

Projet ExFLEM



## Bistabilité via flambement : de la statique à la dynamique

17-18/10/24

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- **ANR ExFLEM – Extreme Flexible Metamaterials**
  - Utiliser les évènements ondulatoire extrême dans des métamatériaux non-linéaires flexibles



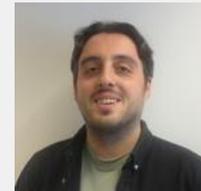
Vincent Tournat



Georgios Theocharis



Vassos Achilleos



Apostolos Paliovaivos



Stéphane Job

- Etudier les ondes non-linéaires dans les métamatériaux souples

**ExFLEM** = Étudier des ondes non linéaires dans des métamatériaux élastiques souples

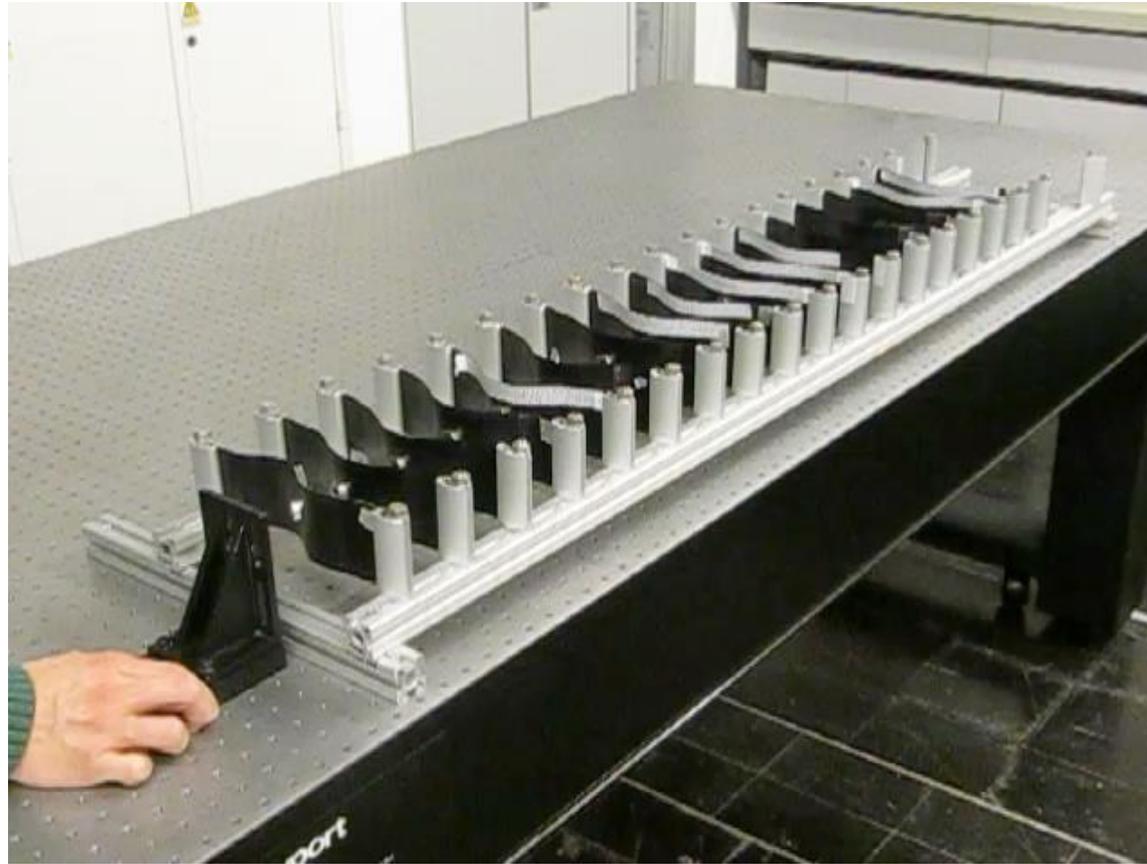
↳ Créer un réseau avec des propriétés élastiques non linéaires

↳ Comprendre et caractériser la cellule unitaire : **Dynamique**

↳ Comprendre et caractériser la cellule unitaire : **Quasi-statique**

# Métamatériaux mécaniques flexibles

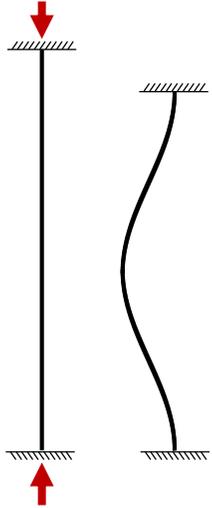
Ex : Ondes de transitions



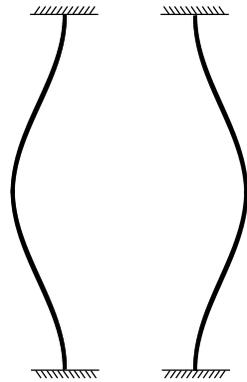
N. Nadkarni et.al., Phys. Rev. Lett.  
116, 244501 (2016)

# Bistabilité via flambement de poutre : statique

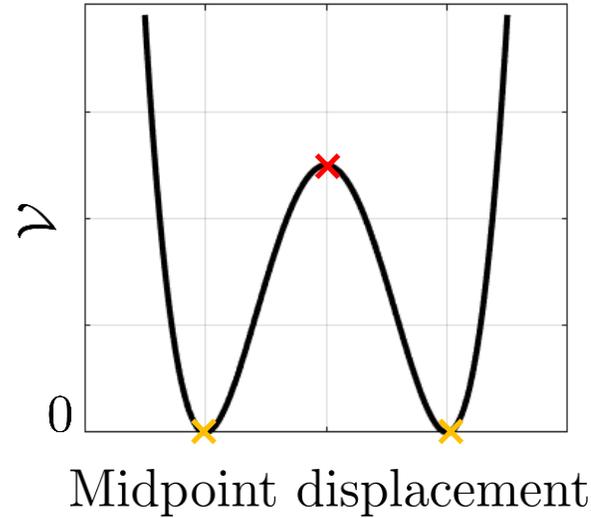
Flambage de poutre



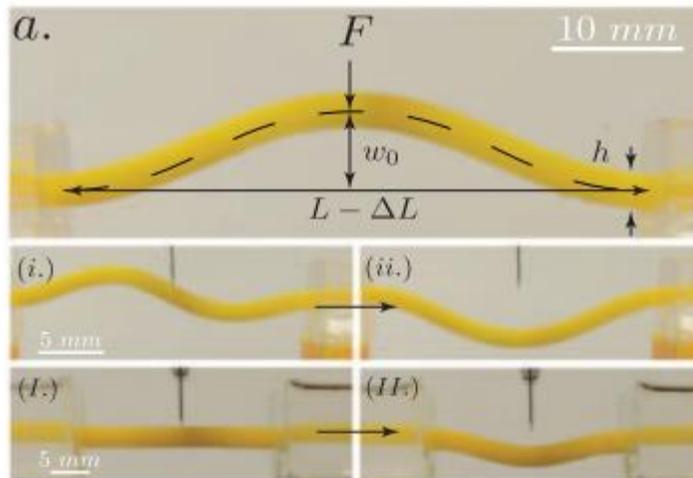
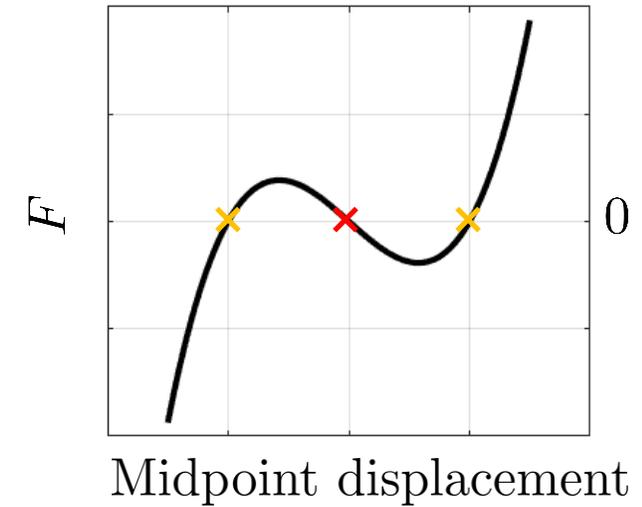
2 positions stables



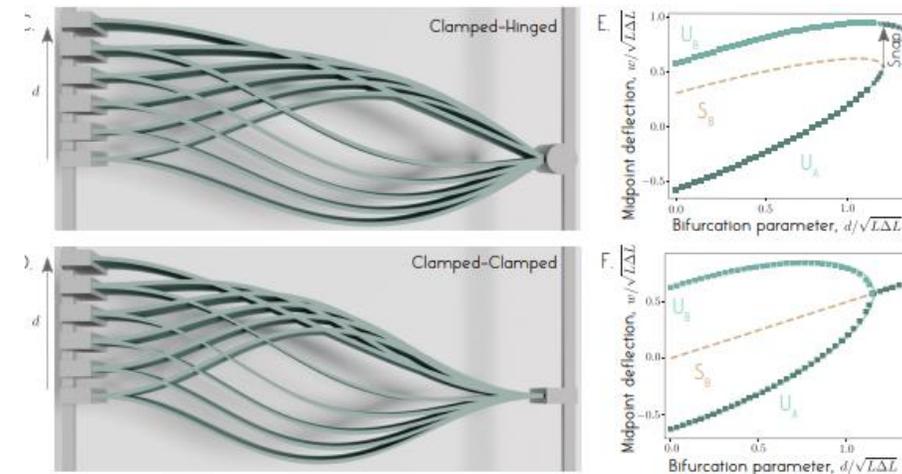
Potential energy



Nonlinear restoring force



Pandey, 2014.



Radisson, 2023.

# Flambement de poutre : dynamique ? Dissipation ?

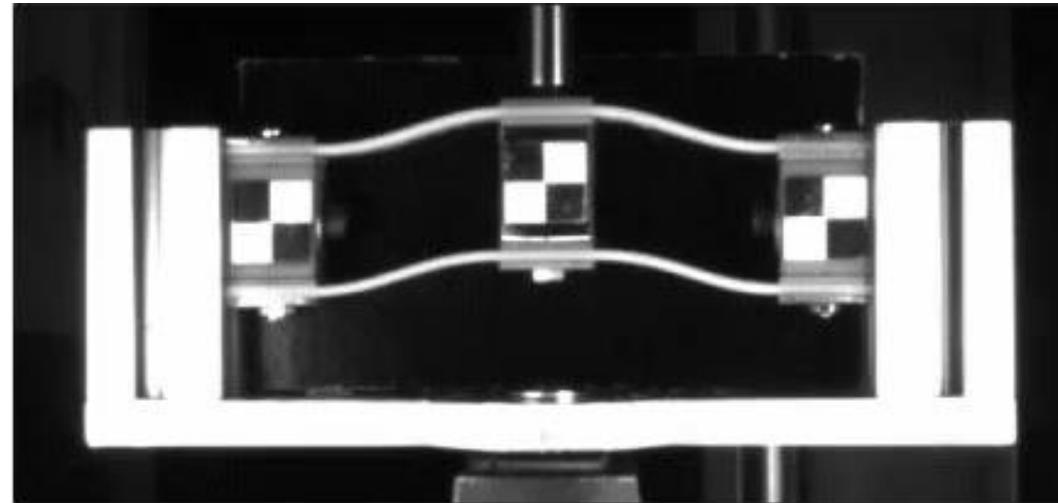
Systeme bi-poutre, encastree-encastree et connectees par une masse

→ pas de rotation du point milieu

Masse  $\gg$  masses poutres  
→ Augmente le facteur de qualite

## Principaux objectifs

- Comprendre les proprietes de la poutre
- Prediction du comportement en regime dynamique
- Construction d'une cellule unitaire adaptee a un reseau



2 materiaux differents

→ Polycarbonate (faibles pertes)

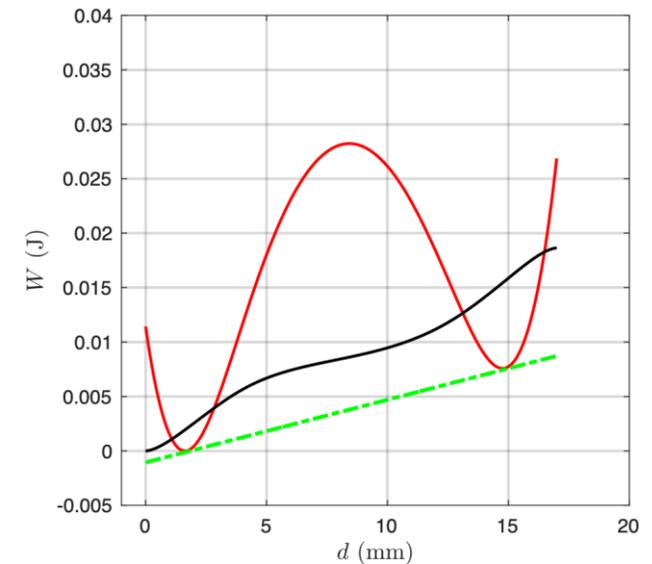
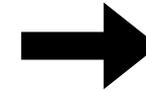
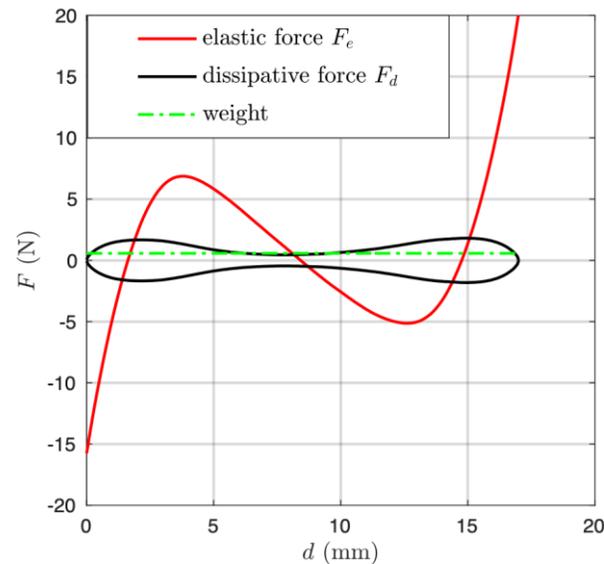
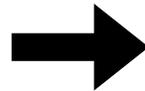
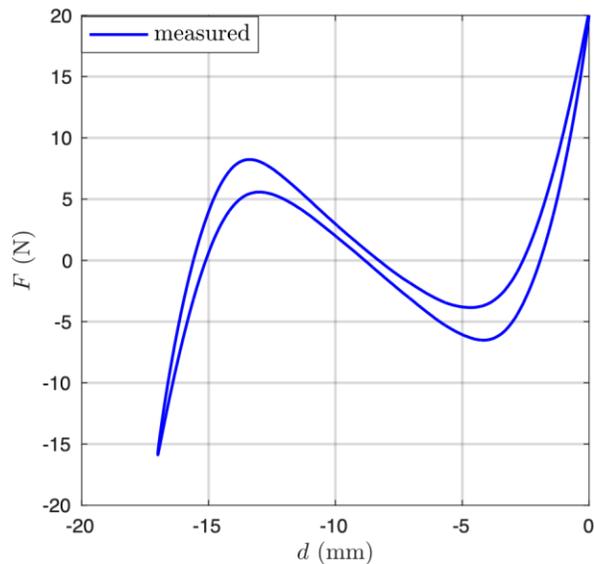
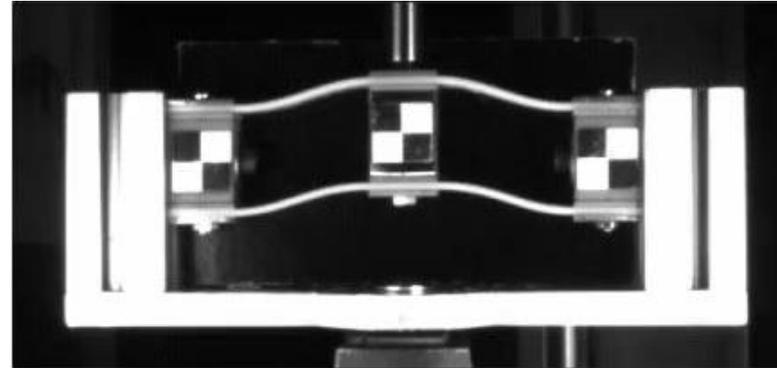
→ TPU Flex93 (pertes elevees)

# Relation force/déplacement

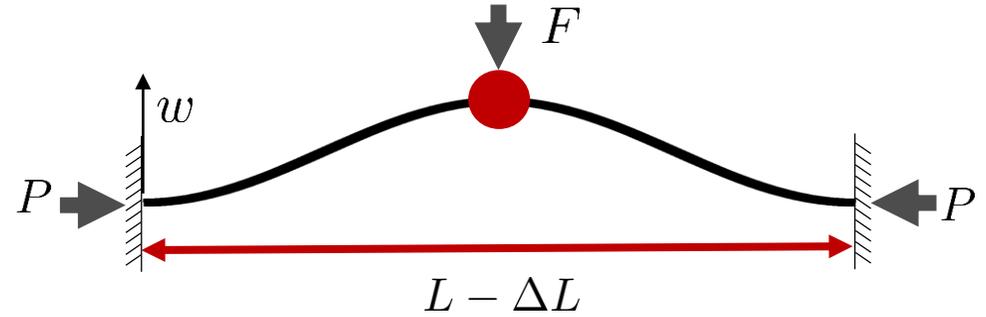
- Essais de traction/compression cycliques en quasi-statique
- Mesure de la relation force déplacement non-linéaire

$$F_d(d) = |F_d(d)|\text{sign}(\dot{d})$$

- Intégration de la force pour obtenir le travail



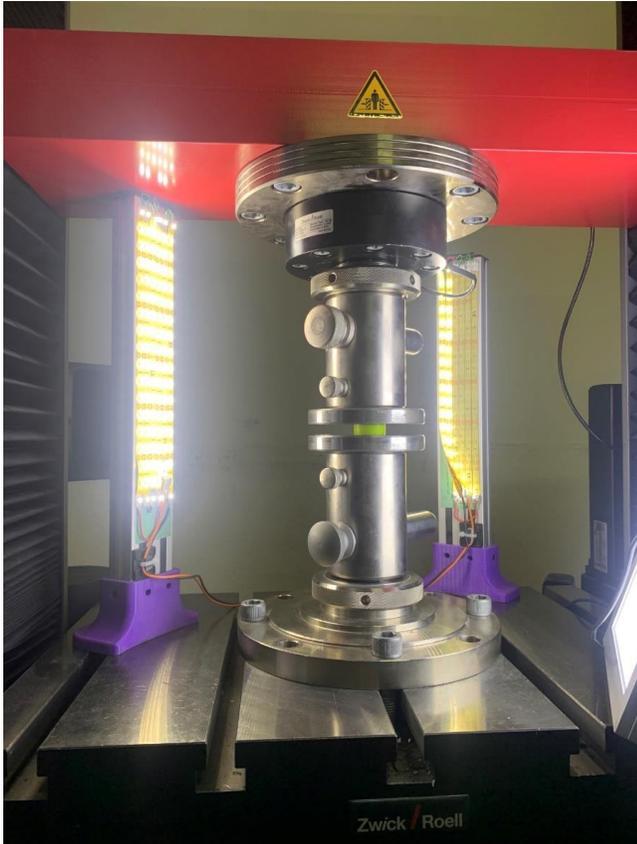
# Modèle de poutre en flambement



- Poutre d'Euler Bernoulli :  $EI \frac{\partial^4 w}{\partial x^4} + P \frac{\partial^2 w}{\partial x^2} = F \delta \left( x - \frac{L}{2} \right), x \in [0, L],$
- Contrainte géométrique (von Karman) :  $P = \frac{EA}{L} \left[ \Delta L - \frac{1}{2} \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 dx \right]$
- Conditions aux limites :  $w|_{x=0} = w|_{x=L} = \frac{\partial w}{\partial x} |_{x=0} = \frac{\partial w}{\partial x} |_{x=L} = \frac{\partial w}{\partial x} |_{x=L/2} = 0.$

Energie de deformation de la poutre : 
$$\mathcal{V} = \underbrace{\frac{1}{2} EI \int_0^L \left( \frac{\partial^2 w}{\partial x^2} \right)^2 dx}_{\text{Flexion}} - \underbrace{\frac{EA \Delta L}{2L} \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 dx + \frac{EA}{8L} \left[ \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 dx \right]^2}_{\text{Compression/élongation}}$$

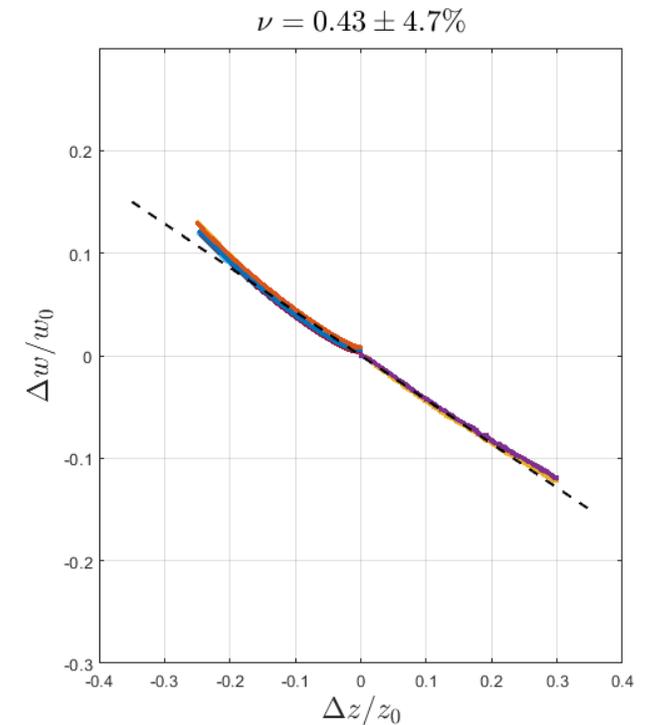
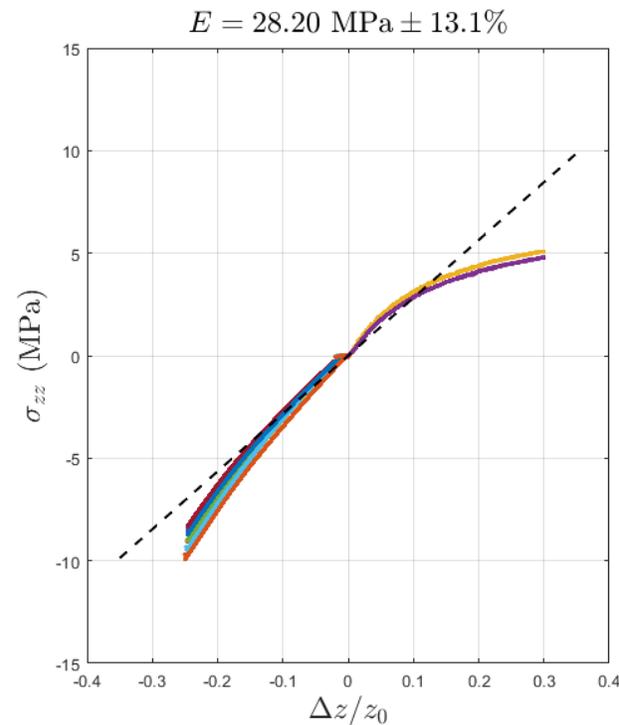
# Module d'Young de Hooke en petites déformations VS comportement de Neo-Hooke incompressible



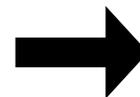
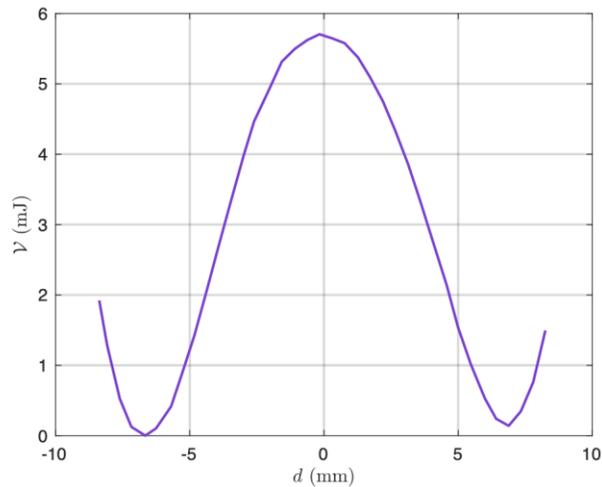
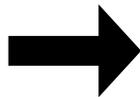
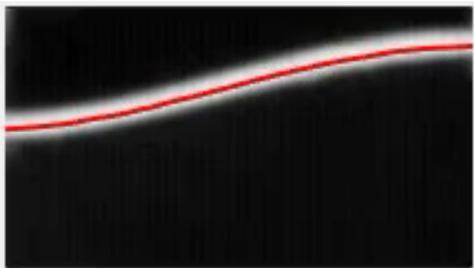
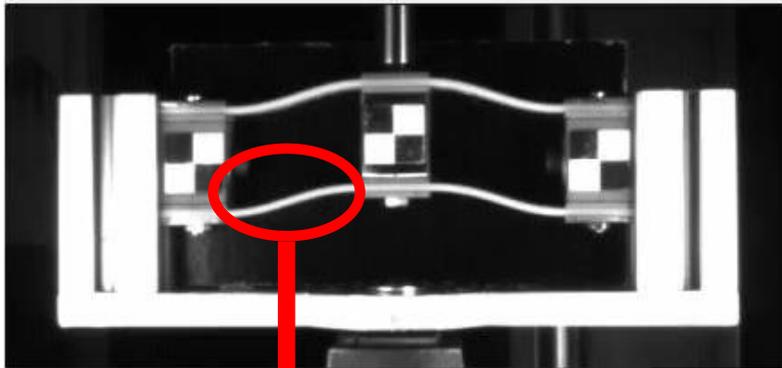
Test de compression sur  
une éprouvette  
cylindrique de Flex93



Eprouvette haltère de traction d'un échantillon de Flex93

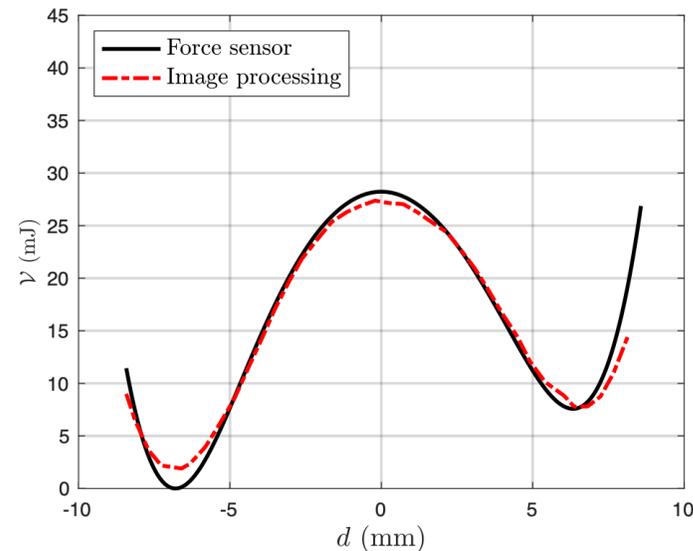
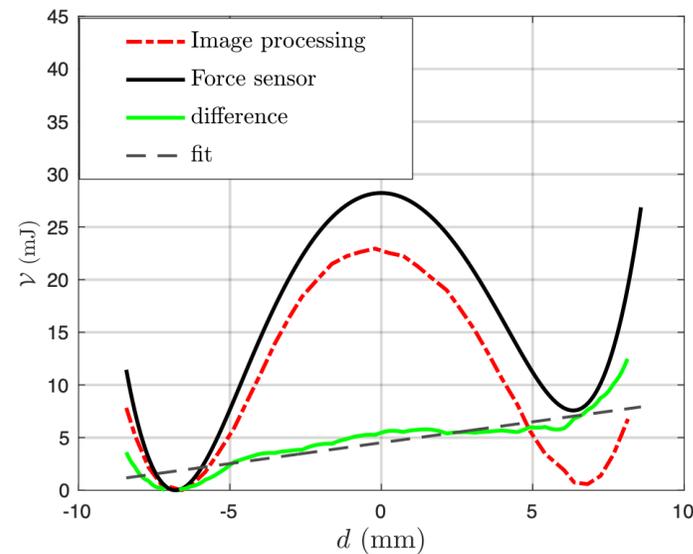


# Energie de déformation via la mesure de champs



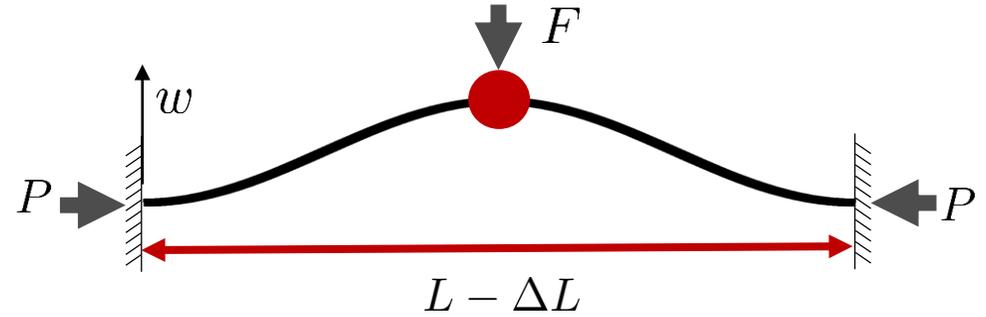
Calcul de l'énergie

$$\mathcal{V} = \left( \frac{\partial w}{\partial x}, \frac{\partial^2 w}{\partial x^2} \right)$$



Extraction de la flèche  $w(x)$   
via une analyse vidéo

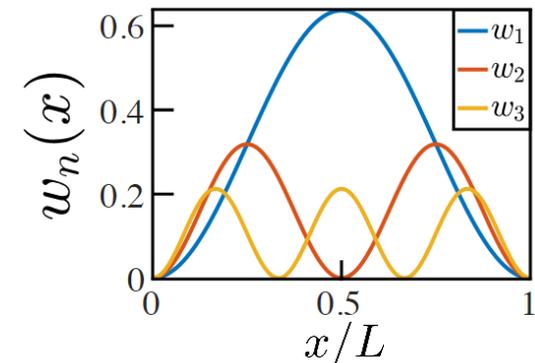
# Modèle de poutre en flambement



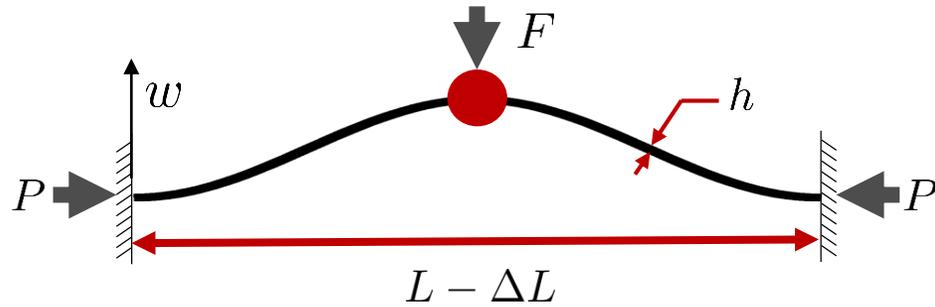
- Poutre d'Euler Bernoulli pré-contrainte :  $EI \frac{\partial^4 w}{\partial x^4} + P \frac{\partial^2 w}{\partial x^2} = F \delta \left( x - \frac{L}{2} \right), x \in [0, L],$
- Contrainte géométrique (von Karman) :  $P = \frac{EA}{L} \left[ \Delta L - \frac{1}{2} \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 dx \right]$
- Conditions aux limites :  $w|_{x=0} = w|_{x=L} = \frac{\partial w}{\partial x} \Big|_{x=0} = \frac{\partial w}{\partial x} \Big|_{x=L} = \frac{\partial w}{\partial x} \Big|_{x=L/2} = 0.$

**Décomposition modale de la solution :**  $w(x) = \sum_{n=1}^{\infty} c_n w_n(x)$

$$w_n(x) = \sqrt{\frac{2EI}{P_n}} [1 - \cos(\kappa_n x)], \quad P_n = EI \left( \frac{2\pi n}{L} \right)^2, \quad \kappa_n = 2\pi n, \quad n = \{1, 2, \dots\}$$



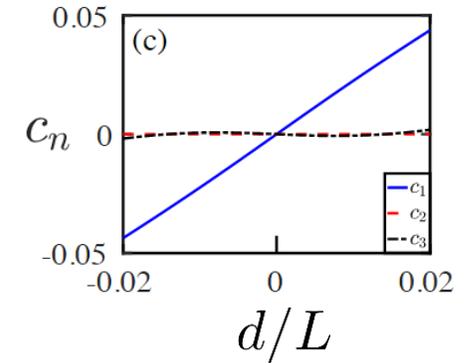
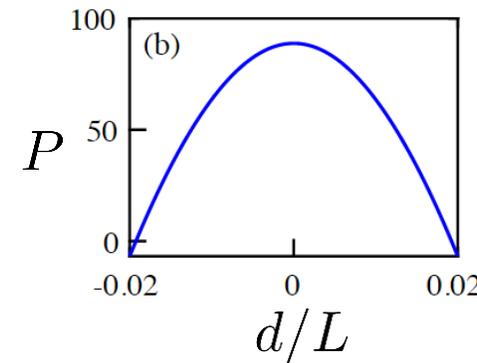
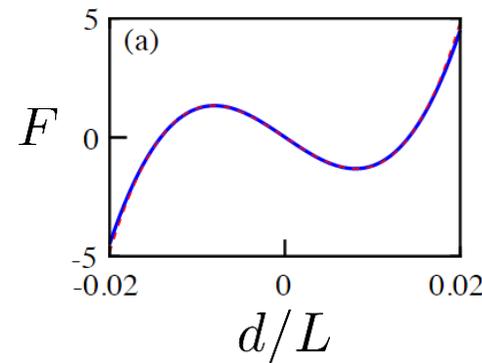
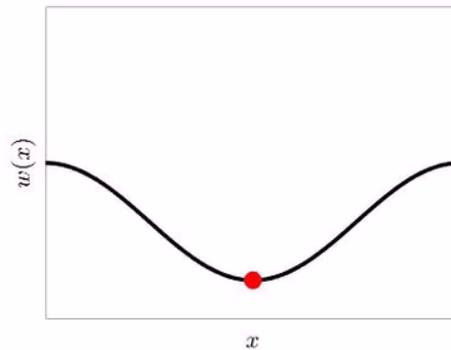
# Expression de la solution sur la base des modes de flambement



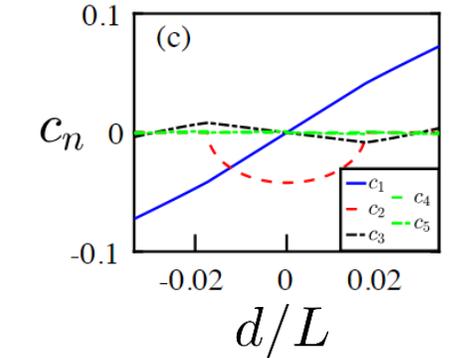
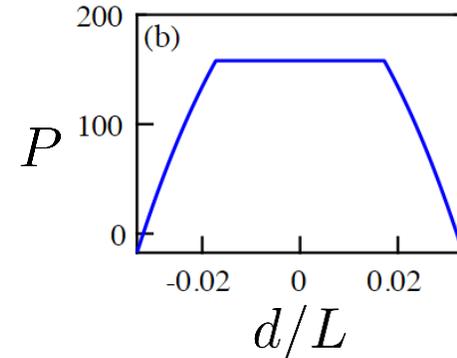
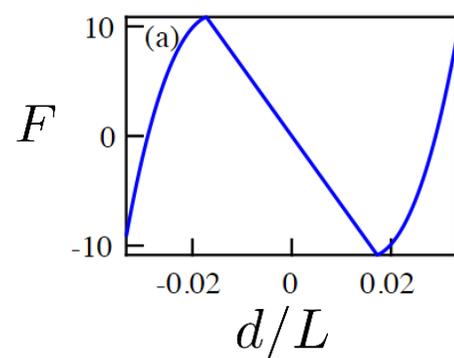
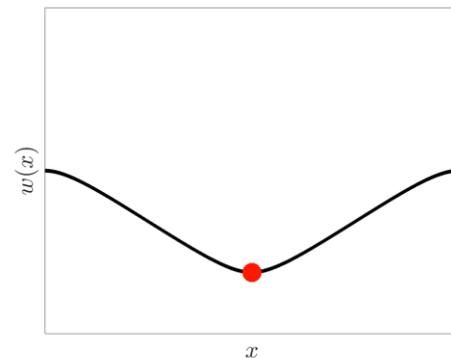
$$w(x) = \sum_{n=1}^{\infty} c_n w_n(x)$$

Déplacement du point milieu  $d = w(x = L/2)$

$\Delta L$  petit  
 $h$  grand

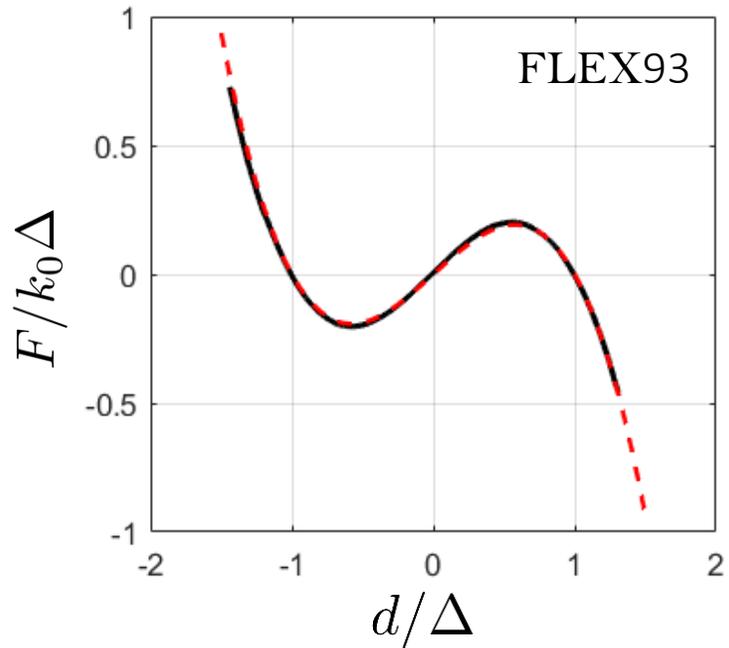


$\Delta L$  grand  
 $h$  petit



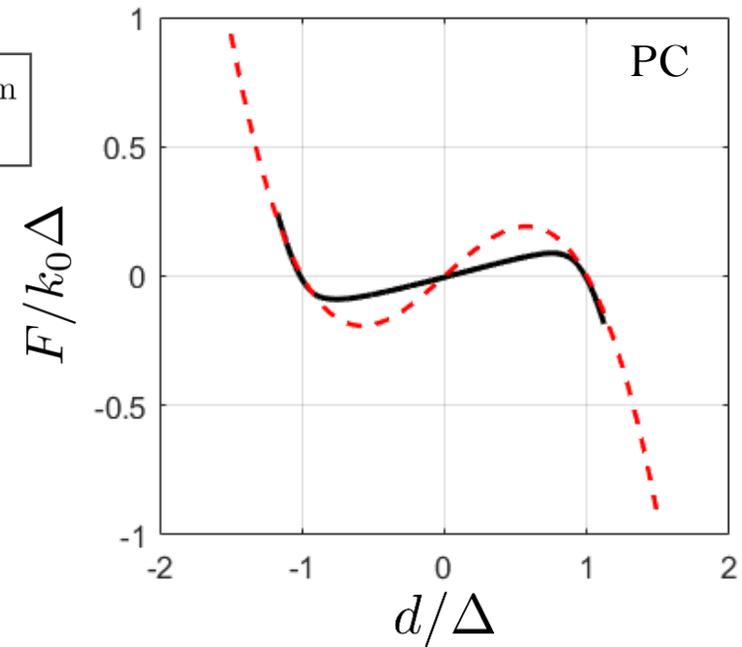
# Validation expérimentale

$\Delta L$  petit,  $h$  grand

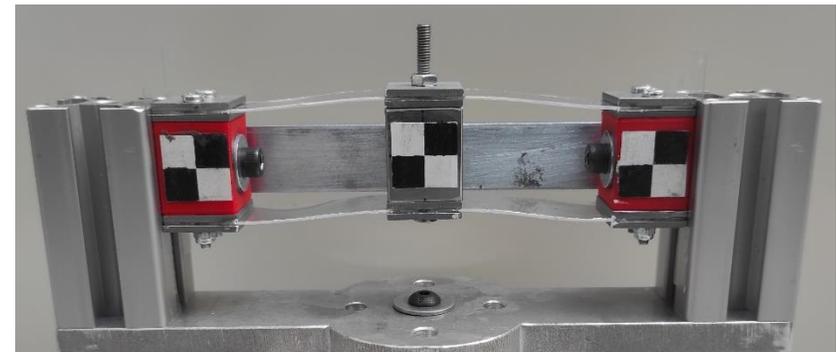
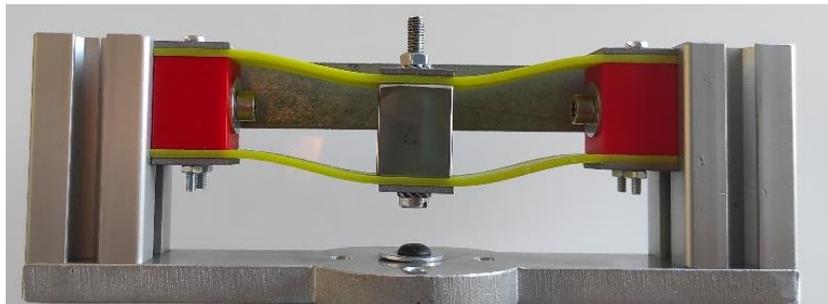


$$w(x) \approx c_1 w_1(x)$$

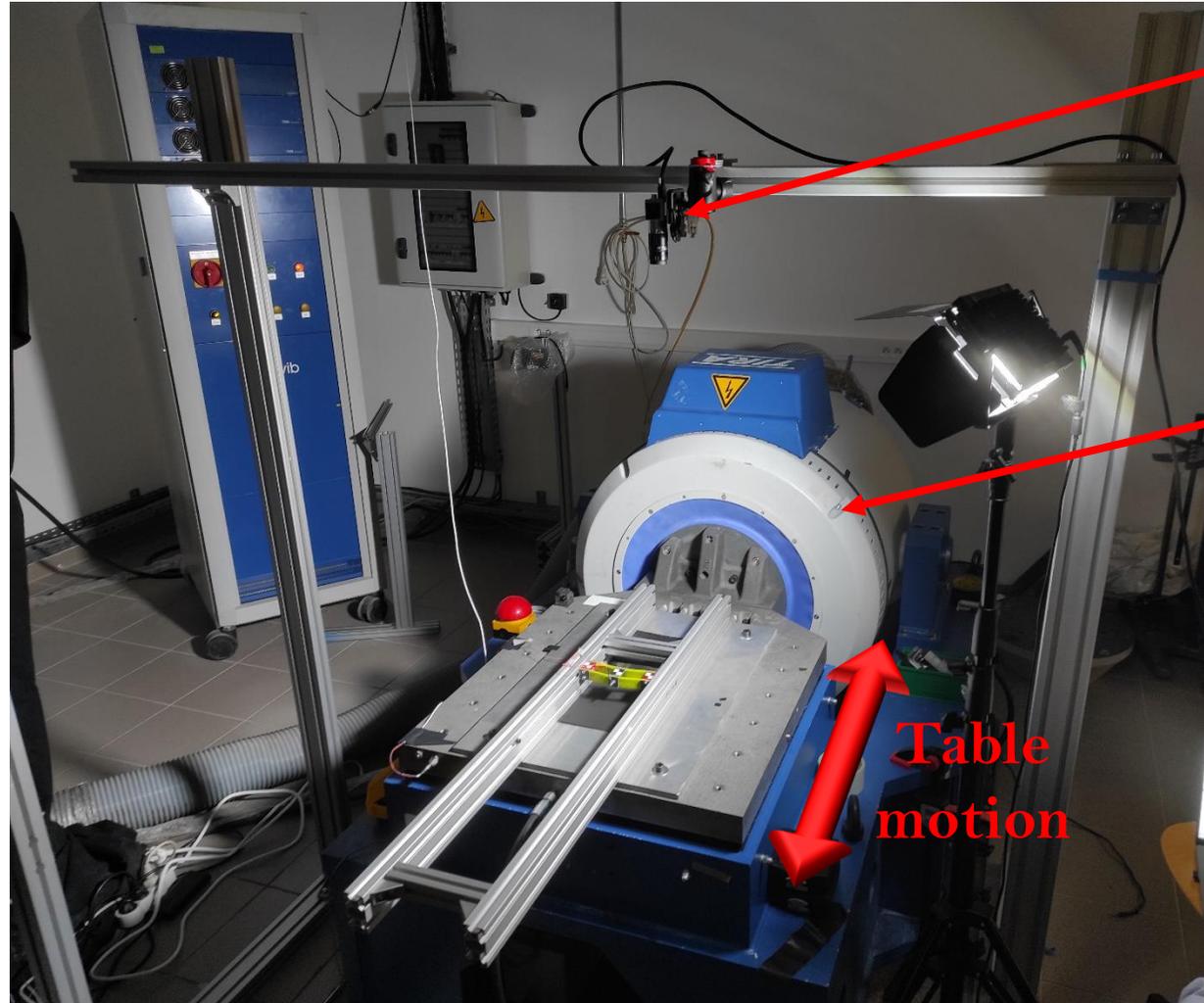
$\Delta L$  grand,  $h$  petit



$$w(x) \approx c_1 w_1(x) + c_2 w_2(x)$$



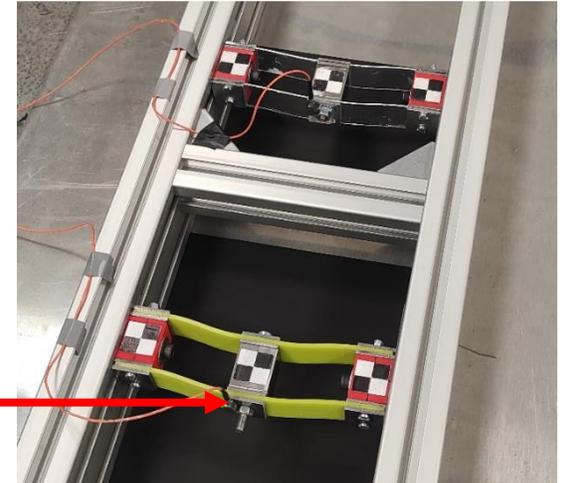
# Essai en dynamique



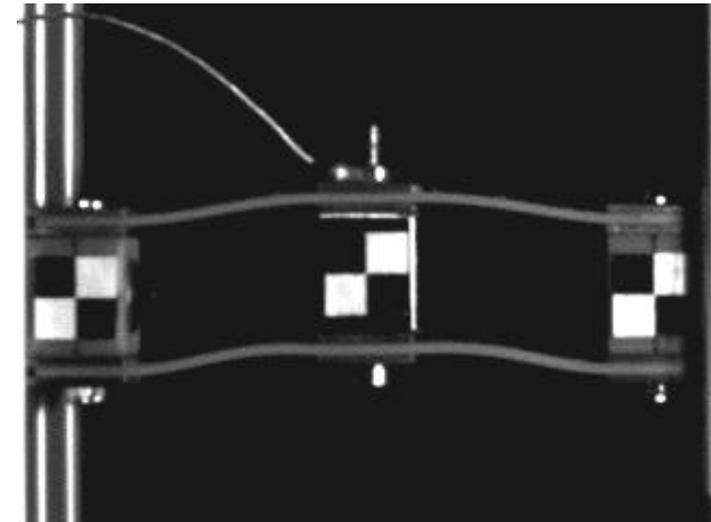
Camera

Pot  
vibrant

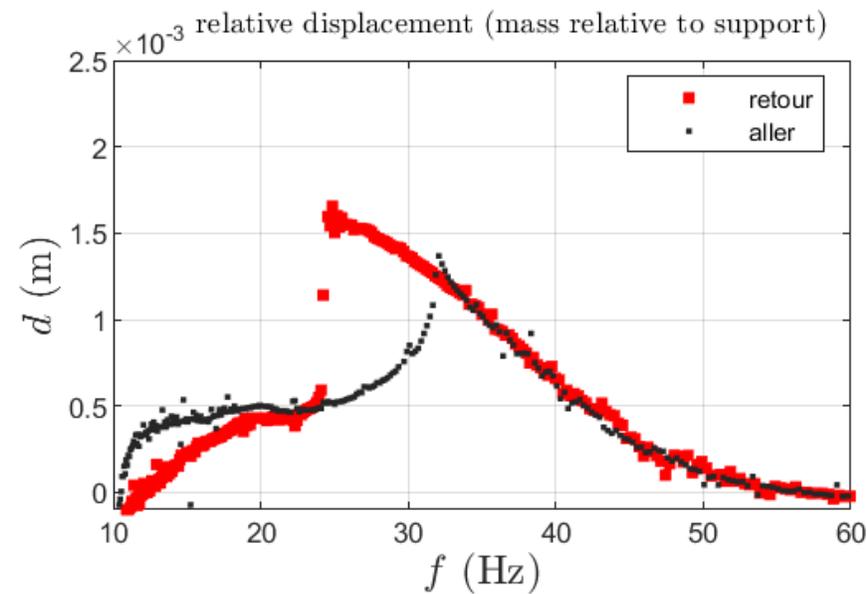
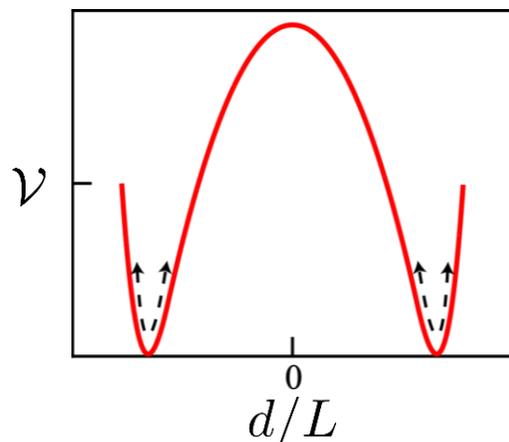
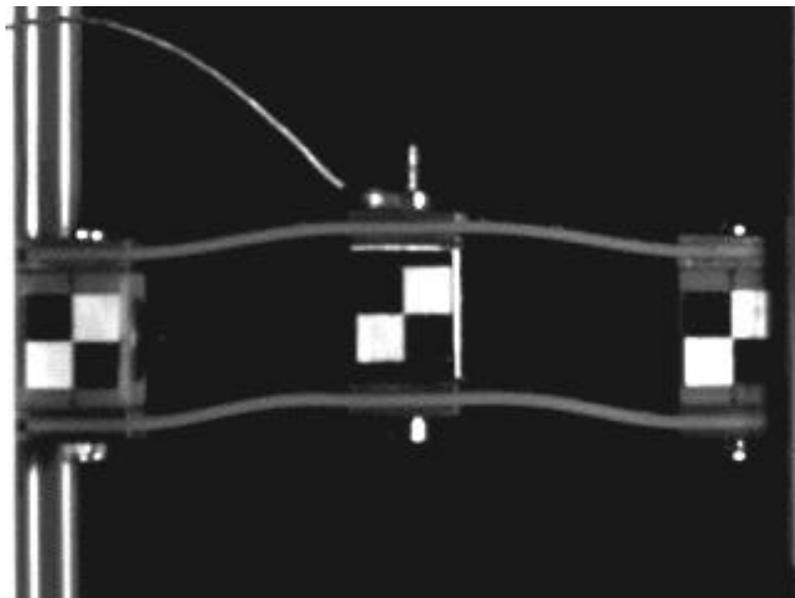
Table  
motion



accéléromètre



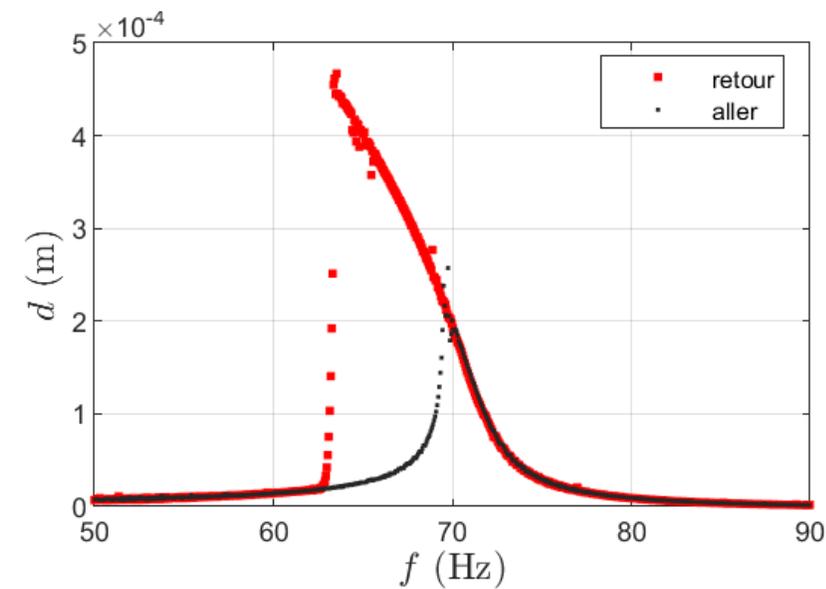
# Réponse fréquentielle



Échantillon de  
TPU FLEX93 à 3g



Échantillon de  
PC à 0.15g



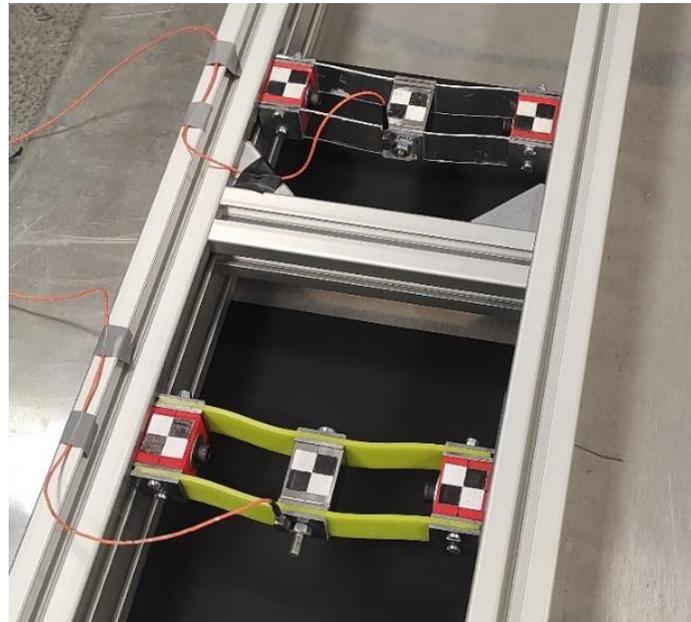
# Conclusion

ExFLEM = Étudier des ondes non linéaires dans des métamatériaux élastiques souples

↳ Créer un réseau avec des propriétés élastiques non linéaires

↳ Comprendre et caractériser la cellule unitaire : **Dynamique**

↳ Comprendre et caractériser la cellule unitaire : **Quasi statique**



Merci pour votre attention