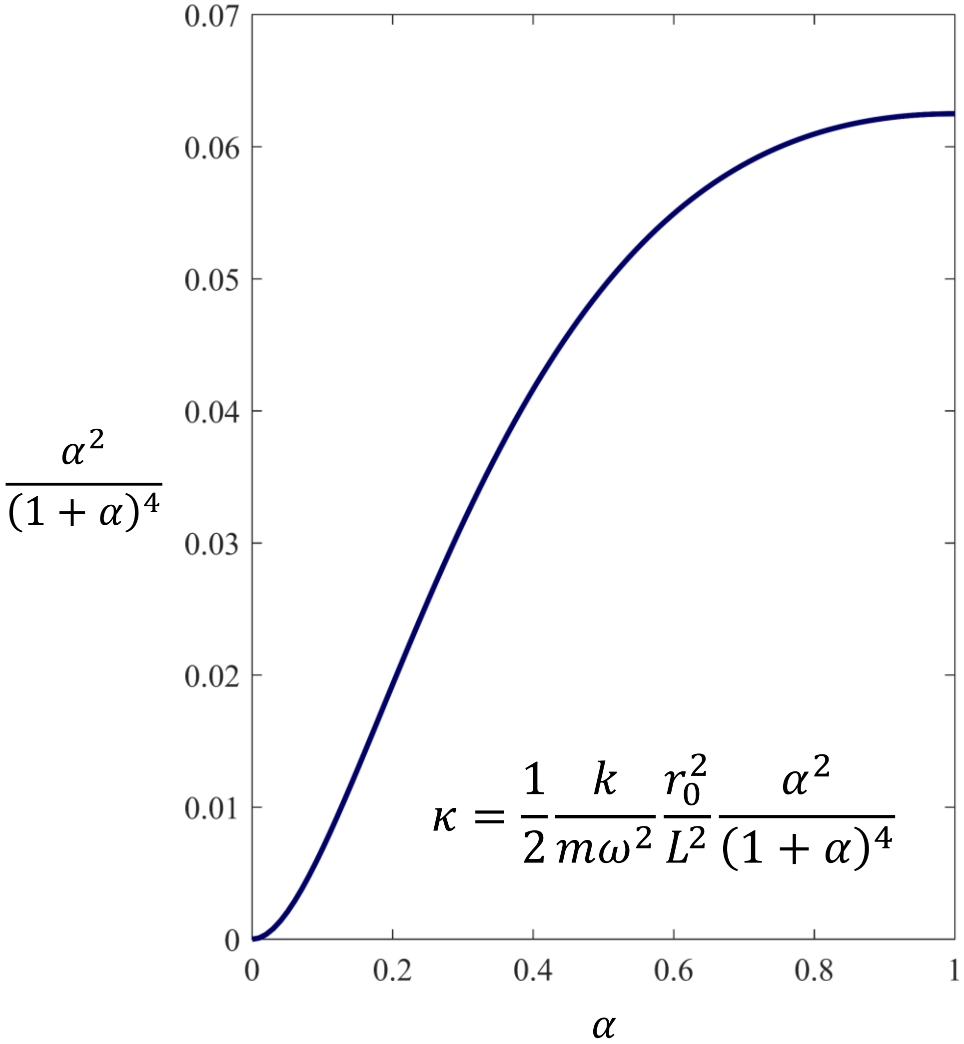
$$\alpha = \frac{l_2}{l_1}$$



$l_1 = l_2 \rightarrow \alpha = 1$

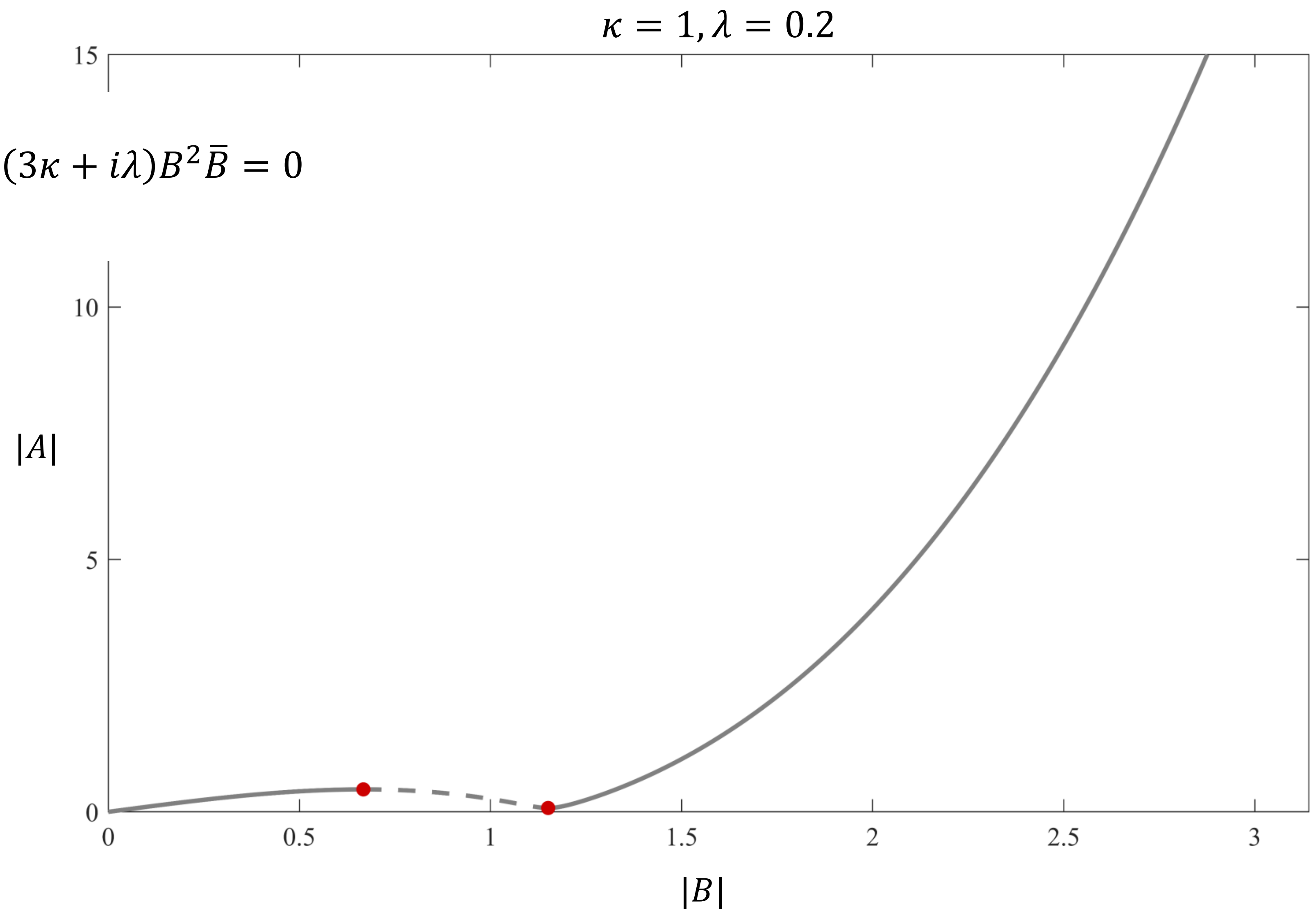


$l_1 > l_2 \rightarrow \alpha < 1$



Dynamical behavior – analysis of the SIM

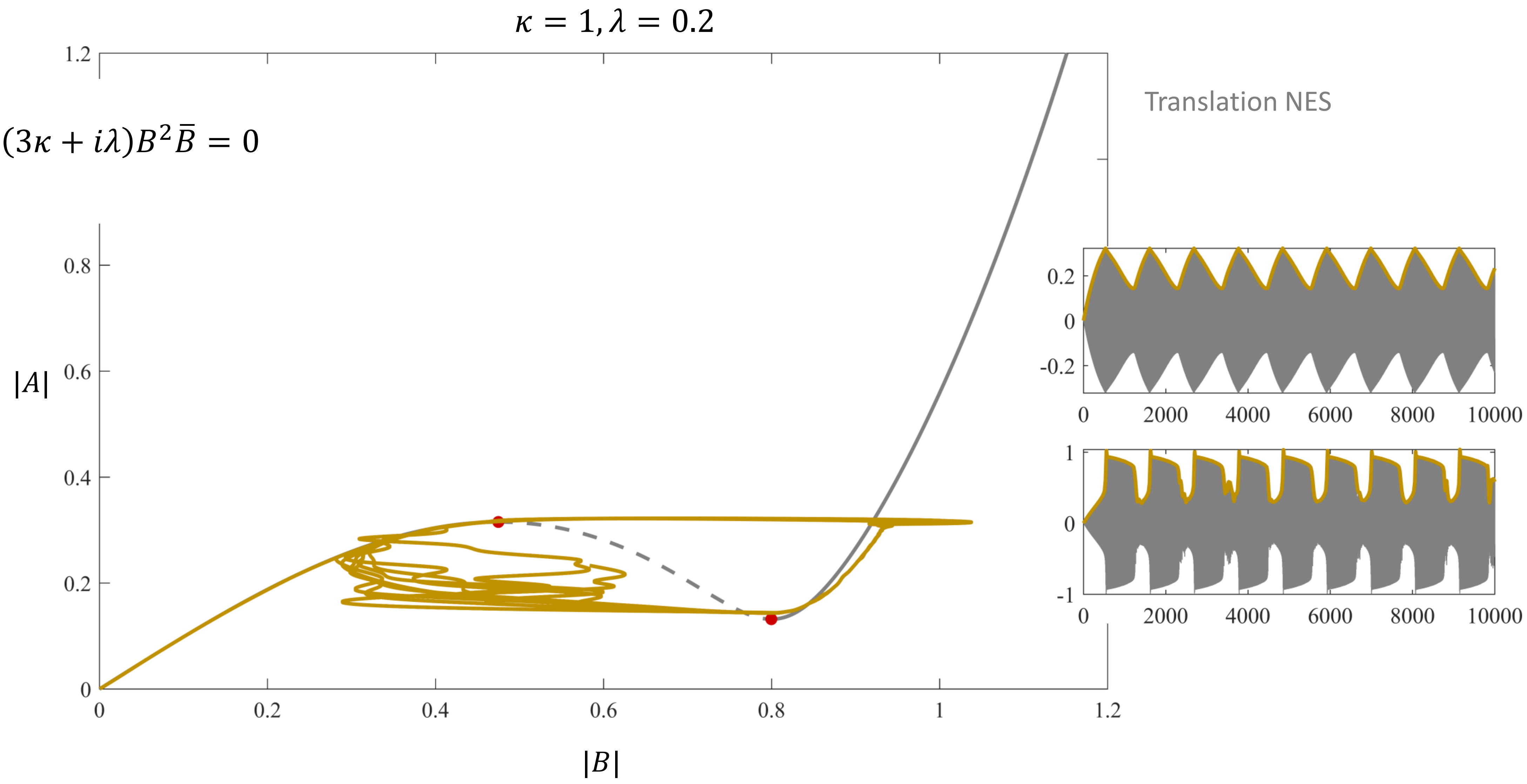
$$A - B + \frac{1}{4}(3\kappa + i\lambda)B^2\bar{B} = 0$$



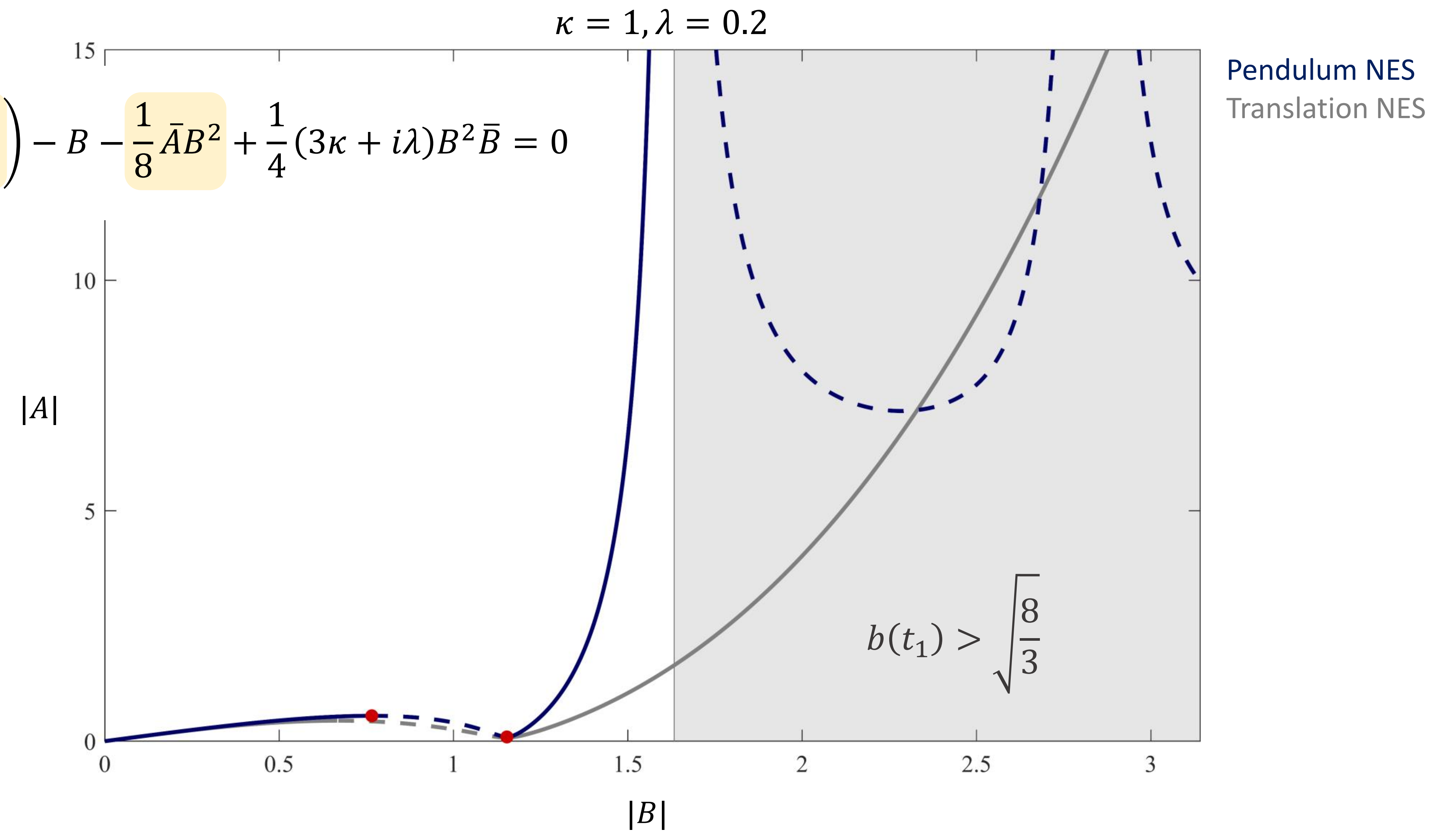
Translation NES

Dynamical behavior – analysis of the SIM

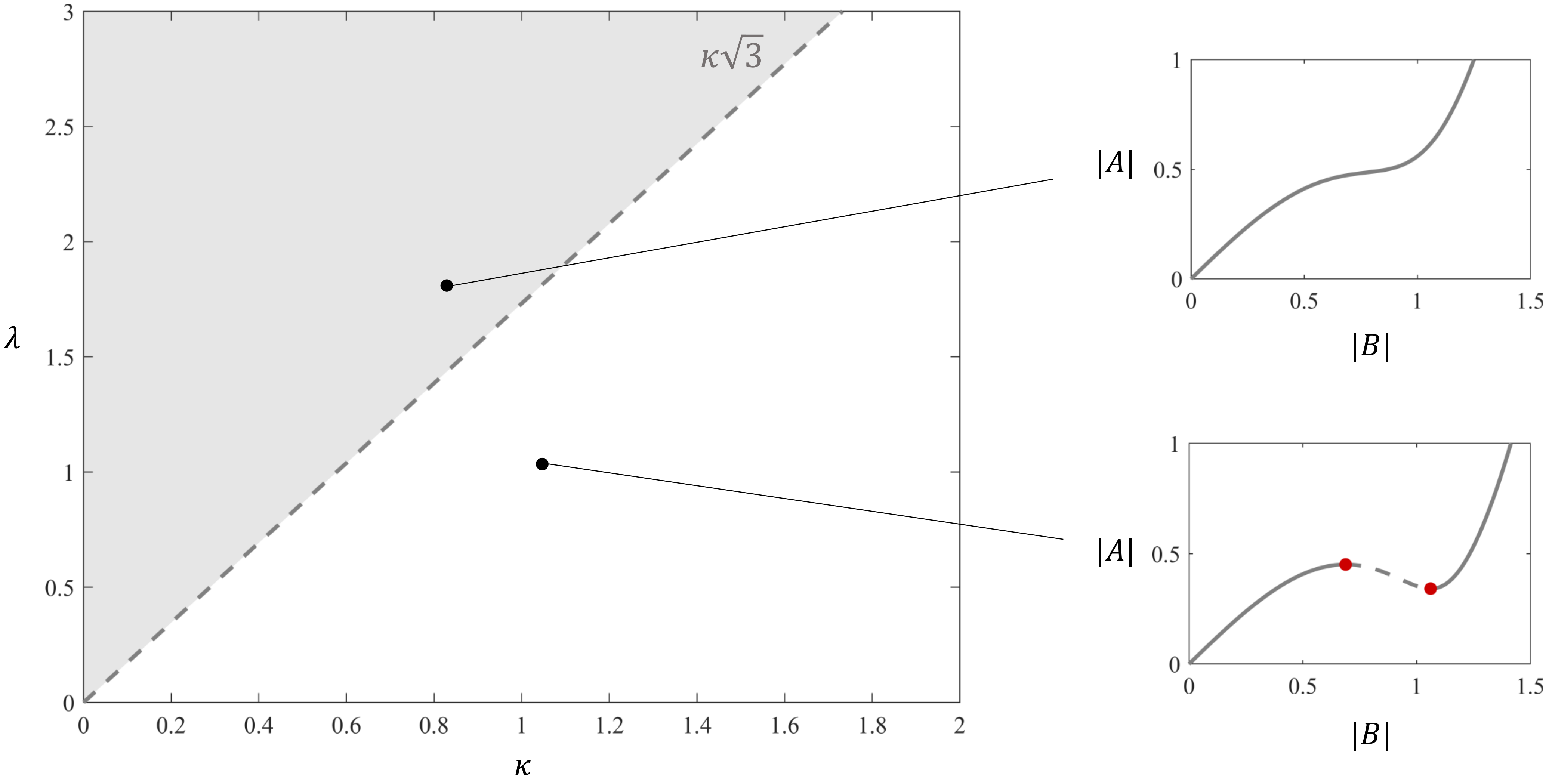
$$A - B + \frac{1}{4}(3\kappa + i\lambda)B^2\bar{B} = 0$$



$$A\left(1-\frac{1}{4}B\bar{B}\right)-B-\frac{1}{8}\bar{A}B^2+\frac{1}{4}(3\kappa+i\lambda)B^2\bar{B}=0$$

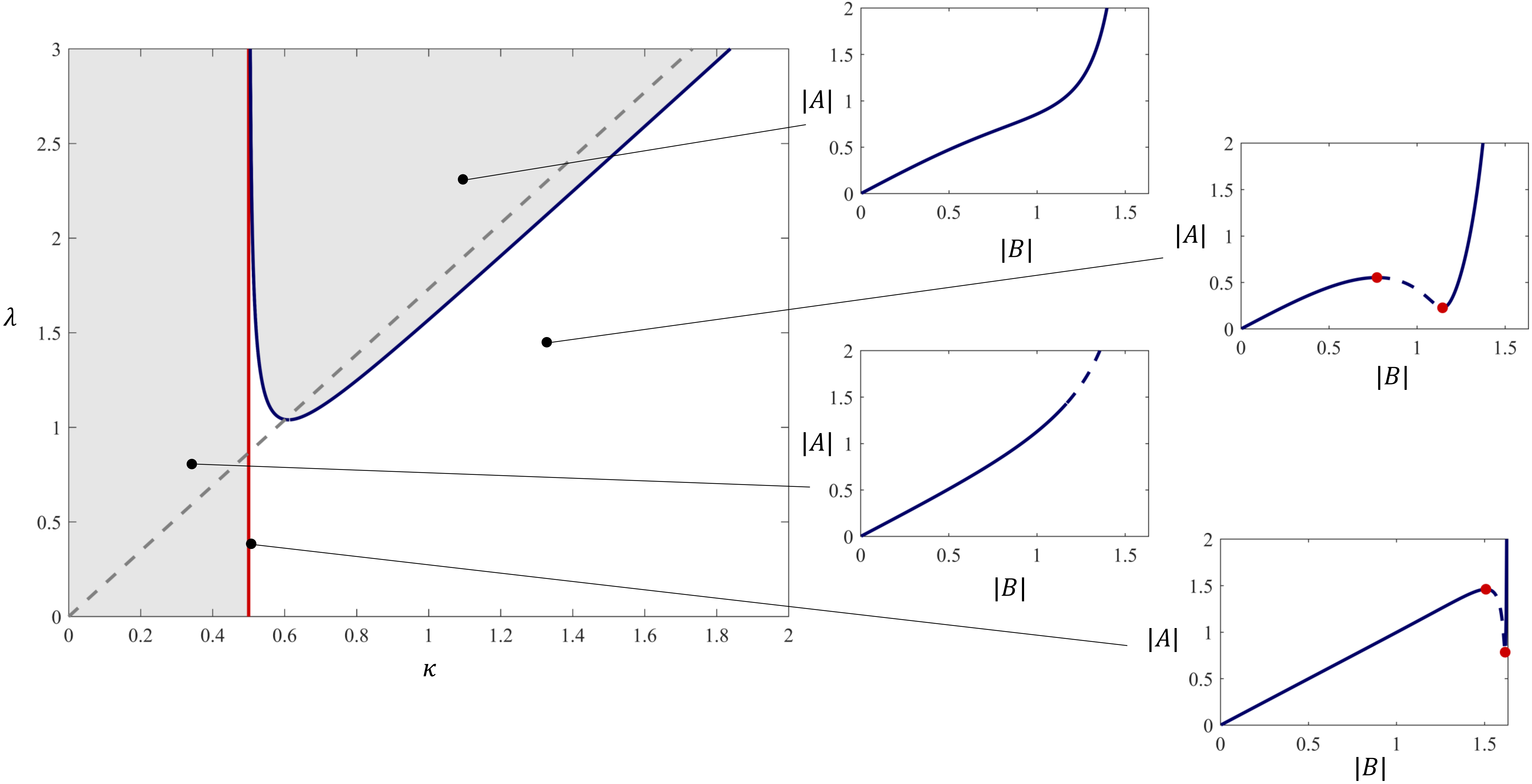


Dynamical behavior – Influence of NES parameters on the SIM



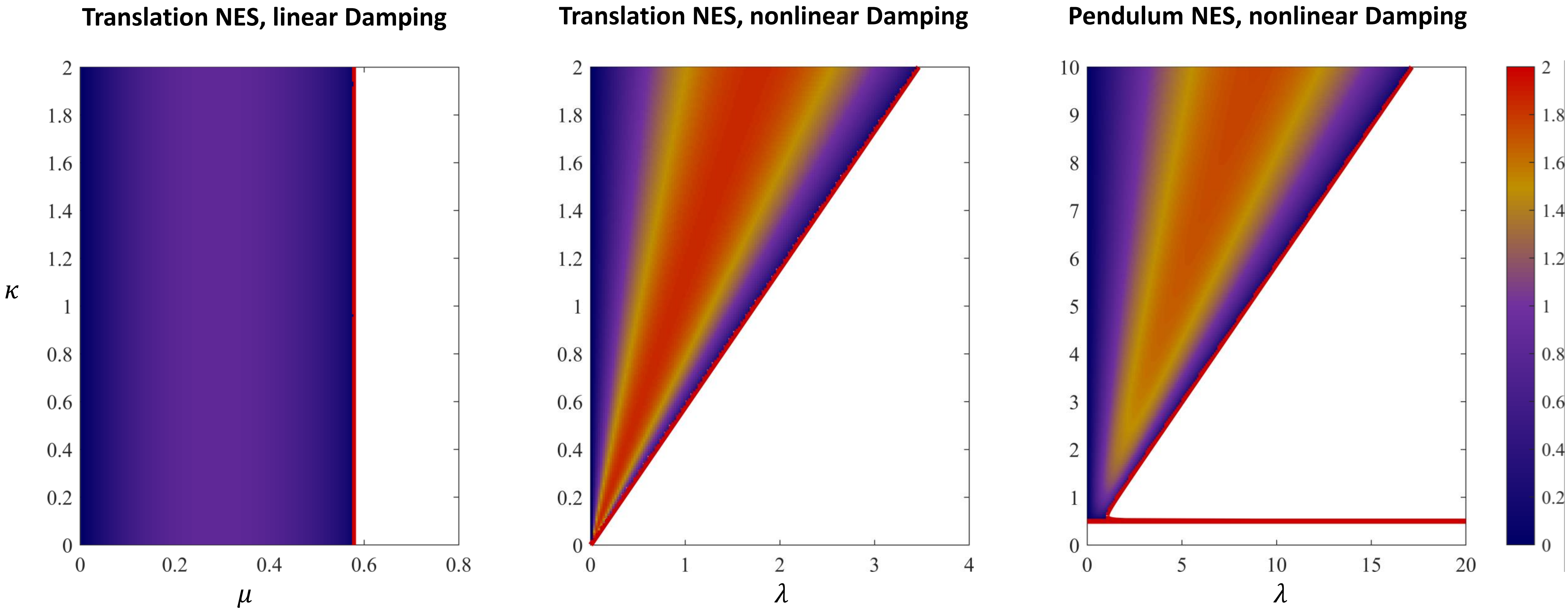
- For a translational NES, the SIM is multi-valued $\forall\lambda < \kappa\sqrt{3}$

Dynamical behavior – Influence of NES parameters on the SIM



Dynamical behavior – Performance comparison

Defining the dynamic range: $\Delta = \frac{\max(G_{isola}, G_{ampl})}{G_{fs}}$



- Nonlinear damping **significantly increase** the dynamic range
- Translation and pendulum NES have similar dynamic range

Dynamical behavior – Experimental setup

NES

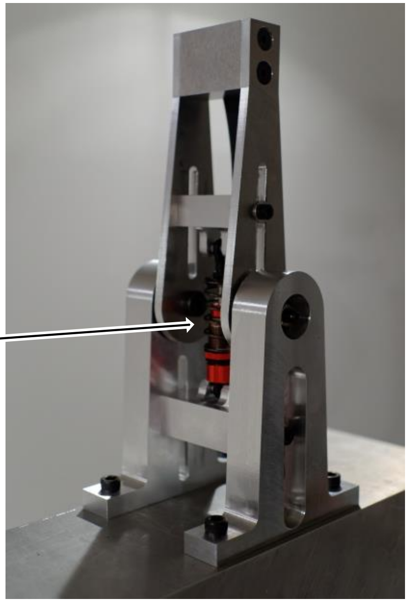
Rotary encoder

RNA
$8.8kg$
$780T$

Mast
$1.5m, 9.5kg$
$122m, 990T$

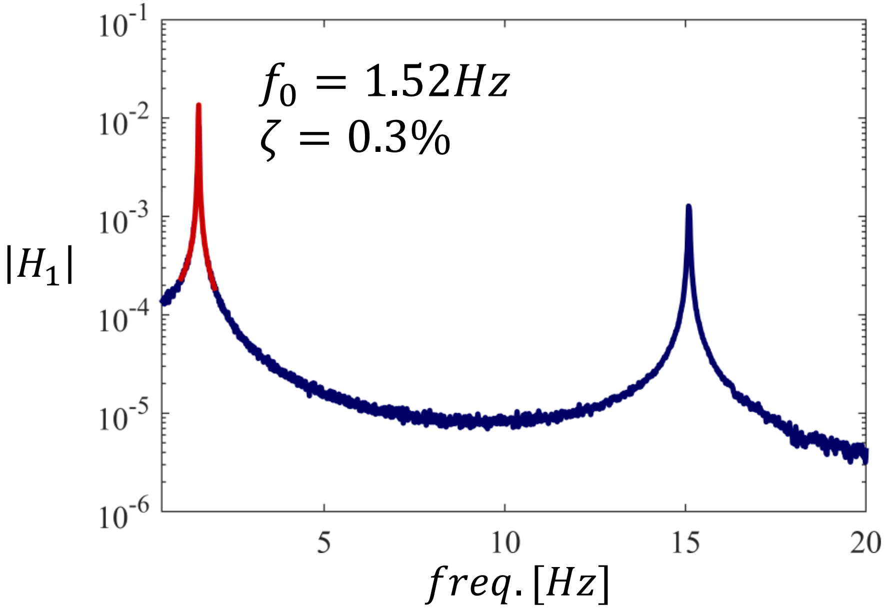
Laser sensor

Shaker

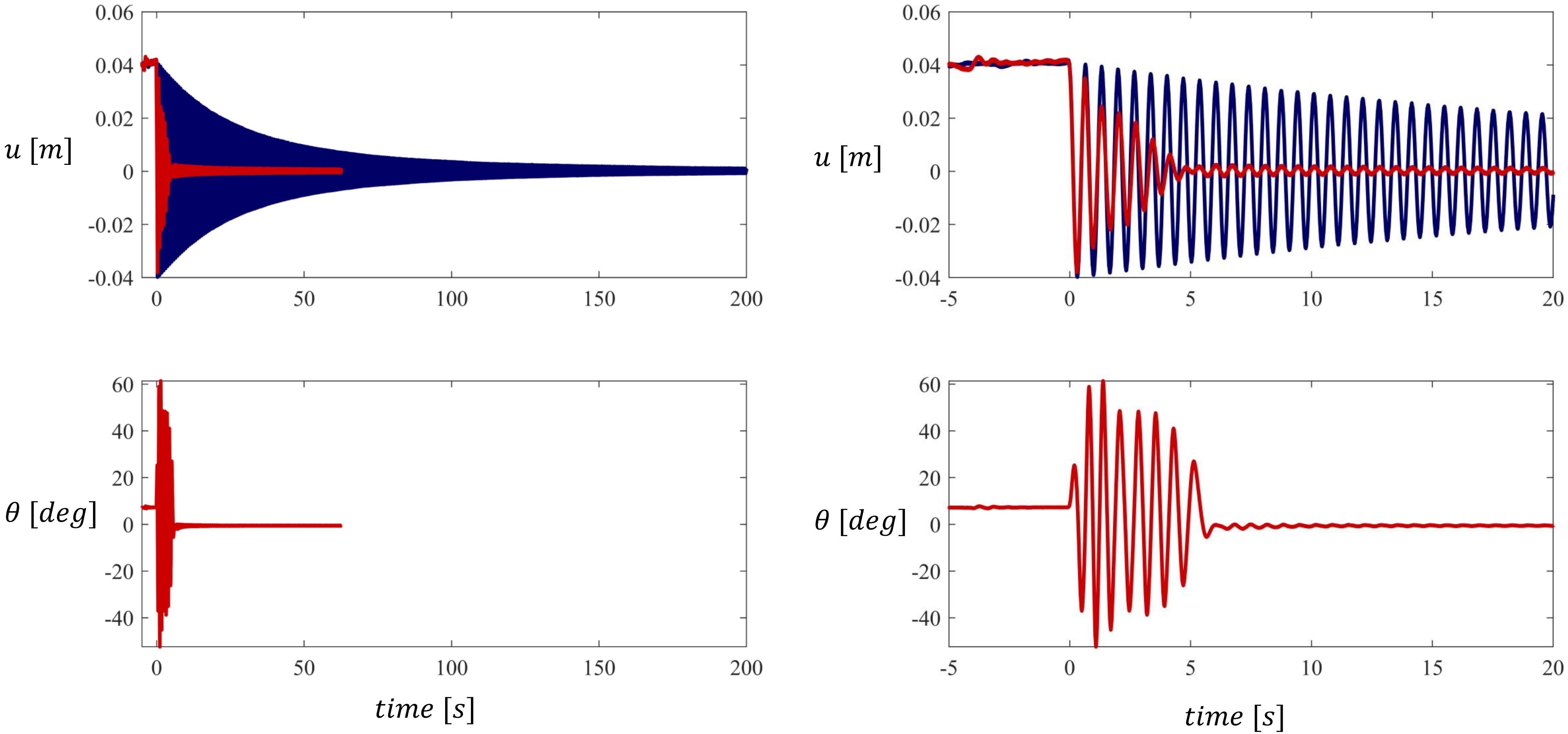


Damper

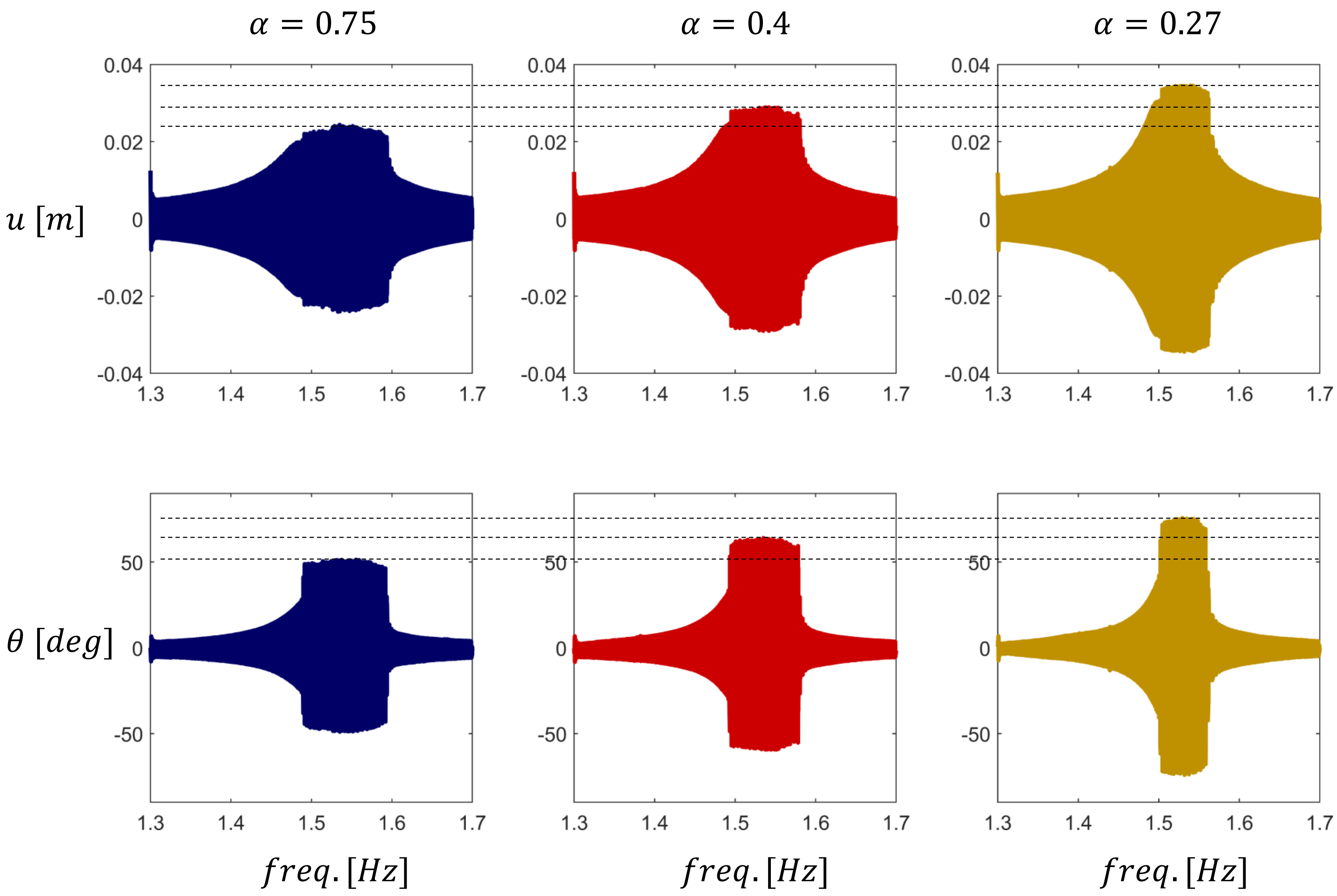
- Moving mass: $323g$
- Equiv. length: $84mm$

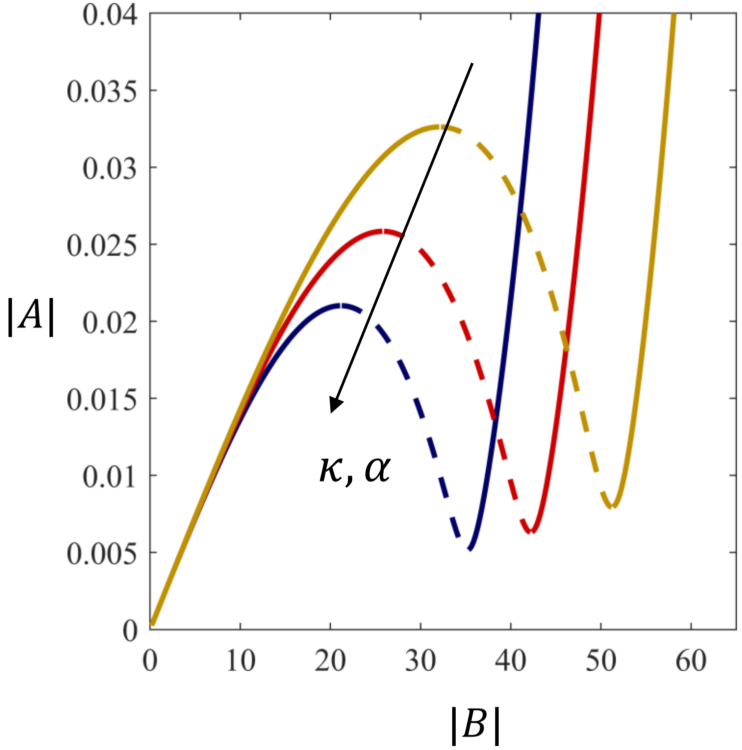


Dynamical behavior – Evidence of 1: 1 resonance capture

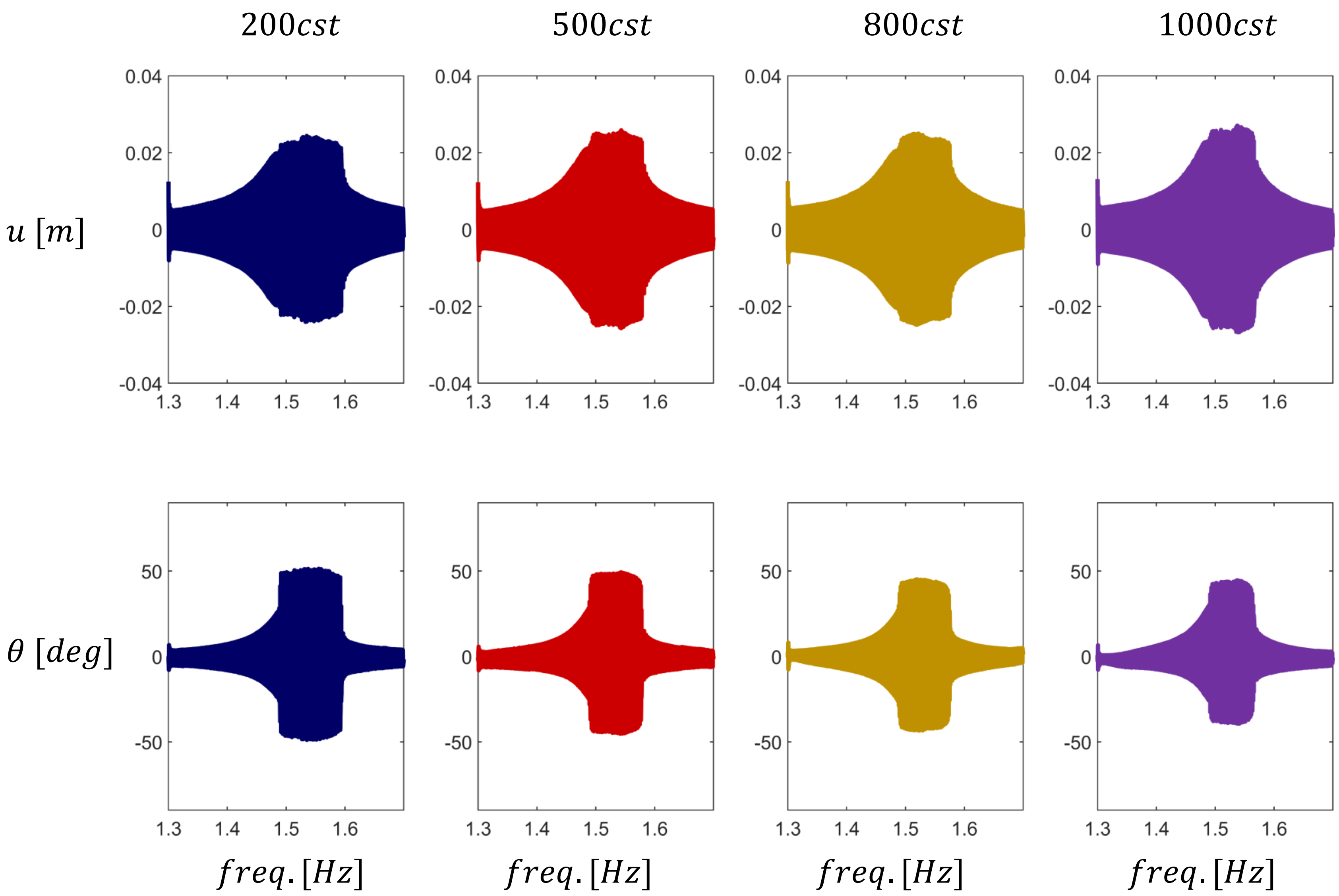


Dynamical behavior – Differents configurations of NES





Dynamical behavior – Influence of nonlinear damping



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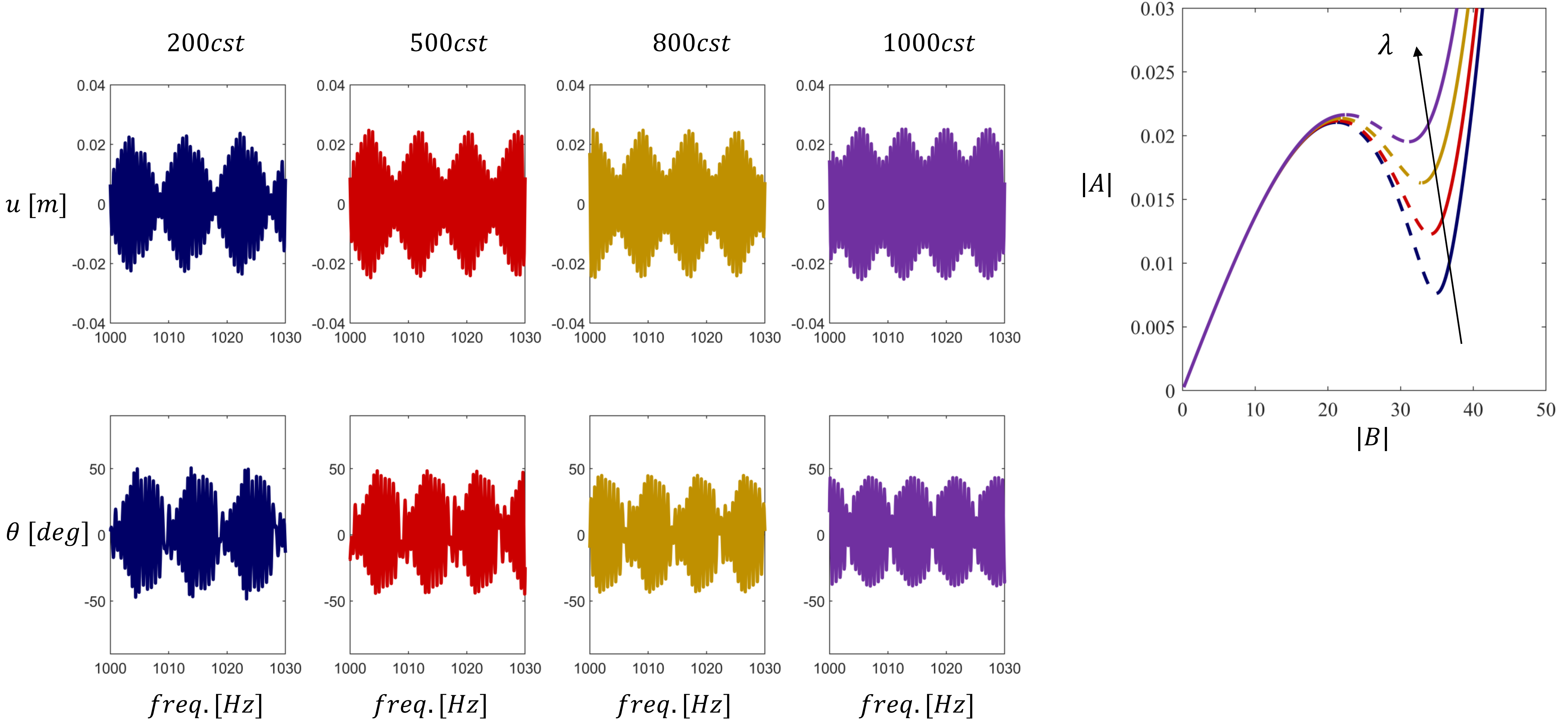
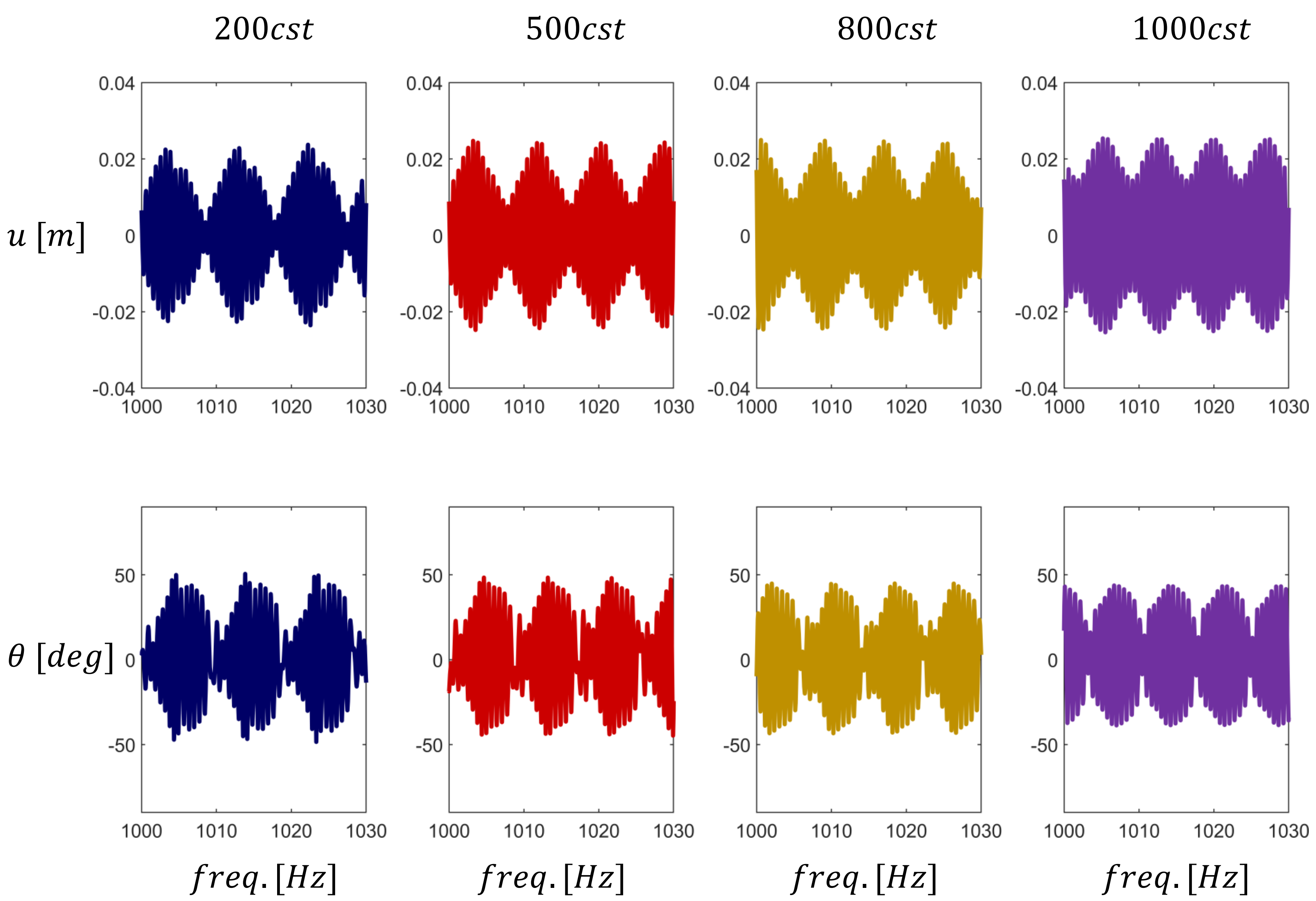
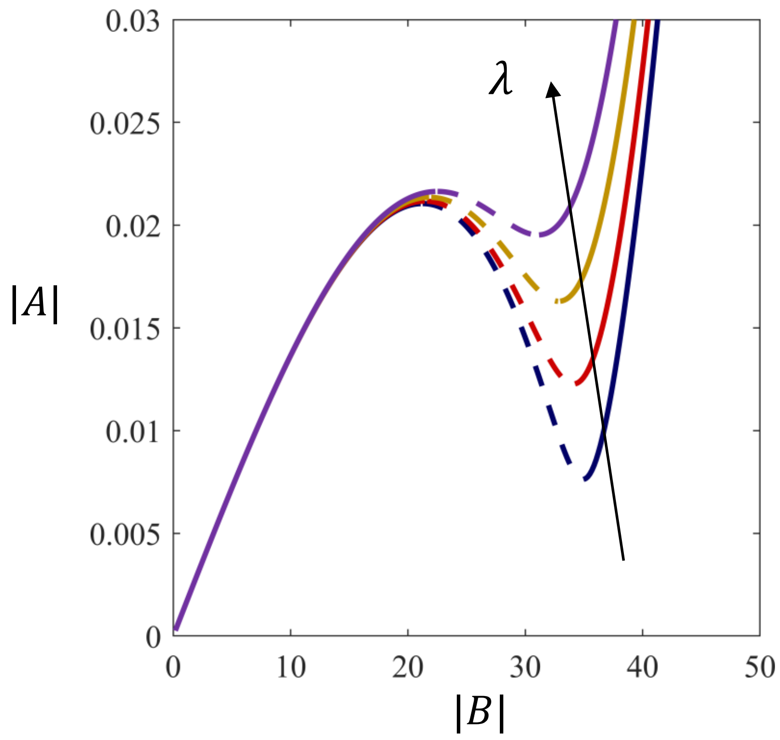
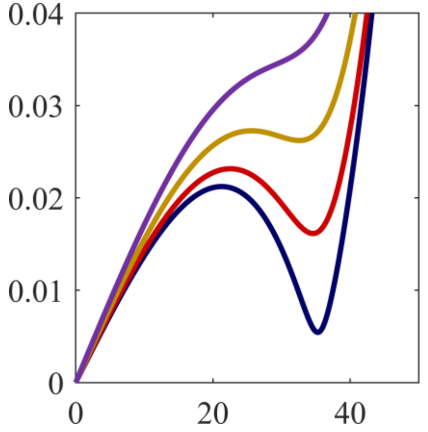


Dynamical behavior – Influence of nonlinear damping



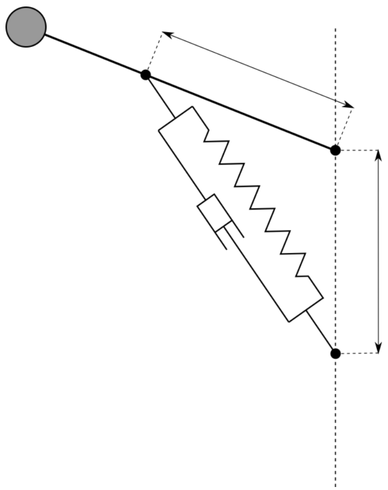


SIM if the damping was linear…

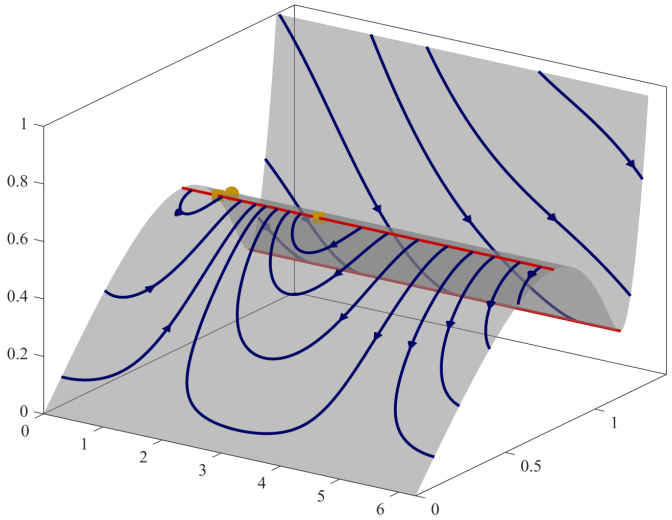


Conclusion & Perspectives

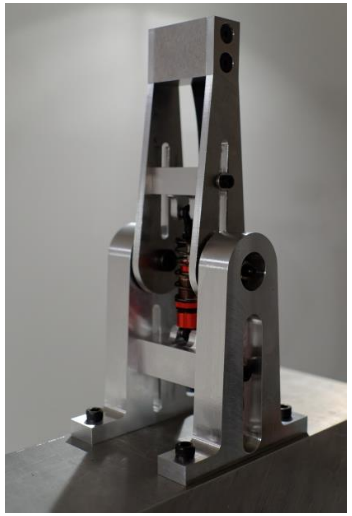
- New type of NES



- Theoretical beahvior using MMS-HBM



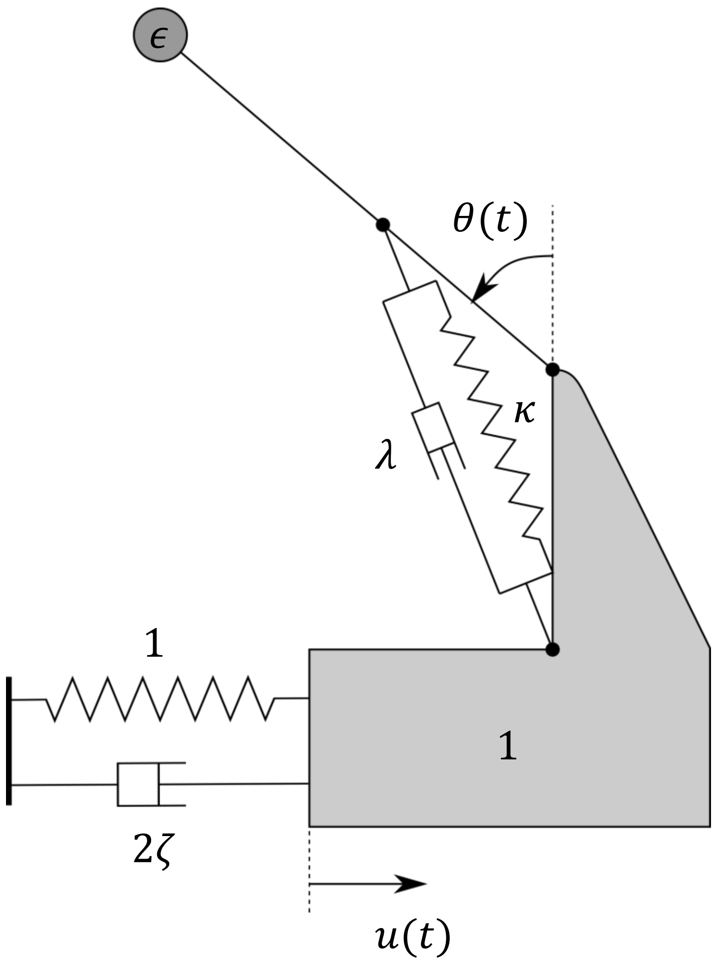
- Experimental validation



Perspectives:

- Experimental caracterization of NES
- Implementation on FOWT numerical twin
- …

Proposed design – adimensional full equation of motion



- Kinematically "exact" EoM

Primary system | Inertial coupling

$$(1+\epsilon)\ddot{u} + u + 2\zeta\dot{u} - \epsilon\frac{d}{dt}\left(\dot{\theta}\cos\theta\right) = F\cos\Omega t$$

Nonlinear stiffness

$$\ddot{\theta} - \ddot{u}\cos\theta + 2\kappa\frac{(1+\alpha)^2}{\alpha}\sin\theta\left(\frac{1+\alpha}{\sqrt{1+\alpha^2+2\alpha\cos\theta}} - 1\right)$$

$$+\lambda\frac{(1+\alpha)^2}{1+\alpha^2+2\alpha\cos\theta}\dot{\theta}\sin^2\theta = 0$$

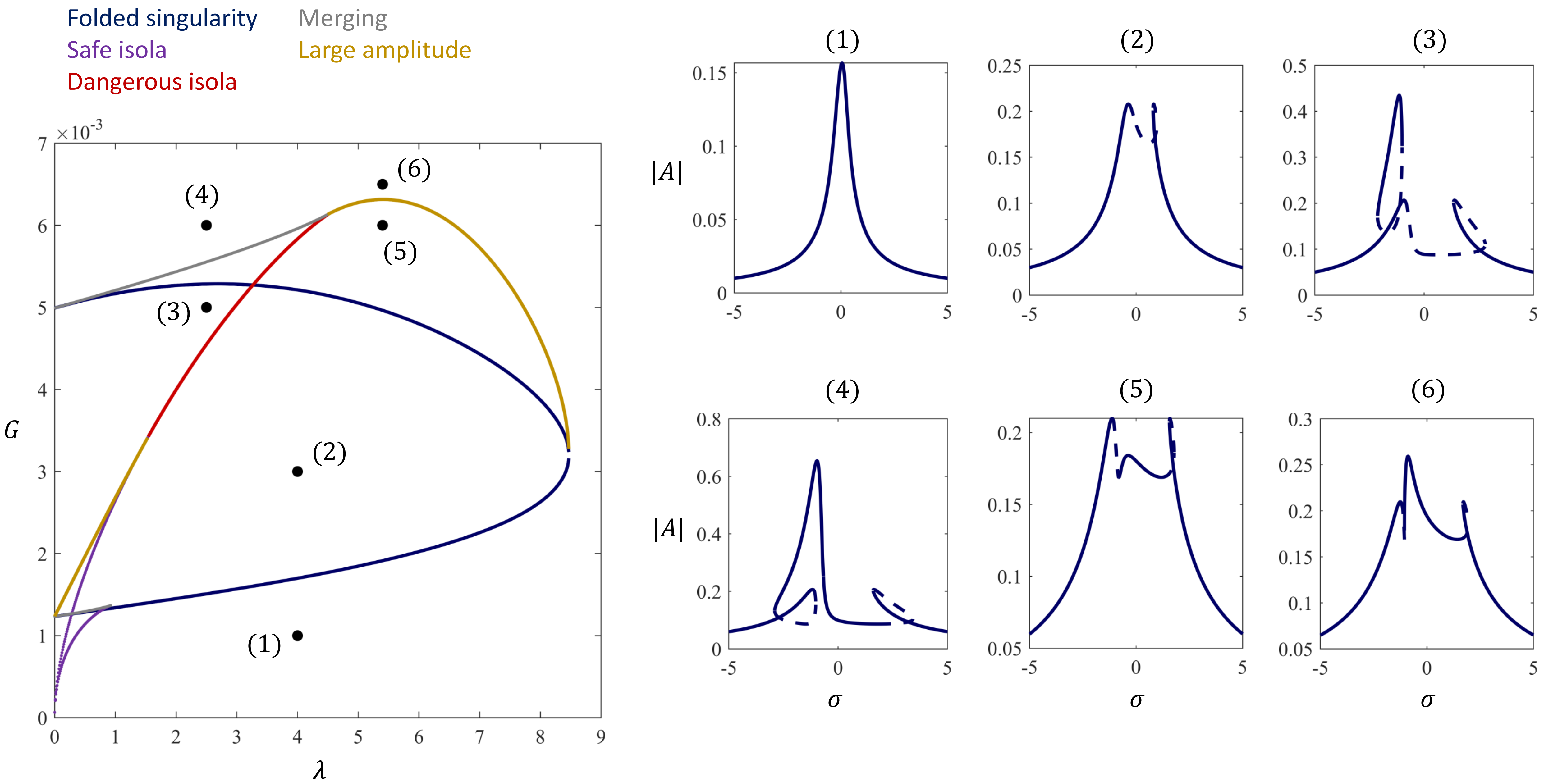
Nonlinear damping

Adimensional parameters

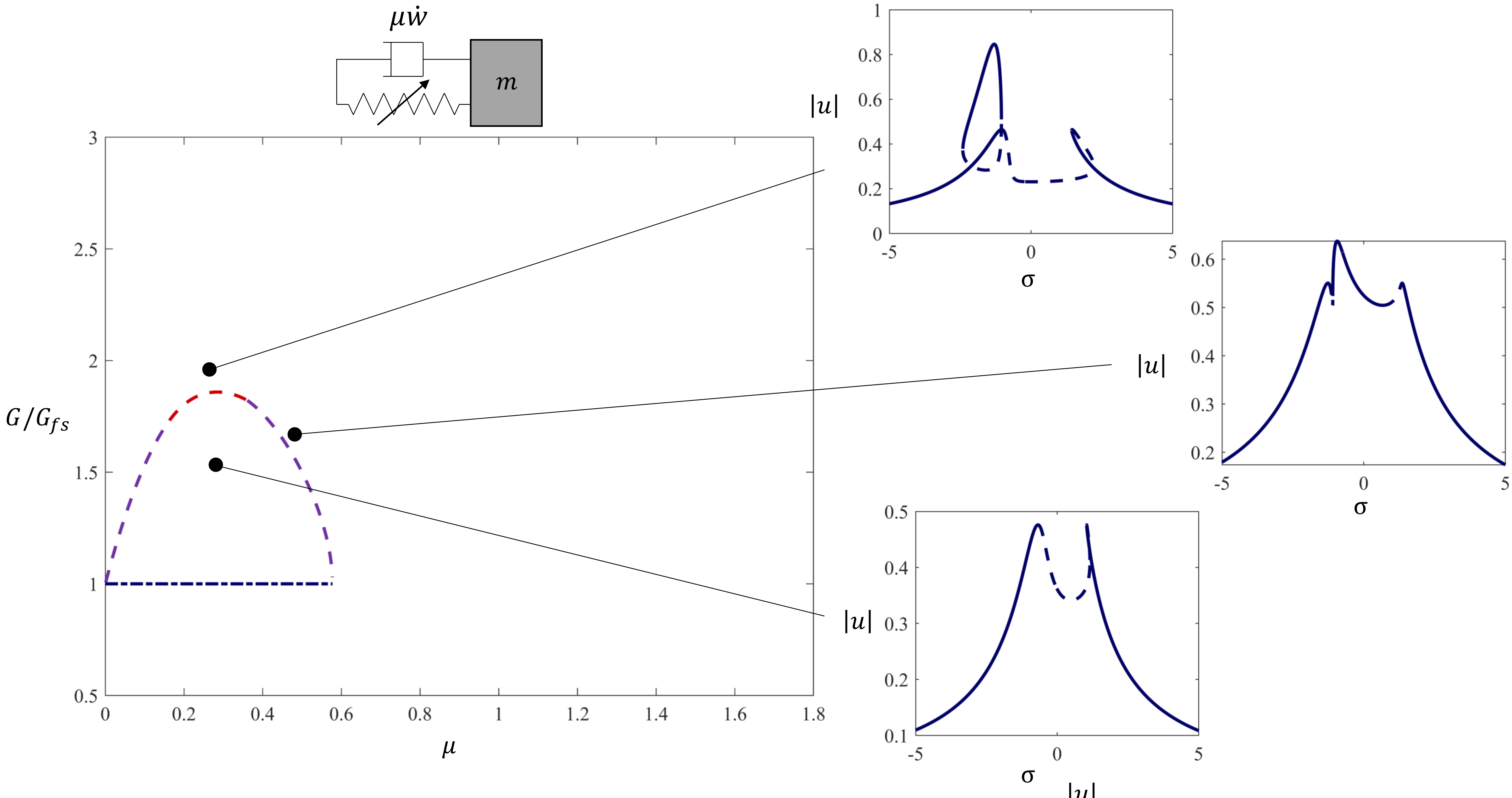
$$\epsilon = \frac{m}{M}, \qquad \kappa = \frac{1}{2}\frac{k}{m\omega^2}\frac{r_0^2}{L^2}\frac{\alpha^2}{(1+\alpha)^4}, \qquad \lambda = \frac{c}{m\omega}\frac{r_0^2}{L^2}\frac{\alpha^2}{(1+\alpha)^4}, \qquad \alpha = \frac{l_2}{l_1}$$

The **linear stiffness vanishes** if the initial length of the spring $r_0 = l_1 + l_2$

Dynamical behavior – Example of sizing diagram



Ex-Modeli Besançon – The "classic" NES



$$u(t) = A(t)e^{it_0} + \bar{A}(t)e^{-it_0}$$

$$\theta(t) = B(t)e^{it_0} + \bar{B}(t)e^{-it_0}$$

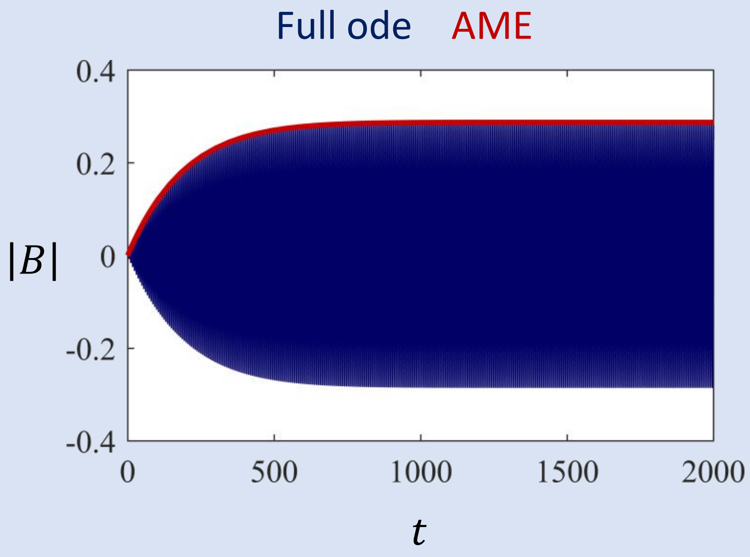
Order ϵ^0: **Slow Invariant Manifold** (SIM)

$$A\left(1 - \frac{1}{4}B\bar{B}\right) - B - \frac{1}{8}\bar{A}B^2 + \frac{1}{4}(3\kappa + i\lambda)B^2\bar{B} = 0$$

Order ϵ^1: **Amplitude Modulation Equation**

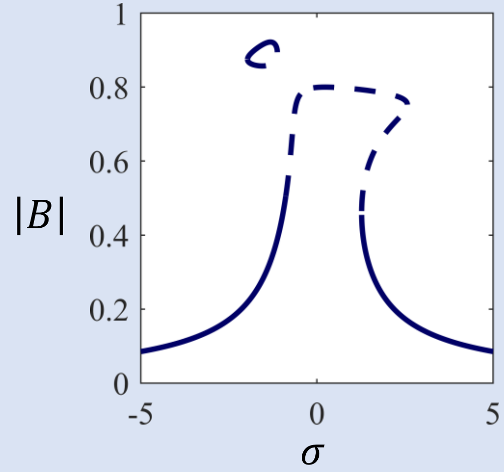
$$|\dot{B}| = f_1(B),$$

$$\arg \dot{B} = f_2(B, \sigma)$$



Fixed points

$$\left.\begin{aligned} f_1(B) &= 0 \\ f_2(B,\sigma) &= 0 \end{aligned}\right\} \quad h(|B|, \sigma) = 0$$



Dynamical behavior – interlude… stability of the SIM!

- Going back to the equation of the NES at $\mathcal{O}(\epsilon^0)$: a **harmonically forced nonlinear oscillator**

$$\frac{1}{2}\left(Ae^{it_0}+\bar{A}e^{-it_0}\right)\left(1-\frac{1}{2}\theta_0^2\right)+d_0^2\theta_0+\kappa\theta_0^3+\lambda\theta_0^2 d_0\theta_0=0$$

- Stability computation using **Floquet theory**

i. Adding **perturbations**: $\theta_0(t_0,t_1)=\frac{1}{2}B(t_1)e^{it_0}+\frac{1}{2}\bar{B}(t_1)e^{-it_0}+y(t_0)$

perdiodic solution — disturbance

ii. **Linearizing** around disturbances: $d_0^2 y+M_1(t)d_0 y+M_2(t)y=0$

periodic coefficient of period 2π

iii. We seek a solution of **Floquet form**: $y(t_0)=\phi(t_0)e^{\gamma t_0}$

Floquet exponent

iv. Expanding $\phi(t_0)$ in **Fourier series** and balancing first harmonic → **Fourth order polynomial in γ**!!!!

Dynamical behavior – analysis using the MMS-HBM

- Introducing **independent time scales**: $t_0 = t,$ $t_1 = \epsilon t,$ $\epsilon \ll 1$

  Fast time Slow time

- Power series expansion of the dependent variable:

$$u(t;\epsilon) = u_0(t_0, t_1) + \epsilon u_1(t_0, t_1)$$

$$\theta(t;\epsilon) = \theta_0(t_0, t_1) + \epsilon \theta_1(t_0, t_1)$$

- Scaling parameters: $\zeta, F \sim \mathcal{O}(\epsilon)$

- Substituting into the equation of motion and balancing term with the same power of ϵ

$\mathcal{O}(\epsilon^0)$: $d_0^2 u_0 + u_0 = 0$ harmonic oscillator $\rightarrow$ $u_0(t_0, t_1) = \frac{1}{2}A(t_1)e^{it_0} + \frac{1}{2}\bar{A}(t_1)e^{-it_0}$

$$d_0^2\theta_0 - d_0^2 u_0\left(1 - \frac{1}{2}\theta_0^2\right) + \kappa\theta_0^3 + \lambda\theta_0^2 d_0\theta_0 = 0$$

No closed form solution

- we seek an approximate solutions using **first harmonic method**: $\theta_0(t_0, t_1) = \frac{1}{2}B(t_1)e^{it_0} + \frac{1}{2}\bar{B}(t_1)e^{-it_0}$

Complex valued **Slow Invariant Manifold** (SIM)

Pendulum NES

$$A\left(1 - \frac{1}{4}B\bar{B}\right) - B - \frac{1}{8}\bar{A}B^2 + \frac{1}{4}(3\kappa + i\lambda)B^2\bar{B} = 0$$

Translational NES

$$A - B + \frac{1}{4}(3\kappa + i\lambda)B^2\bar{B} = 0$$

Dynamical behavior – Amplitude modulation equation

$\mathcal{O}(\epsilon^1)$ equation:

$$d_0^2 u_1 + u_1 = -2d_0 d_1 u_0 - d_0^2 u_0 + d_0^2 \theta_0 - \theta_0 (d_0 \theta_0)^2 - \frac{1}{2}\theta_0^2 d_0^2 \theta_0 - 2\zeta d_0 u_0 + G\cos\Omega t_0$$

- The excitation frequency is **close** to the frequency of the primary system: $\Omega = 1 + \epsilon\sigma$
- Substituting solutions at $\mathcal{O}(\epsilon^0)$:

$$d_0^2 u_1 + u_1 = \left(-id_1 A - i\zeta A + \frac{1}{2}(A - B) + \frac{1}{16}B^2\bar{B} + \frac{1}{2}Ge^{i\sigma t_1}\right)e^{it_0} + N.S.T. + c.c.$$

- Elimination of secular terms:

$$-id_1 A - i\zeta A + \frac{1}{2}(A - B) + \frac{1}{16}B^2\bar{B} + \frac{1}{2}Ge^{i\sigma t_1} = 0$$

- Projection of the dynamics on the SIM (**invariance property of the SIM**): $A(t_1) = g\big(B(t_1)\big)$
- … after *some manipulations* (polar form, reabsorbing ϵ, change of phase variable)

Amplitude modulation equation (AME)

$$\dot{b} = f_1(b, \theta), \qquad \dot{\theta} = f_2(b, \theta, \sigma)$$

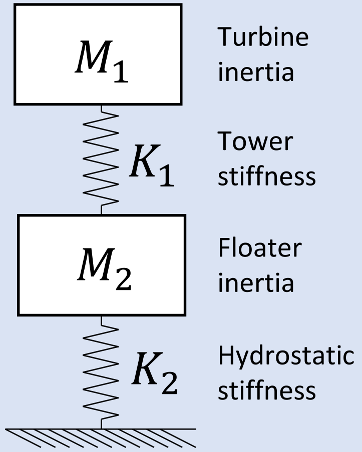
Context

Impact of turbine size and floating support

Larger turbine

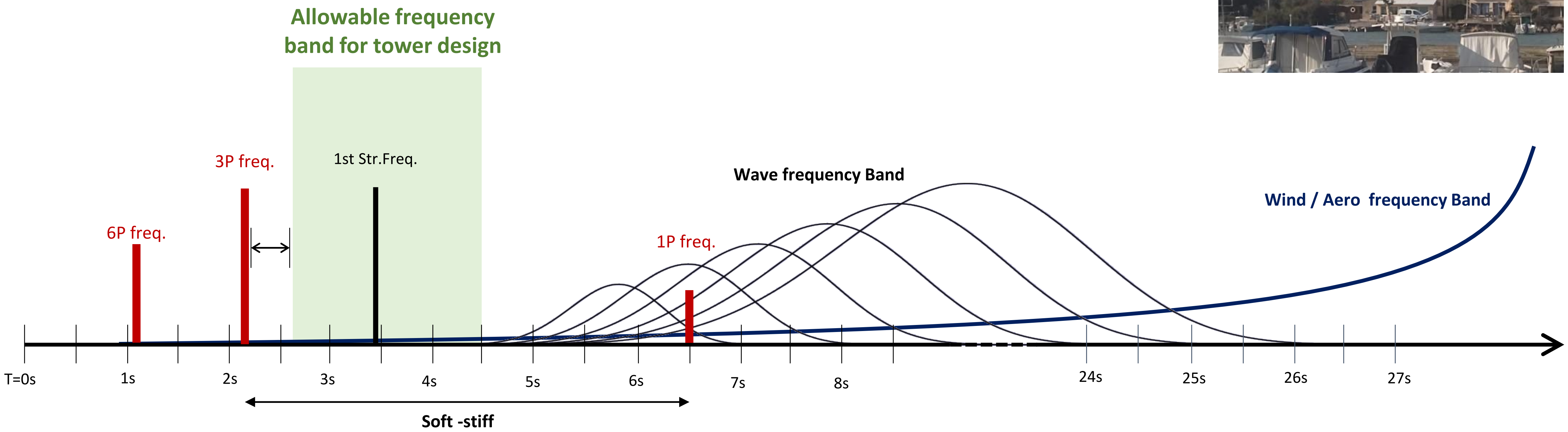
⇩

3P freq ↘

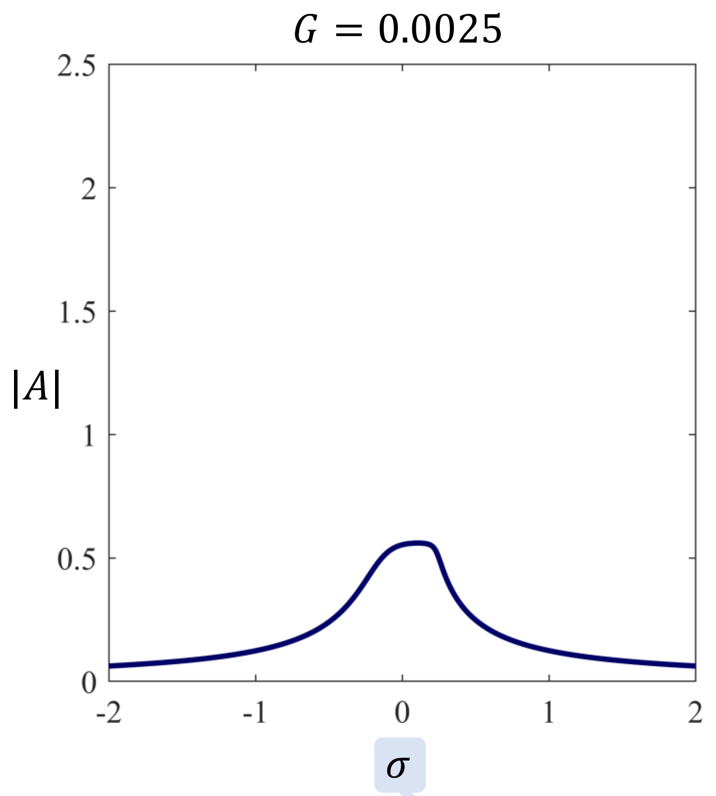


$$K_1 \gg K_2$$

$$\omega \approx \sqrt{\frac{K_1}{M_1} + \frac{K_1}{M_2}}$$

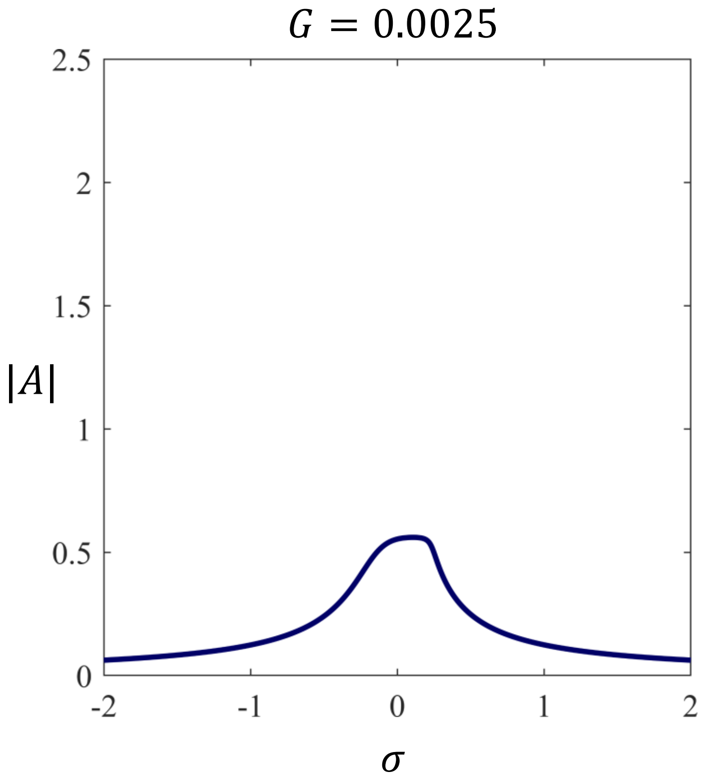


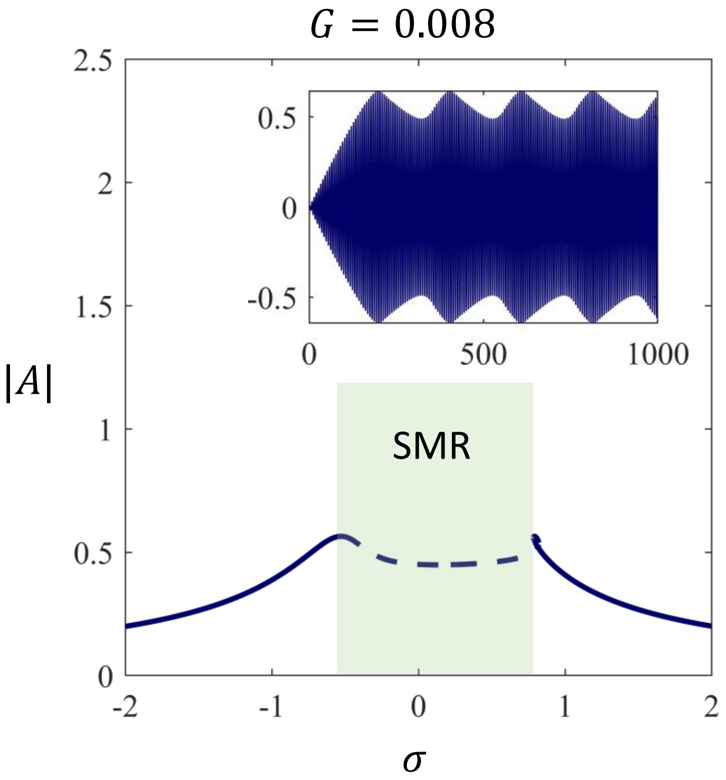
Dynamical behavior – Influence of the forcing amplitude



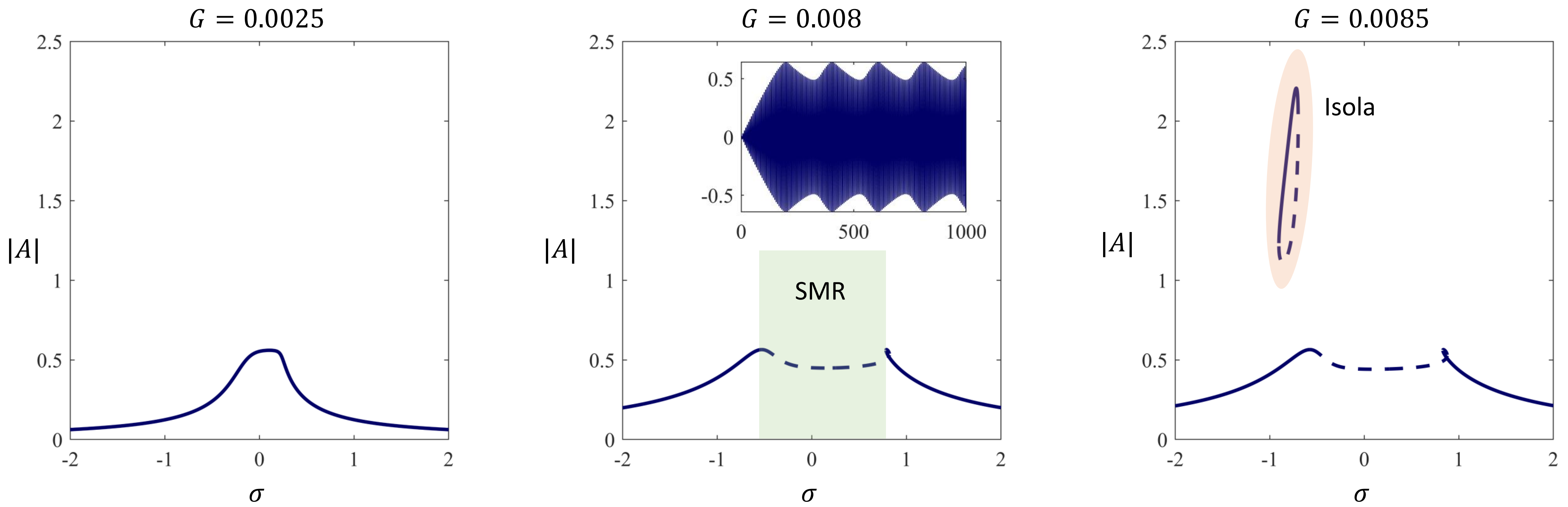
$$\Omega = 1 + \epsilon\sigma$$

Dynamical behavior – Influence of the forcing amplitude





Dynamical behavior – Influence of the forcing amplitude

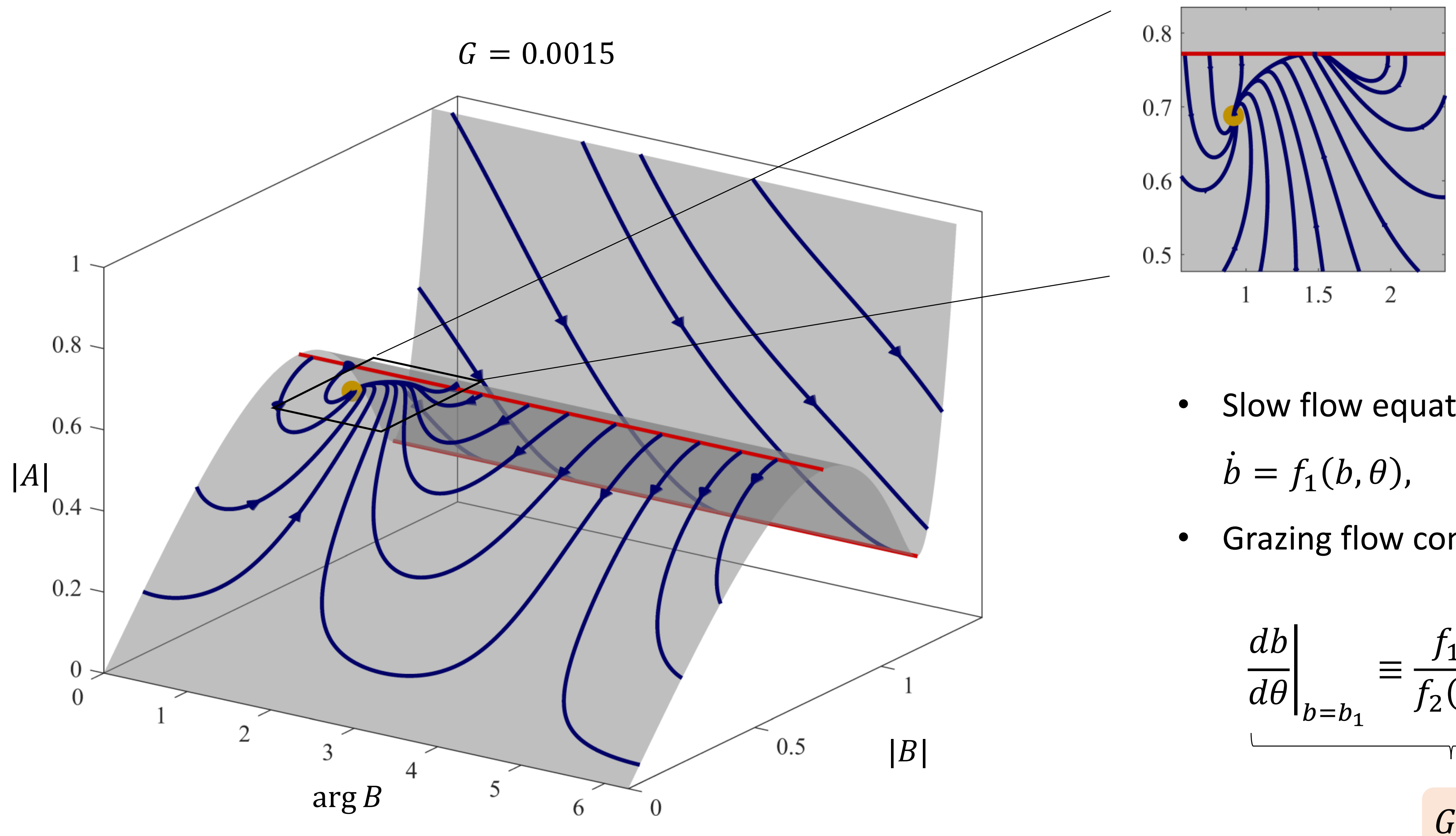


We need to know:

At which forcing amplitude:

- SMR triggers?
- Isola appears?

Dynamical behavior – SMR triggered by grazing flow



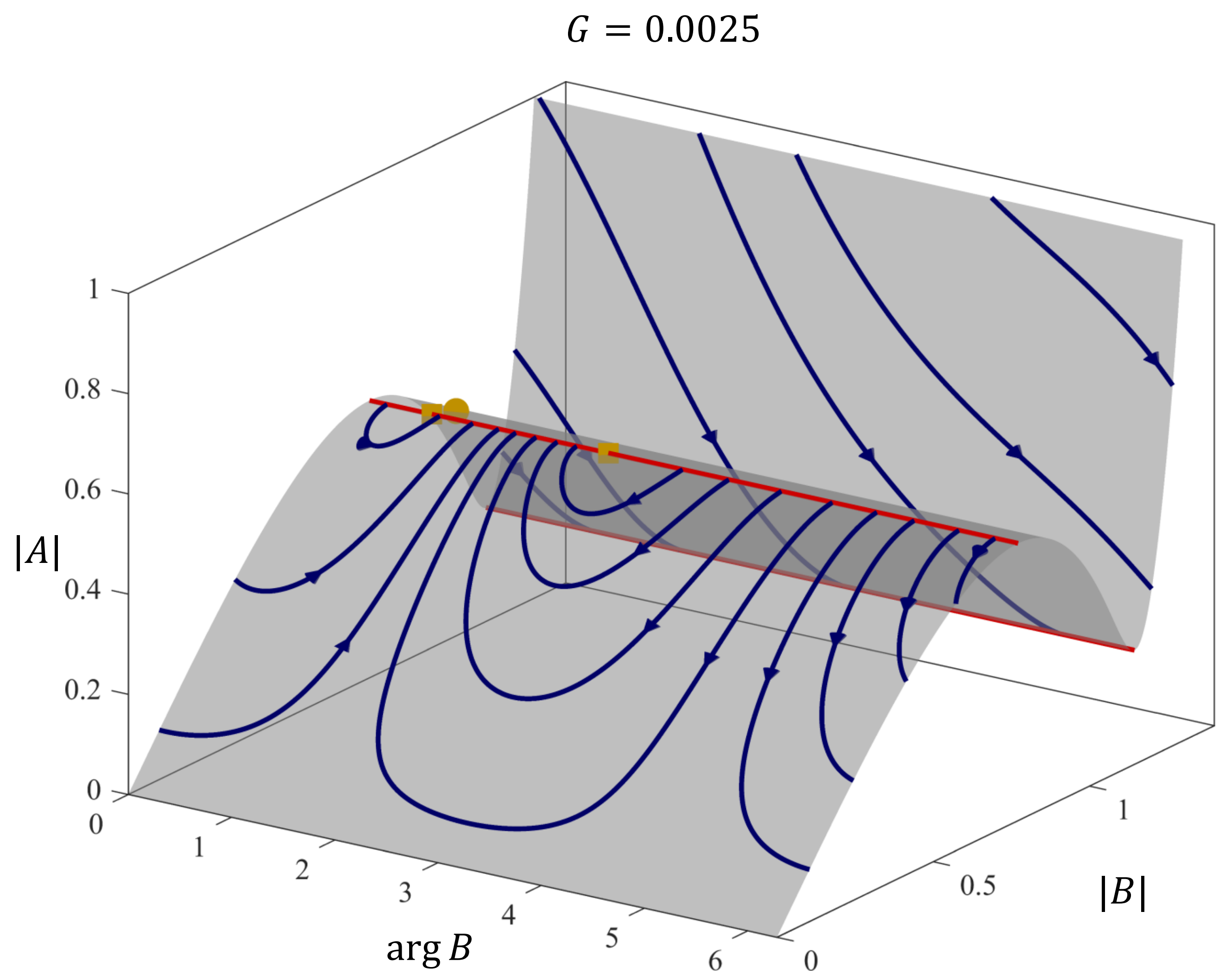
- Slow flow equation

$$\dot{b} = f_1(b, \theta), \qquad \dot{\theta} = f_2(b, \theta, \sigma)$$

- Grazing flow condition

$$\left.\frac{db}{d\theta}\right|_{b=b_1} \equiv \frac{f_1(b_1, \theta)}{f_2(b_1, \theta, \sigma)} = 0$$

G_{fs}



Creation of a pair of **folded singularities**

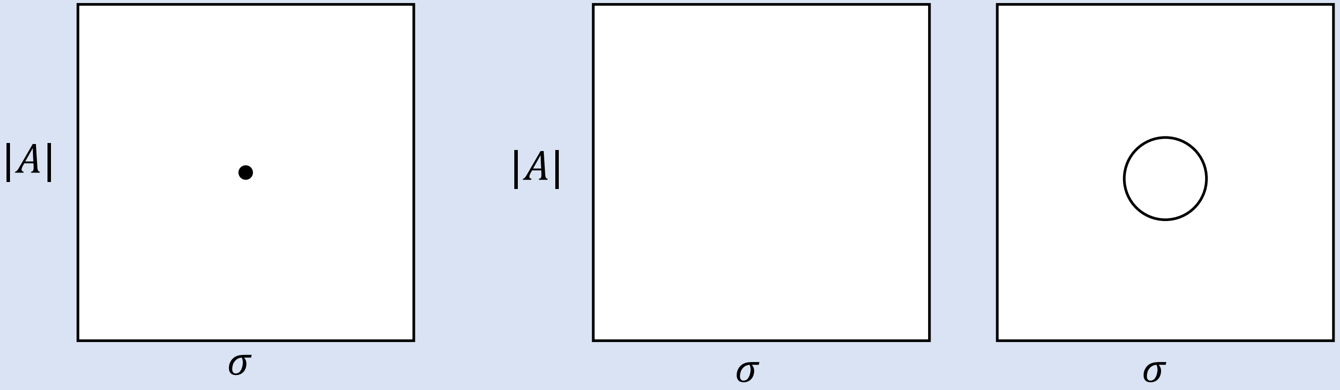
Dynamical behavior – Singularity theory to detect isola

Singularity theory

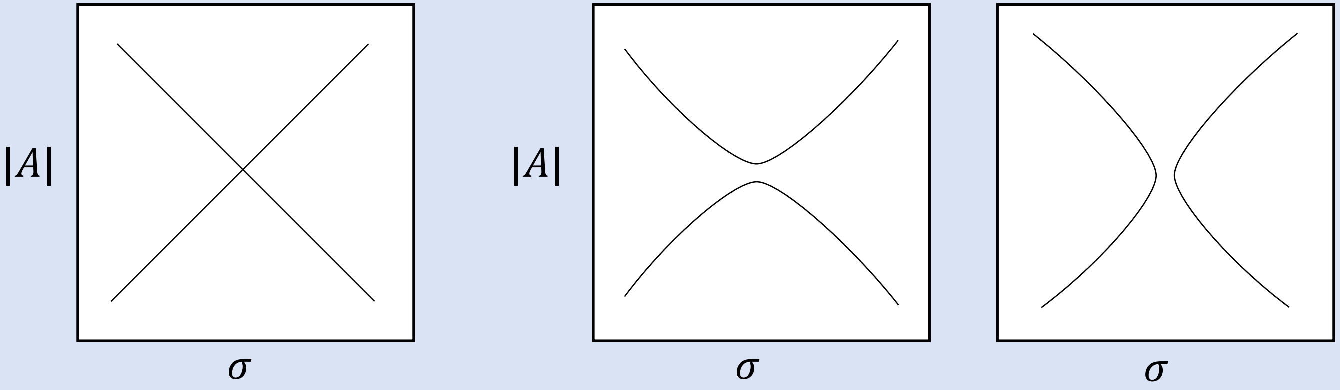
Tool to detect **topological** modifications of a **manifold** $h = 0$

- Isola singularity

Unperturbed diagram   Perturbed diagrams



- Simple bifurcation singularity (isola merging)



G_{isola}

$$h = 0, \quad \frac{\partial h}{\partial \sigma} = \frac{\partial h}{\partial b} = 0, \quad \det(d^2 h) > 0$$

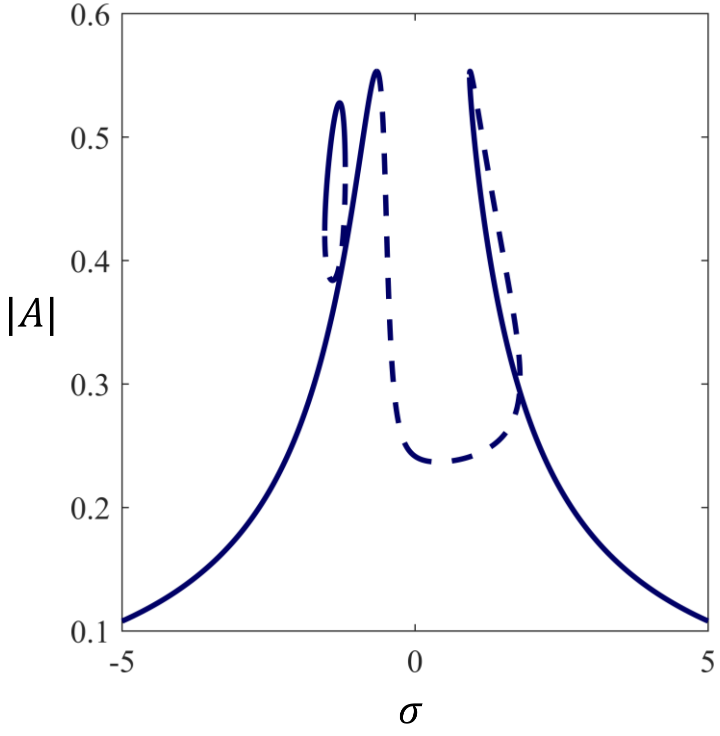
Fixed point expression	Defining condition	Non-degeneracy condition

$$h = 0, \quad \frac{\partial h}{\partial \sigma} = \frac{\partial h}{\partial b} = 0, \quad \det(d^2 h) < 0$$

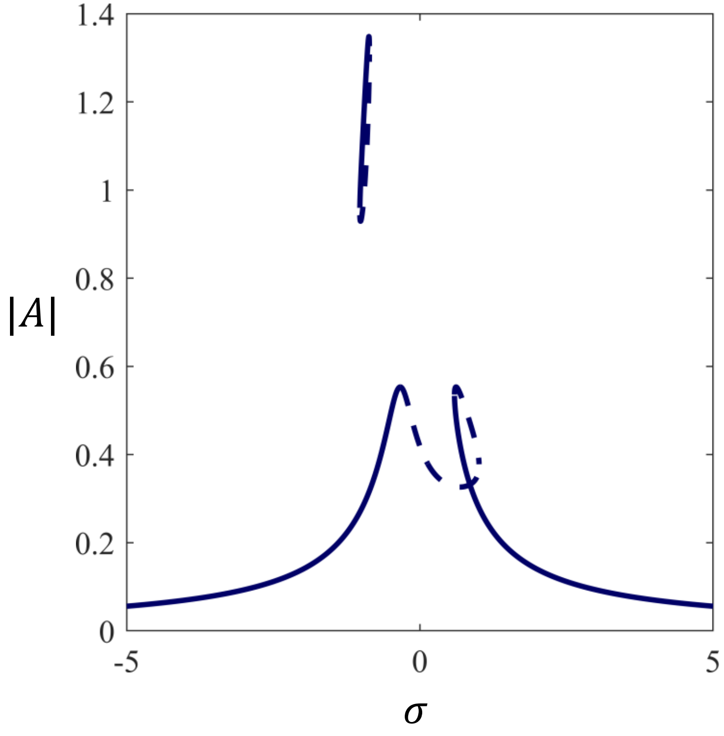
G_{merge}

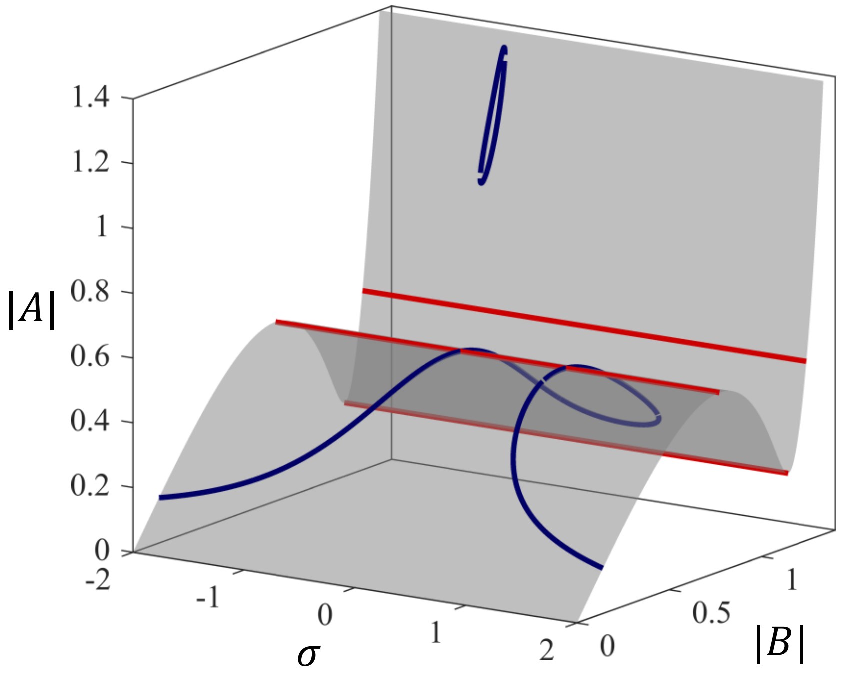
Dynamical behavior – All isolas are not dangerous!

Safe isola



Dangerous isola





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