Computational methods for whirl flutter analysis
urban air mobility vehicles

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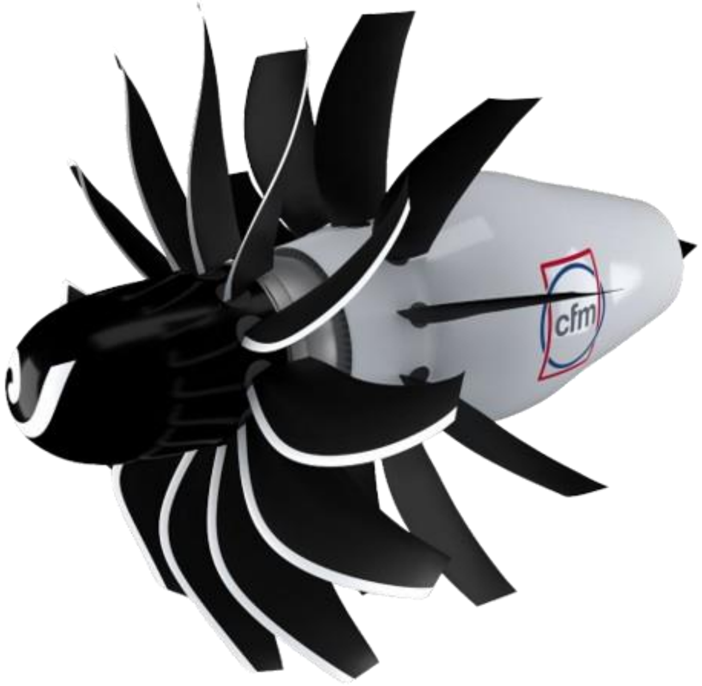
Context

- Work started in 2020 at Imperial College London with Professor Vahdati
- Since 2020, I have been trying to work on this topic…

https://vertical-aerospace.com/wp-content/uploads/2024/07/Vertical-Aerospace-Begins-Testing.pdf

Other applications

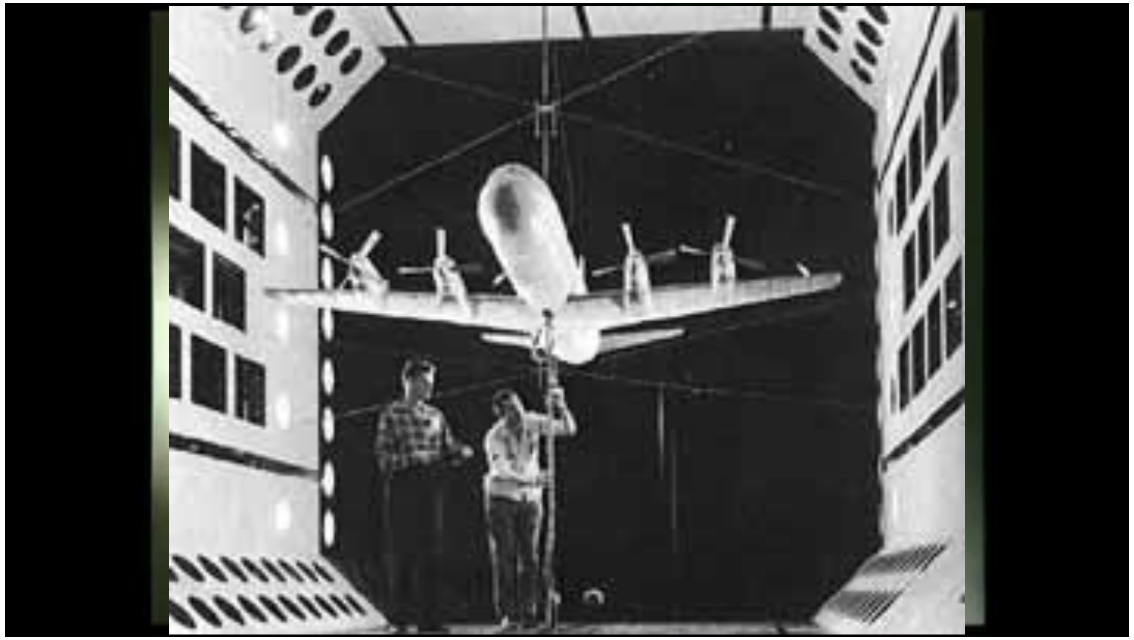
- Future aircraft engine
- Floating wind turbine
- Industrial air-cooling system
- …



What is Whirl Flutter

- Whirl Flutter is an aeroelastic instability in rotating systems, where the coupling between the aerodynamic forces and gyroscopic effects causes self-excited oscillations.
- This instability arises from the interaction between the rotor's lateral and torsional motions, with gyroscopic effects playing a critical role.

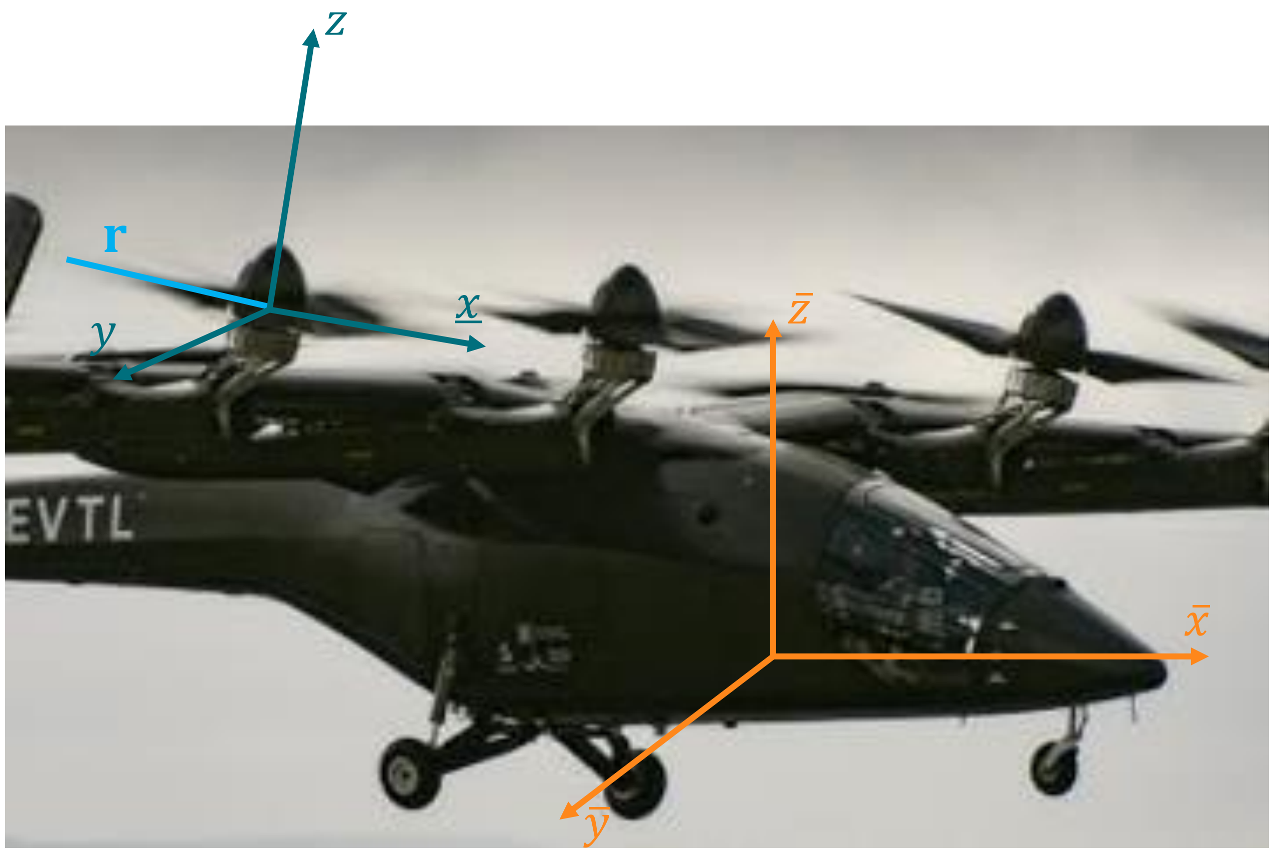
Lockheed L-188 Electra

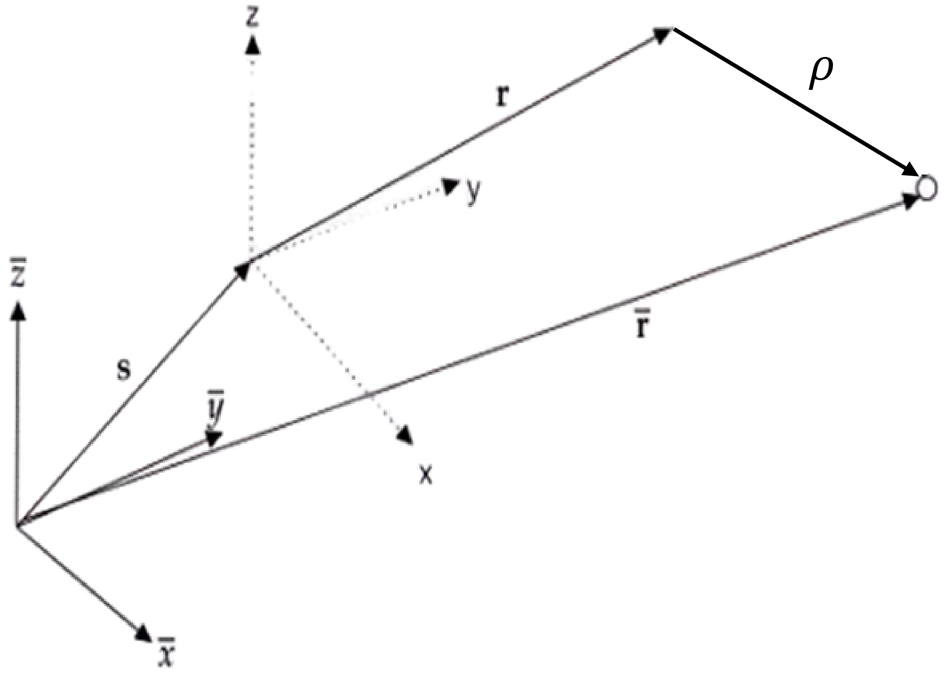


Outline

- Introduction: context, objectives, methods
- Formulation of rotating structures using the floating frame concept
- Implementation and verification
- Time integration using Newmark schemes
- Preliminary results
- Conclusions and Future work

Structure in Rotation coupled to a stationary frame





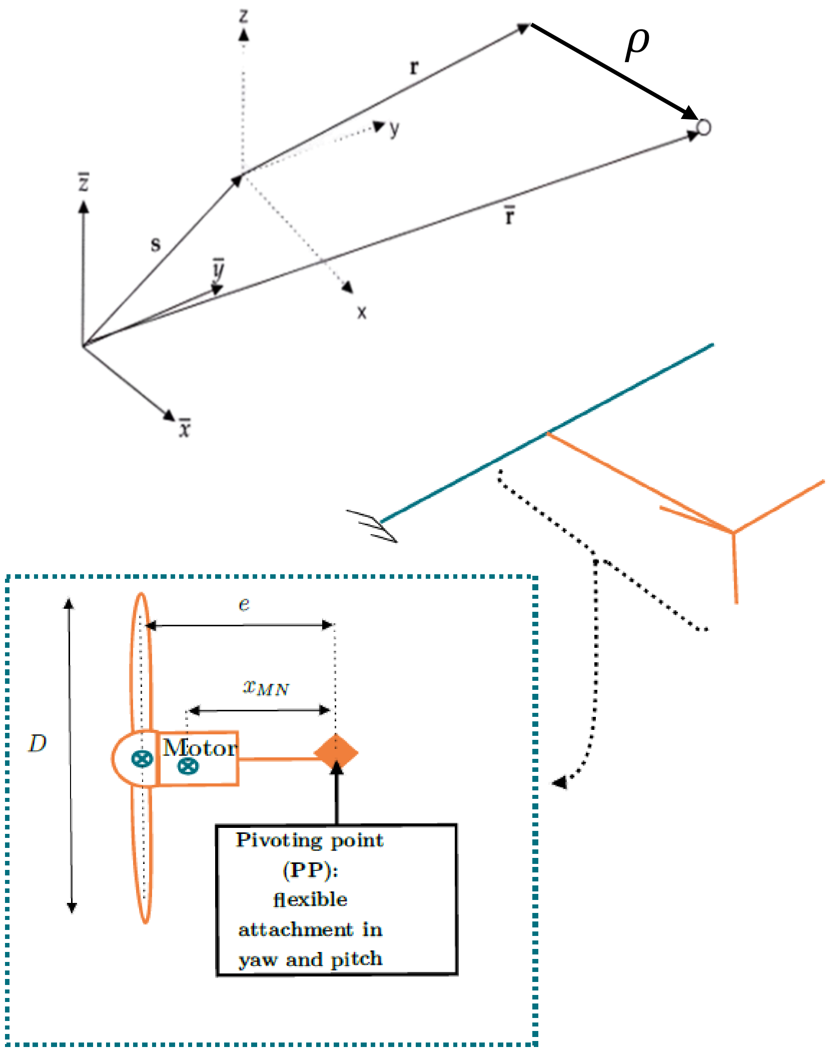
Structure in Rotation – Different approaches

Which formulation to use?

- Rigid Multibody Dynamics

- Finite element model for rotating structure
  - Floating Frame Reference
  - Absolute Nodal Coordinate Formulation
  - Quaternion
  - Lie Group
  - …

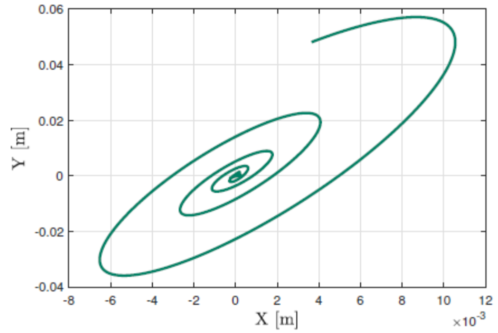
- In this work: Beam element



Stability analysis – Strategy 1

- Ordinary Differential Equation with periodic coefficients

$$\mathbf{M}(t)\ddot{\mathbf{q}} + \mathbf{D}(t)\dot{\mathbf{q}} + \mathbf{K}(t)\mathbf{q} = \mathbf{0}$$



- List of approaches
  - Numerical time integration
  - Linearised system at each time step (Qblade)
  - Method based on Floquet's theory using numerical time scheme
  - Coleman Transformationor multiblade coordinate transformation

$$q_0 = \frac{1}{N}\sum_{b=1}^{N} q_b\,, \qquad q_n^c = \frac{2}{N}\sum_{b=1}^{N} q_b \cos n\psi_b\,, \qquad q_n^s = \frac{2}{N}\sum_{b=1}^{N} q_b \sin n\psi_b$$

SIEMENS

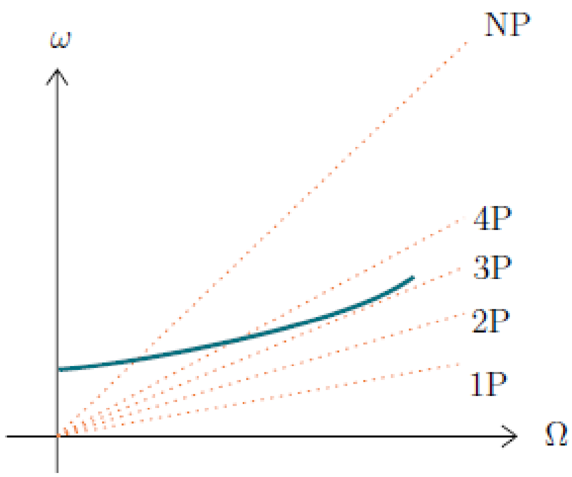
SOL414 Rotor Dynamics

  - Hill's method

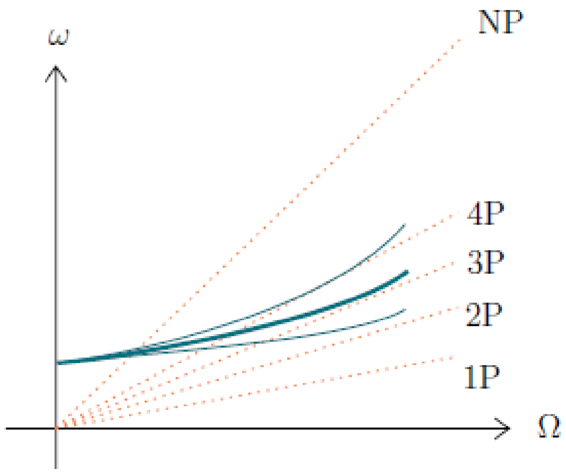
Stability analysis – Strategy 2

- Build a finite element model of the time coupling ODE between the stationary and rotating structures.
- We will use the Floquet theory and study the stability thanks to the eigenvalues of the monodromy matrix computed using a Newmark scheme

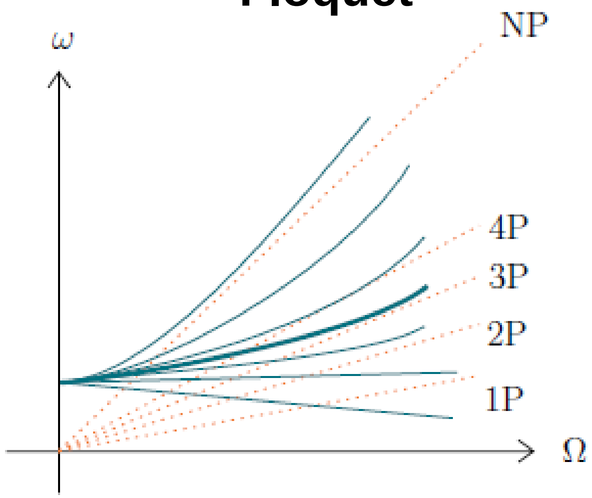
LTI



Coleman



Floquet



Theory of time dependent mechanical coupling

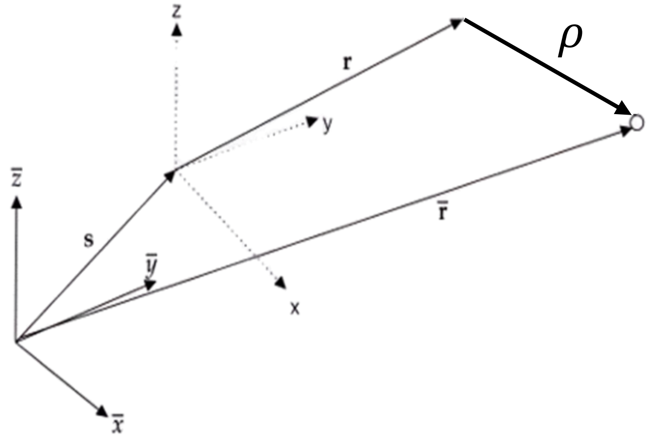
- Each of the rotating components is expressed in a floating frame of reference.

- The position vector of a rotating structural component with respect to a stationary component is written as

$$\bar{\mathbf{r}} = \mathbf{s} + (\mathbf{I} + \mathbf{B}(\beta))\mathbf{H}(\boldsymbol{\Omega}\mathbf{t})(\mathbf{r} + \boldsymbol{\rho})$$

- The potential energy is

$$V = \frac{1}{2}\int_V \boldsymbol{\sigma}_0 : \boldsymbol{\epsilon}(\boldsymbol{\rho})dV + \frac{1}{2}\int_V \boldsymbol{\sigma}_d^T : \boldsymbol{\epsilon}(\boldsymbol{\rho})dV$$



- No time coupling on elastic and geometric stiffness matrices

Theory of time dependent mechanical coupling

$$\bar{\mathbf{r}} = \mathbf{s} + (\mathbf{I} + \mathbf{B}(\boldsymbol{\beta}))\mathbf{H}(\Omega\mathrm{t})(\mathbf{r} + \boldsymbol{\rho})$$

- We introduce

$$B_0(\Omega t)\beta = B(\beta)H(\Omega t)r$$

$$\bar{\mathbf{r}} = \mathbf{s} + \mathbf{B_0}\boldsymbol{\beta} + \mathbf{H}(\Omega\mathrm{t})\boldsymbol{\rho} + \mathbf{H}(\Omega\mathrm{t})\boldsymbol{r} + \boldsymbol{B}(\boldsymbol{\beta})\boldsymbol{H}(\boldsymbol{\Omega}\mathbf{t})\boldsymbol{\rho}$$

- The time derivative of the position derivative is given by:

$$\dot{\bar{r}} = \dot{s} + B_0\dot{\beta} + \dot{B}_0\beta + \dot{H}\rho + H\dot{\rho} + BH\dot{\rho} + \dot{H}r$$

- We define the operator

$$\mathbf{P} = [\mathbf{I}\ \mathbf{B_0}\ \mathbf{H}\ \mathbf{H}]$$

Time coupling on structural mass matrix, structural gyroscopic matrix and centrifugal stiffness matrix.

$$T = \frac{1}{2}m\left(\dot{\mathbf{w}}\mathbf{P}^{\mathbf{T}}\mathbf{P}\dot{\mathbf{w}} + \mathbf{w}\dot{\mathbf{P}}^{\mathbf{T}}\dot{\mathbf{P}}\mathbf{w} + 2\mathbf{w}^{\mathbf{T}}\dot{\mathbf{P}}^{\mathbf{T}}\mathbf{P}\dot{\mathbf{w}}\right)$$

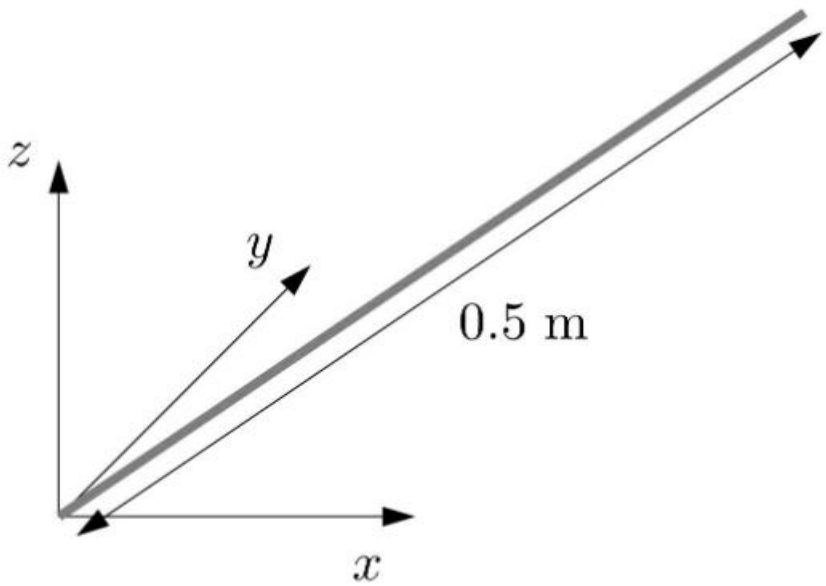
Theory of time dependent mechanical coupling

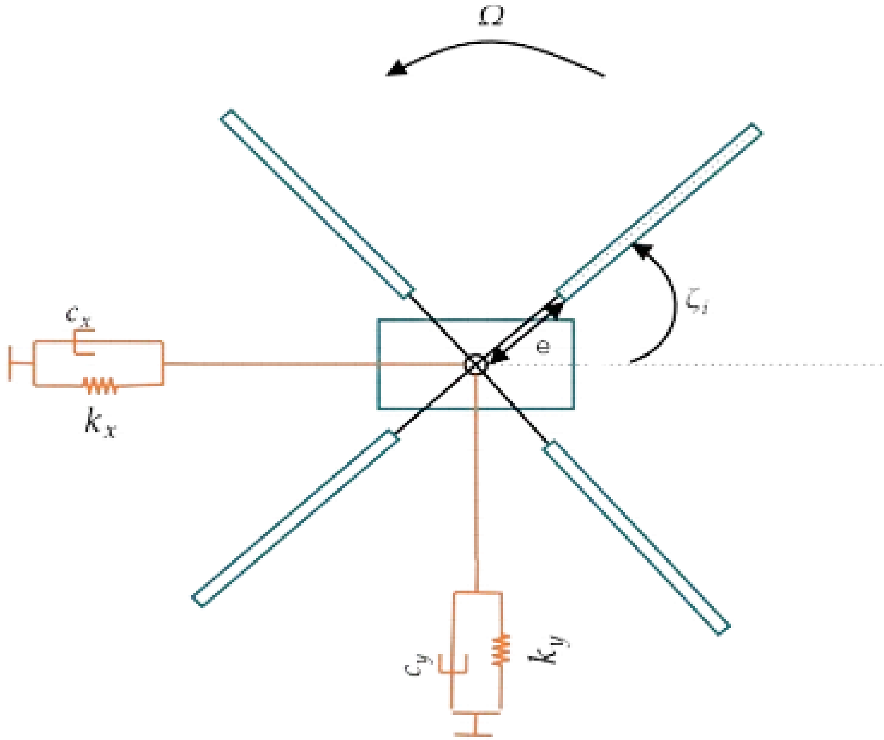
- The fully coupled Lagrange equations gives the following equation of motion without damping

$$m\begin{bmatrix} \mathbf{I} & \mathbf{B}_0 & \mathbf{H} \\ \mathbf{B}_0 & \mathbf{B}_0^T\mathbf{B}_0 & \mathbf{B}_0^T\mathbf{H} \\ \mathbf{H}^T & \mathbf{H}^T\mathbf{B}_0 & \mathbf{I} \end{bmatrix}\begin{Bmatrix} \ddot{\mathbf{s}} \\ \ddot{\boldsymbol{\beta}} \\ \ddot{\boldsymbol{\rho}} \end{Bmatrix} + 2m\Omega\begin{bmatrix} 0 & \overline{\mathbf{B}}_0 & \overline{\mathbf{H}} \\ 0 & \mathbf{B}_0^T\overline{\mathbf{B}}_0 & \mathbf{B}_0^T\overline{\mathbf{H}} \\ 0 & \mathbf{H}^T\overline{\mathbf{B}}_0 & \mathbf{H}^T \end{bmatrix}\begin{Bmatrix} \dot{\mathbf{s}} \\ \dot{\boldsymbol{\beta}} \\ \dot{\boldsymbol{\rho}} \end{Bmatrix} + m\Omega^2\begin{bmatrix} 0 & \overline{\overline{\mathbf{B}}}_0 & \overline{\overline{\mathbf{H}}} \\ 0 & \mathbf{B}_0^T\overline{\overline{\mathbf{B}}}_0 & \mathbf{B}_0^T \\ 0 & \mathbf{H}^T\overline{\overline{\mathbf{B}}}_0 & \mathbf{H}^T\overline{\overline{\mathbf{H}}} \end{bmatrix}\begin{Bmatrix} \mathbf{s} \\ \boldsymbol{\beta} \\ \boldsymbol{\rho} \end{Bmatrix} + \mathbf{K}_T\begin{Bmatrix} \mathbf{s} \\ \boldsymbol{\beta} \\ \boldsymbol{\rho} \end{Bmatrix} = \begin{Bmatrix} -m\overline{\overline{\mathbf{H}}}\mathbf{r} \\ -m\mathbf{B}_0^T\overline{\overline{\mathbf{H}}}\mathbf{r} \\ \mathbf{f}_\rho \end{Bmatrix}$$

Verification on the partial coupling

- Simple Rotating beam* and ground resonance model

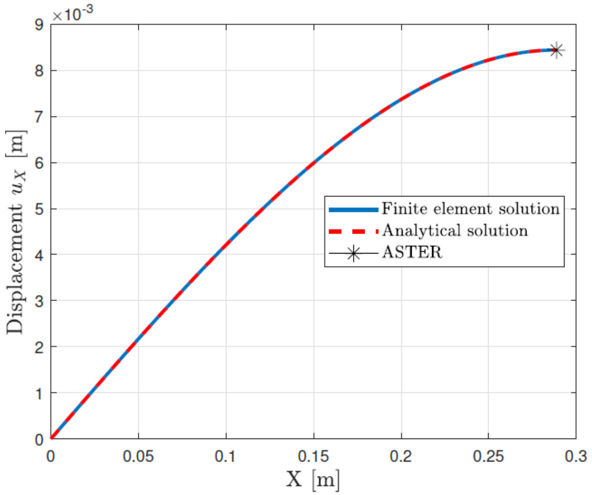




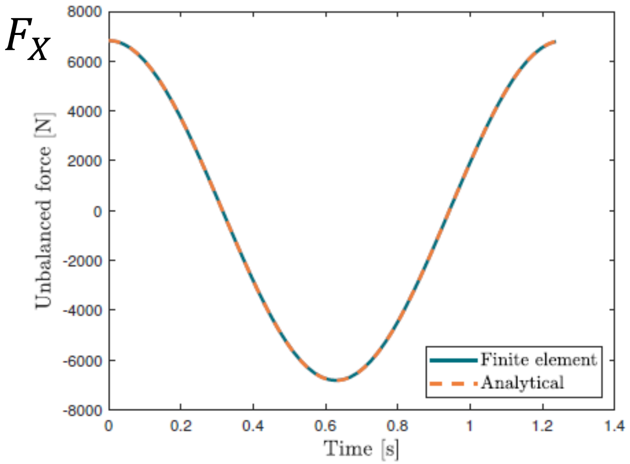
Verification of the different terms

$$m\begin{bmatrix}\mathbf{I} & \mathbf{B}_0 & \mathbf{H}\\ \mathbf{B}_0 & \mathbf{B}_0^T\mathbf{B}_0 & \mathbf{B}_0^T\mathbf{H}\\ \mathbf{H}^T & \mathbf{H}^T\mathbf{B}_0 & \mathbf{I}\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{s}}\\ \ddot{\boldsymbol{\beta}}\\ \ddot{\boldsymbol{\rho}}\end{Bmatrix} + 2m\Omega\begin{bmatrix}0 & \overline{\mathbf{B}}_0 & \overline{\mathbf{H}}\\ 0 & \mathbf{B}_0^T\overline{\mathbf{B}}_0 & \mathbf{B}_0^T\overline{\mathbf{H}}\\ 0 & \mathbf{H}^T\overline{\mathbf{B}}_0 & \mathbf{H}^T\end{bmatrix}\begin{Bmatrix}\dot{\mathbf{s}}\\ \dot{\boldsymbol{\beta}}\\ \dot{\boldsymbol{\rho}}\end{Bmatrix} + m\Omega^2\begin{bmatrix}0 & \overline{\overline{\mathbf{B}}}_0 & \overline{\overline{\mathbf{H}}}\\ 0 & \mathbf{B}_0^T\overline{\overline{\mathbf{B}}}_0 & \mathbf{B}_0^T\\ 0 & \mathbf{H}^T\overline{\overline{\mathbf{B}}}_0 & \mathbf{H}^T\overline{\overline{\mathbf{H}}}\end{bmatrix}\begin{Bmatrix}\mathbf{s}\\ \boldsymbol{\beta}\\ \boldsymbol{\rho}\end{Bmatrix} + \mathbf{K}_T\begin{Bmatrix}\mathbf{s}\\ \boldsymbol{\beta}\\ \boldsymbol{\rho}\end{Bmatrix}$$

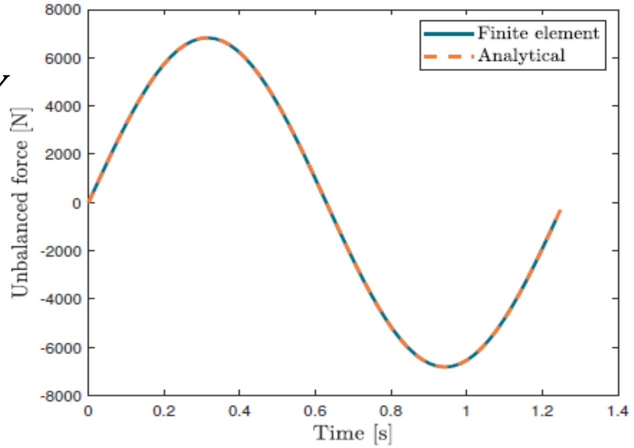
$$= \begin{Bmatrix}-m\overline{\overline{\mathbf{H}}}\mathbf{r}\\ -m\mathbf{B}_0^T\overline{\overline{\mathbf{H}}}\mathbf{r}\\ \mathbf{f}_\rho\end{Bmatrix}$$



F_X



F_Y



Time integration

$$\mathbf{M}(t)\ddot{\mathbf{q}} + \mathbf{D}(t)\dot{\mathbf{q}} + \mathbf{K}(t)\mathbf{q} = \mathbf{0}$$

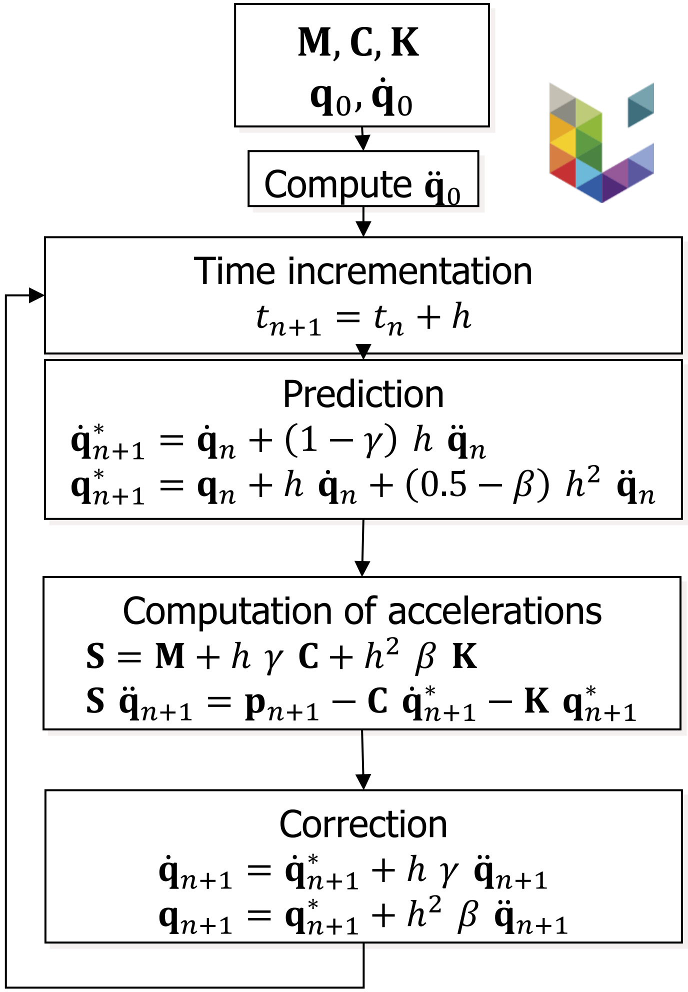
- Newmark scheme between t_n and t_{n+1}

$$\mathbf{Q}_{n+1} = \mathbf{D}_n \mathbf{Q}_n$$

- Transition matrix $\mathbf{D}_n = \mathbf{H}_1^{-1}\mathbf{H}_0$

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{M} + \beta h^2 \mathbf{K}_{n+1} & \beta h^2 \mathbf{D}_{n+1} \\ \gamma h \mathbf{K}_{n+1} & \mathbf{M} + \gamma h \mathbf{D}_{n+1} \end{bmatrix}$$

$$\mathbf{H}_0 = \begin{bmatrix} \mathbf{M} - \left(\frac{1}{2} - \beta\right) h^2 \mathbf{K}_n & h\mathbf{M} - \left(\frac{1}{2} - \beta\right) h^2 \mathbf{D}_n \\ -(1-\gamma) h \mathbf{K}_n & \mathbf{M} - (1-\gamma) h \mathbf{D}_n \end{bmatrix}$$



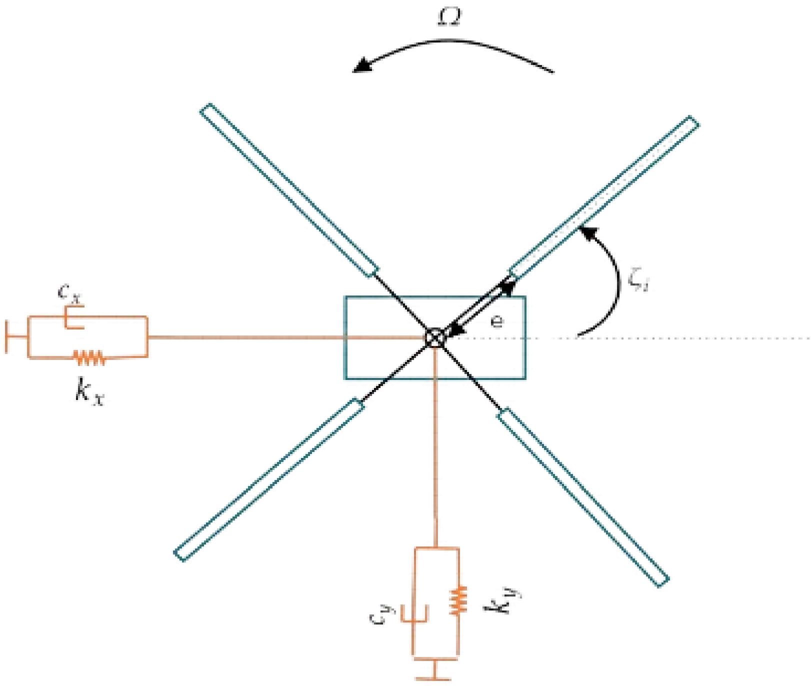
Verification: Ground Resonance model

Rigid model

Parameter	*Value*	*Units*
Number of blade N	4	-
Rotor radius R	5.64	m
Operational rotor speed Ω_0	31.42	rads^{-1}
Blade mass m_b	94.9	kg
Blade mass moment S_b	281.1	kg m
Blade mass moment of inertia I_b	1084.7	kg m^2
Lag hinge offset e	0.3048	m
Longitudinal hub mass M_x	8026.6	kg
Lateral hub mass M_y	3283.6	kg
Longitudinal hub spring k_x	1240481.08	N/m
Lateral hub spring k_y	1240481.08	N/m
Lag damper c_b	4067.5	mNs/rad

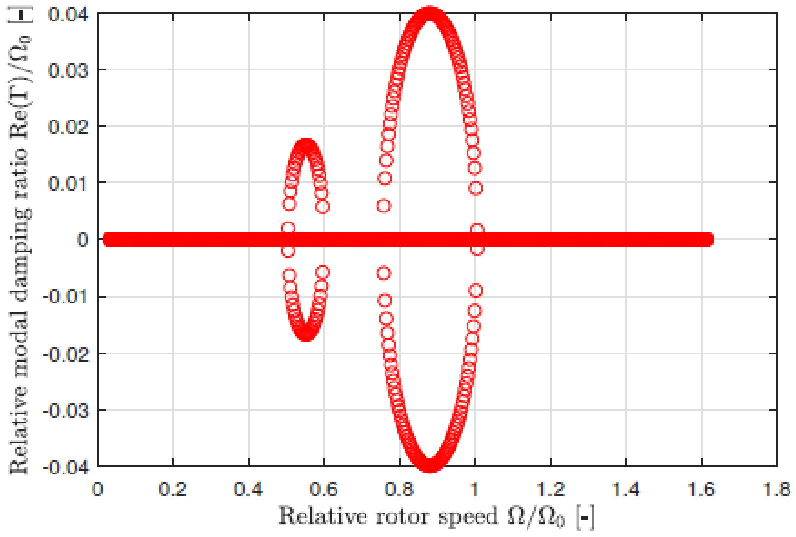
FE model parameters

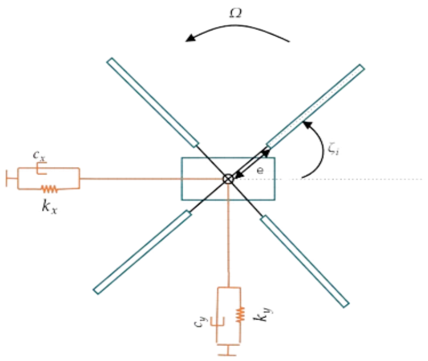
Parameter	*Value*	*Units*
Number of blade N	4	-
Rotor radius R	5.64	m
Operational rotor speed Ω_0	31.42	rads^{-1}
Beam diameter D	0.026	m
Young modulus	210	GPa
Density	7800	kgm^3
Poisson coefficient	0.3	-



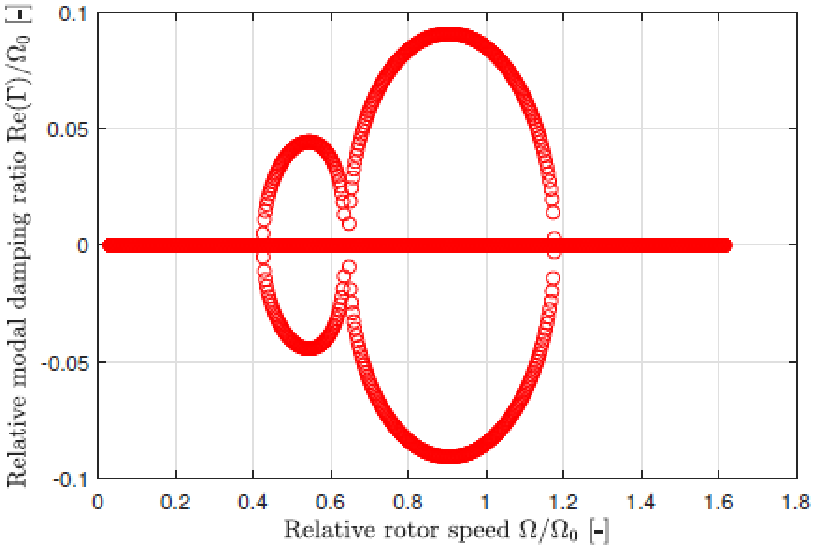
Ground Resonance with a rigid model

Newmark



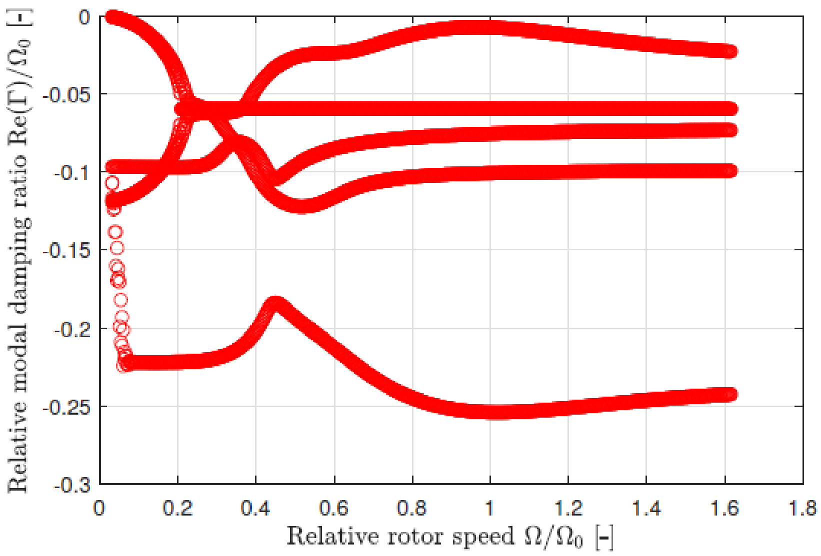


Runge-Kutta

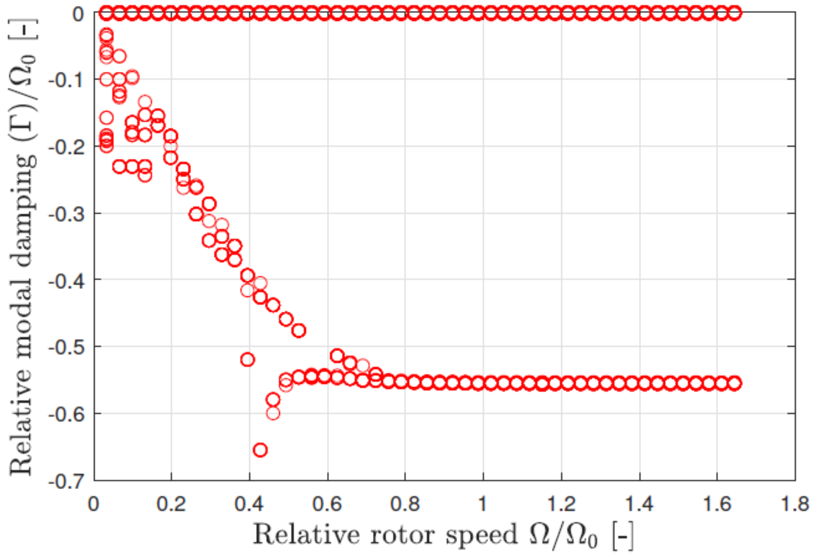


GR model – Results (1)

Rigid model – Runge-Kutta



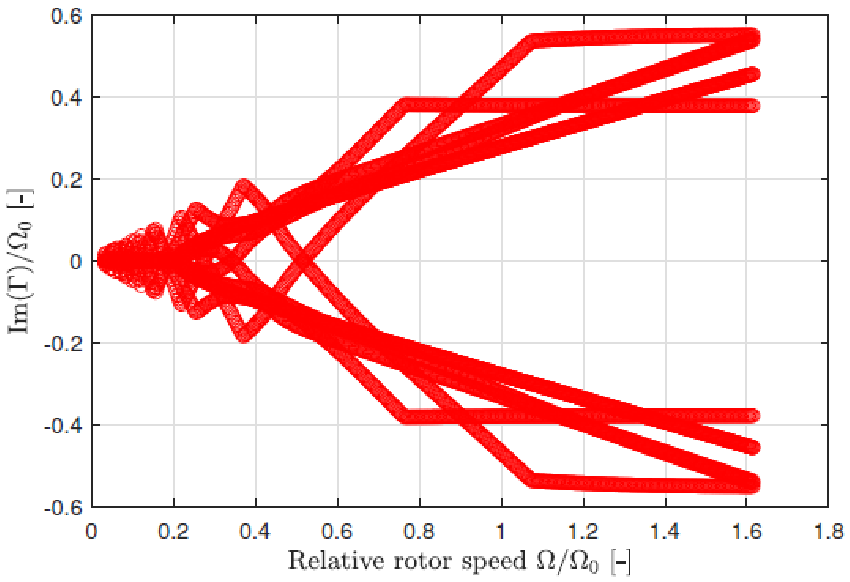
FE model – Runge-Kutta



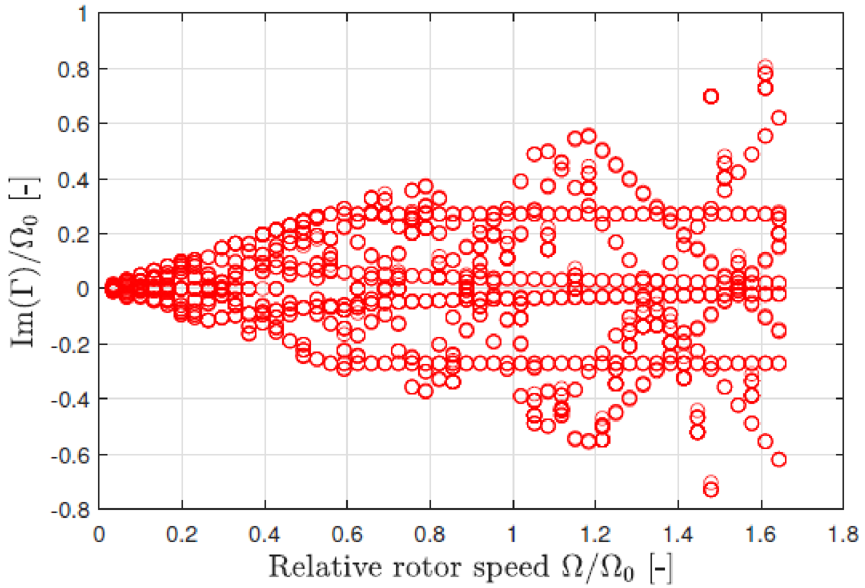
- Stability of the GR model
- Equal stability prediction

GR model – Results (2)

Rigid model – Runge-Kutta



FE model – Runge-Kutta

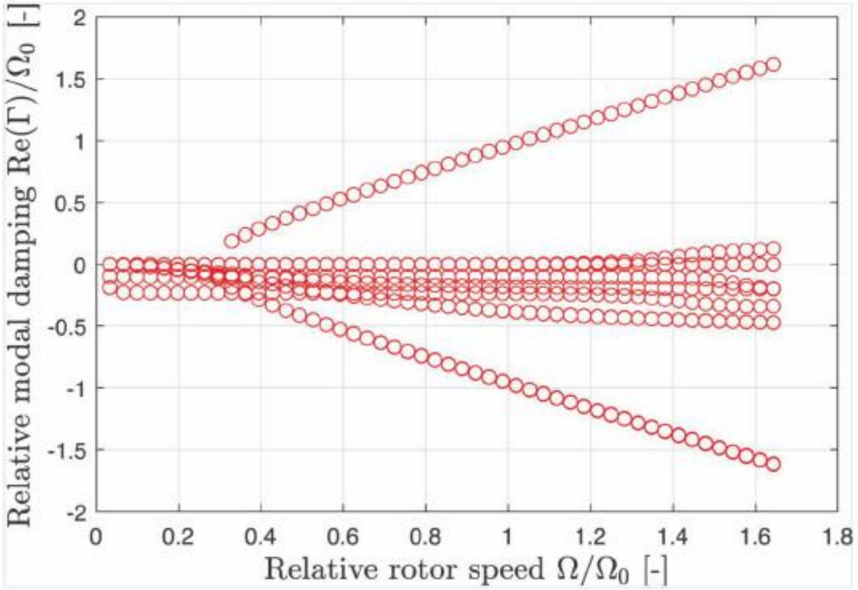


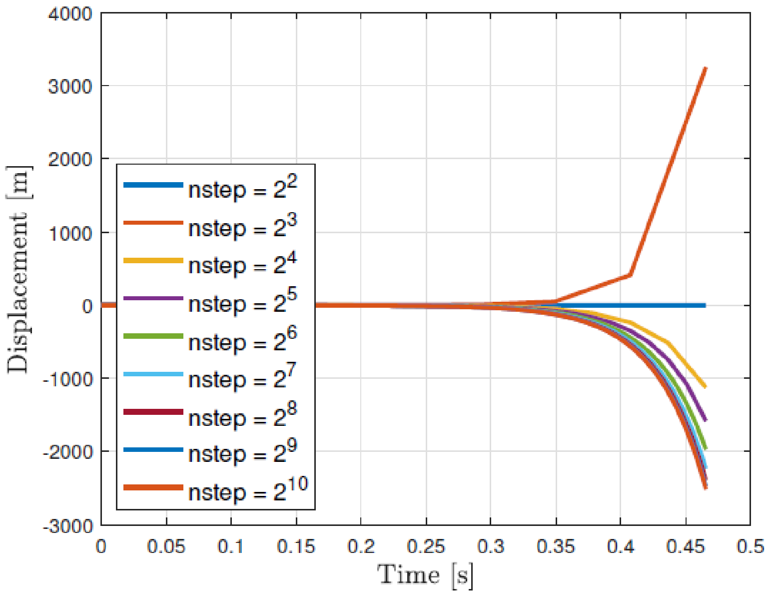
- Equivalent behaviour
- More crossing with 0 axis with the FE model due to higher number of dof
- Validation of the partial coupling using the established FE model using Runge-Kutta integration scheme

GR model – Results (3)

FE model – Newmark

$$\frac{\Omega}{\Omega_0} = 0.8$$





- Problem with each of the related Newmark algorithm to find consistent results
- Conditionning of the coupled matrices ?
- High modal frequencies ?

To be further investigated in a future work.

Conclusions

- Formulation of system with coupling between rotating frame and static frame
- Implementation in a finite element framework
- Numerical scheme for time integration
- Preliminary results on stability analysis of coupling system

Future work

- Find out more about why the Newmark scheme and relative do not work.
- Use reduction method such as Craig-Bampton to reduce computation time.
- Use aerodynamic force to get a better prediction of the whirl-flutter instability.
- Validate the code using an experimental setup.

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