ETH zürich

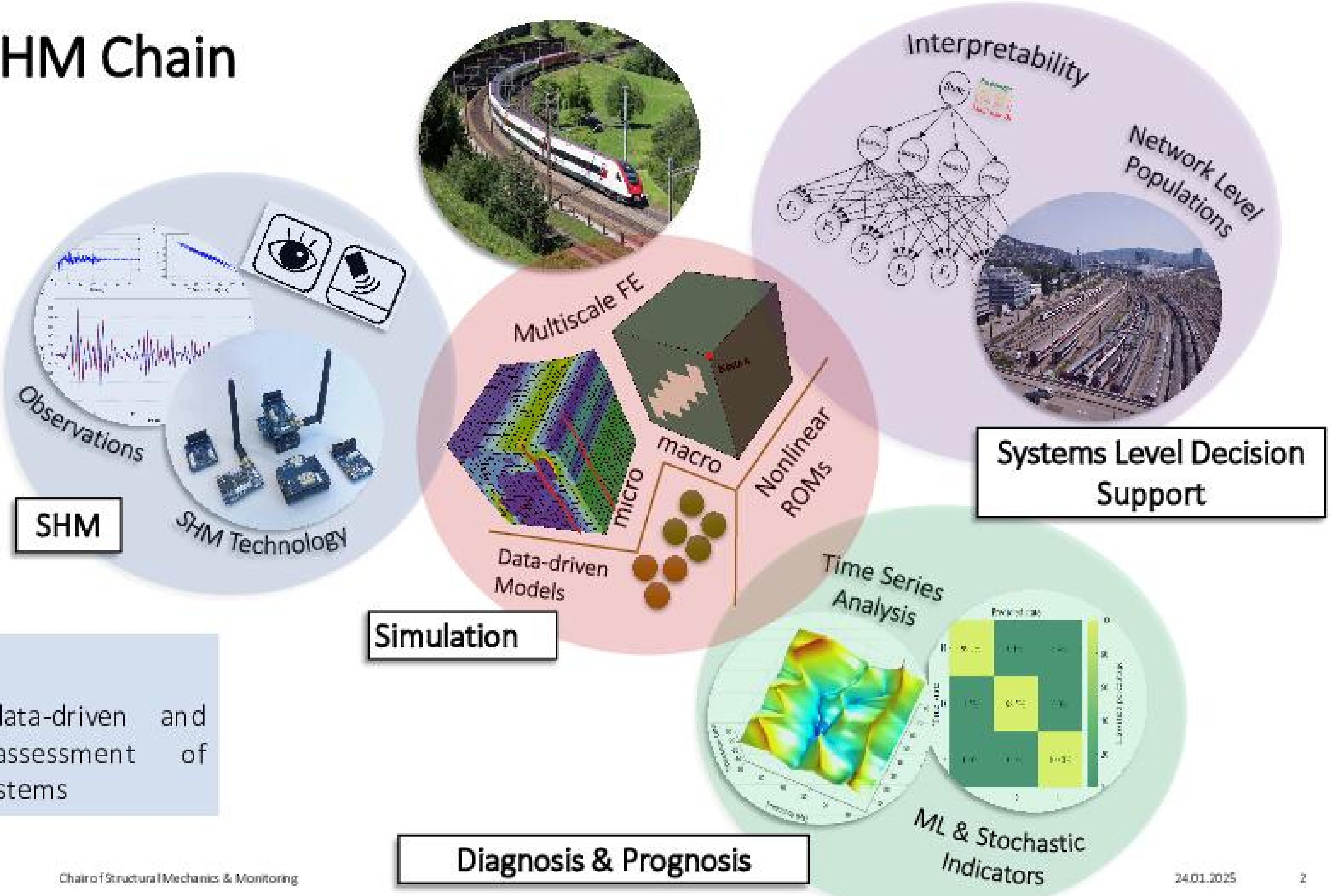
Chair of Structural
Mechanics & Monitoring

A Physics-Enhanced Approach to Modelling and Tracking Nonlinear Behavior

Eleni Chatzi
ETH Zürich

D BAUG

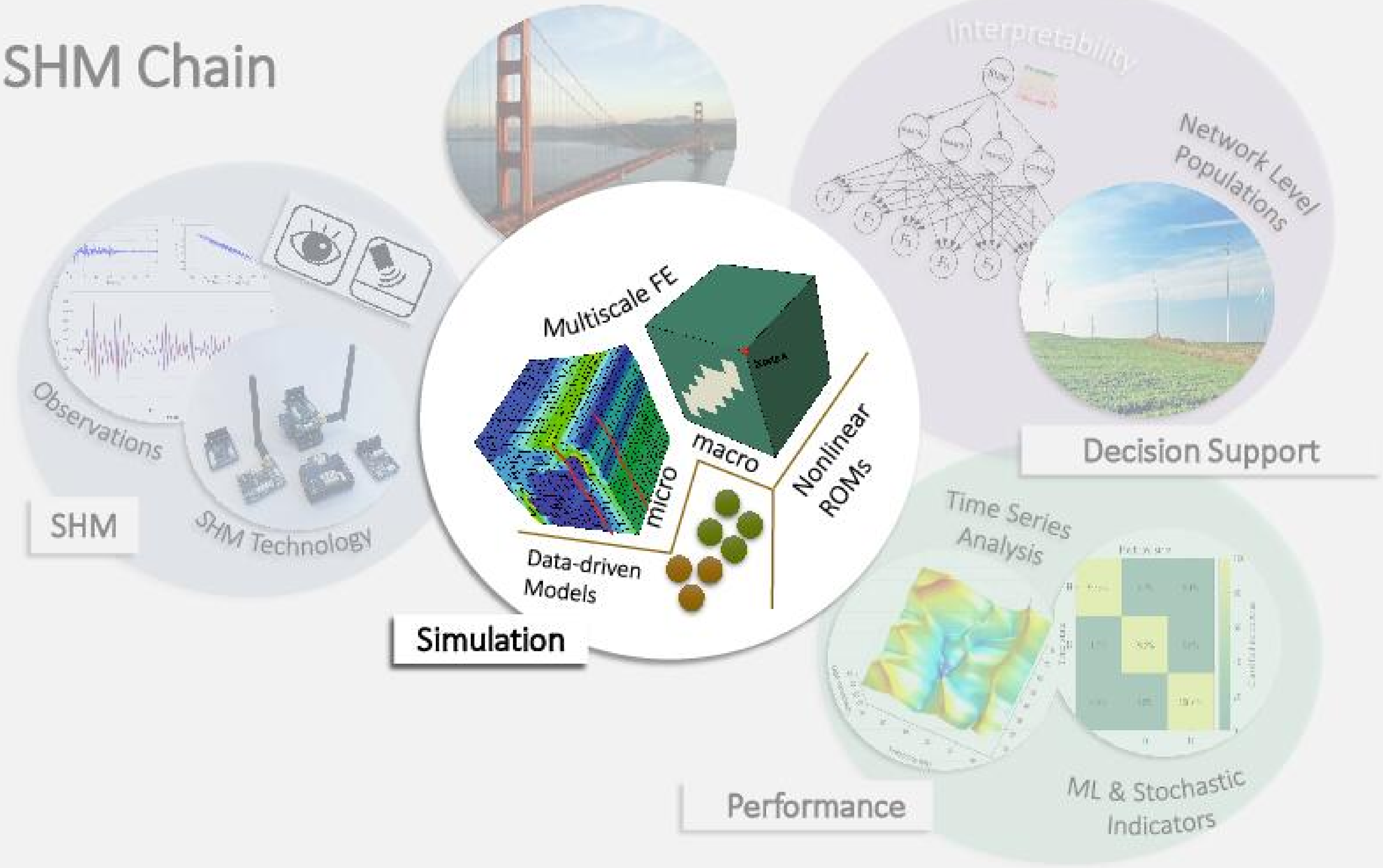
The SHM Chain



Scope:

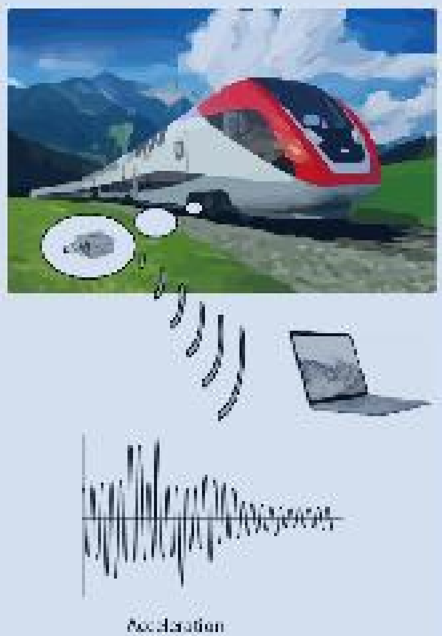
embodied, data-driven and intelligent assessment of engineered systems

The SHM Chain



Learning purely from Data

Data gathering - Domains of Activity of the SMM Chair



on board monitoring for roadway/railway

Transport infrastructure

PTOON, Wind Farm Greece

SMM owned Aventa Turbine

Wind Energy Infrast.

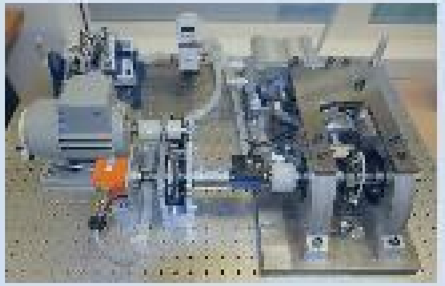
Monitoring under Demolition

Haus Du Pont, Zürich

SBB Bridge, Sempach

Steinavötn Bridge, Iceland

Built Environment



PRONOSTIA ball bearings

train seats

Industrial Assets

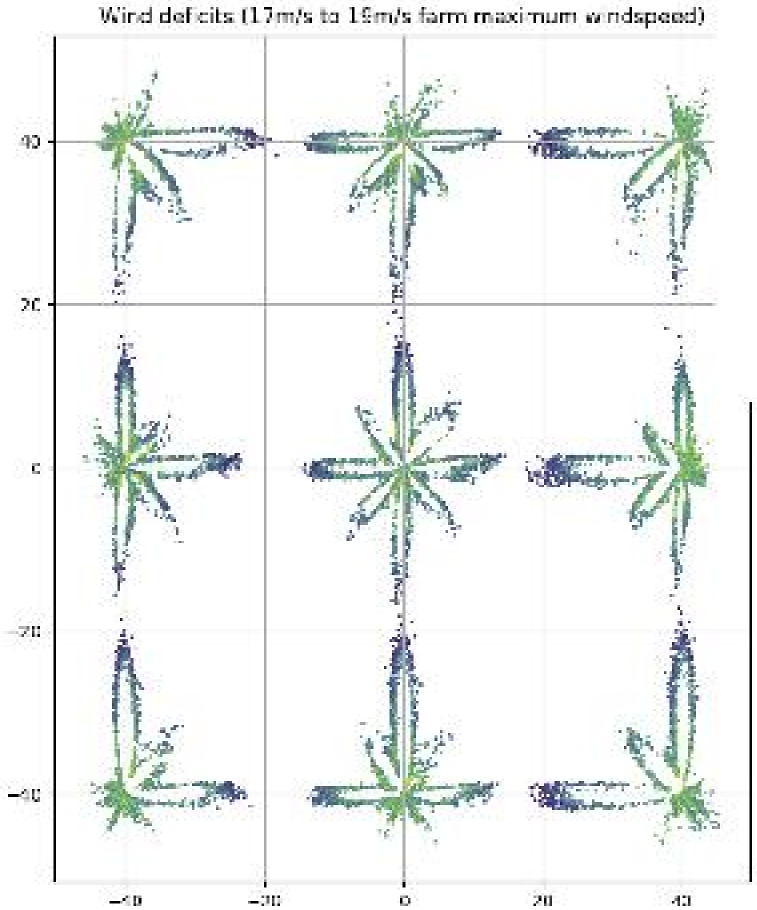
Learning from Data

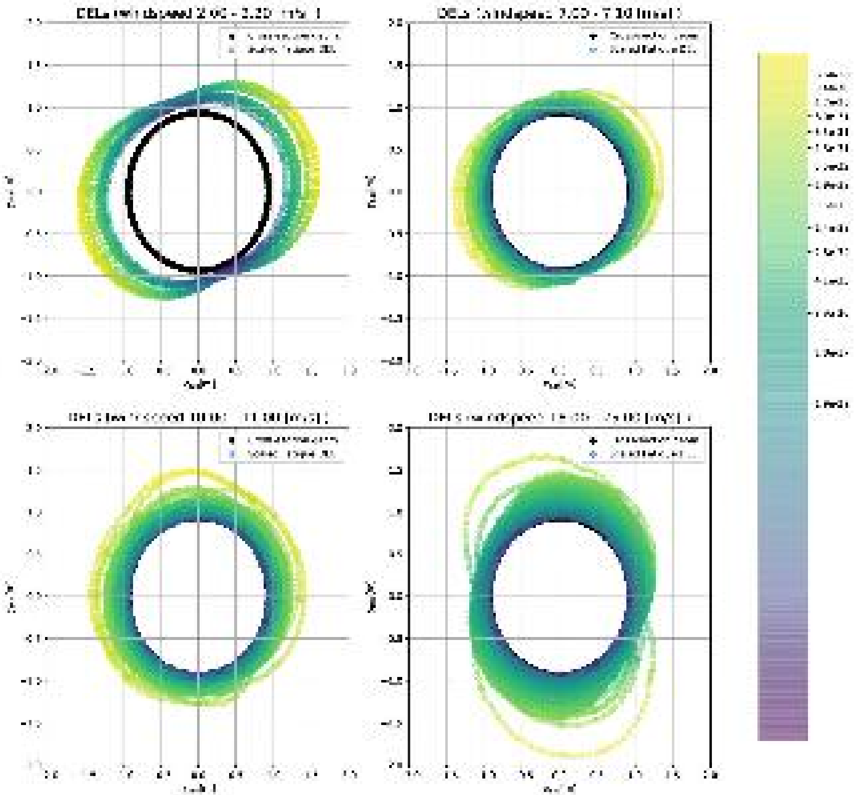
Modelling Wake Effects

Smyth and Elliott, 2014

Motivation

- Obtain a representation of the data that is easier to manipulate and visualize
- Capture QoIs at the farm level, conditioned on operational variables



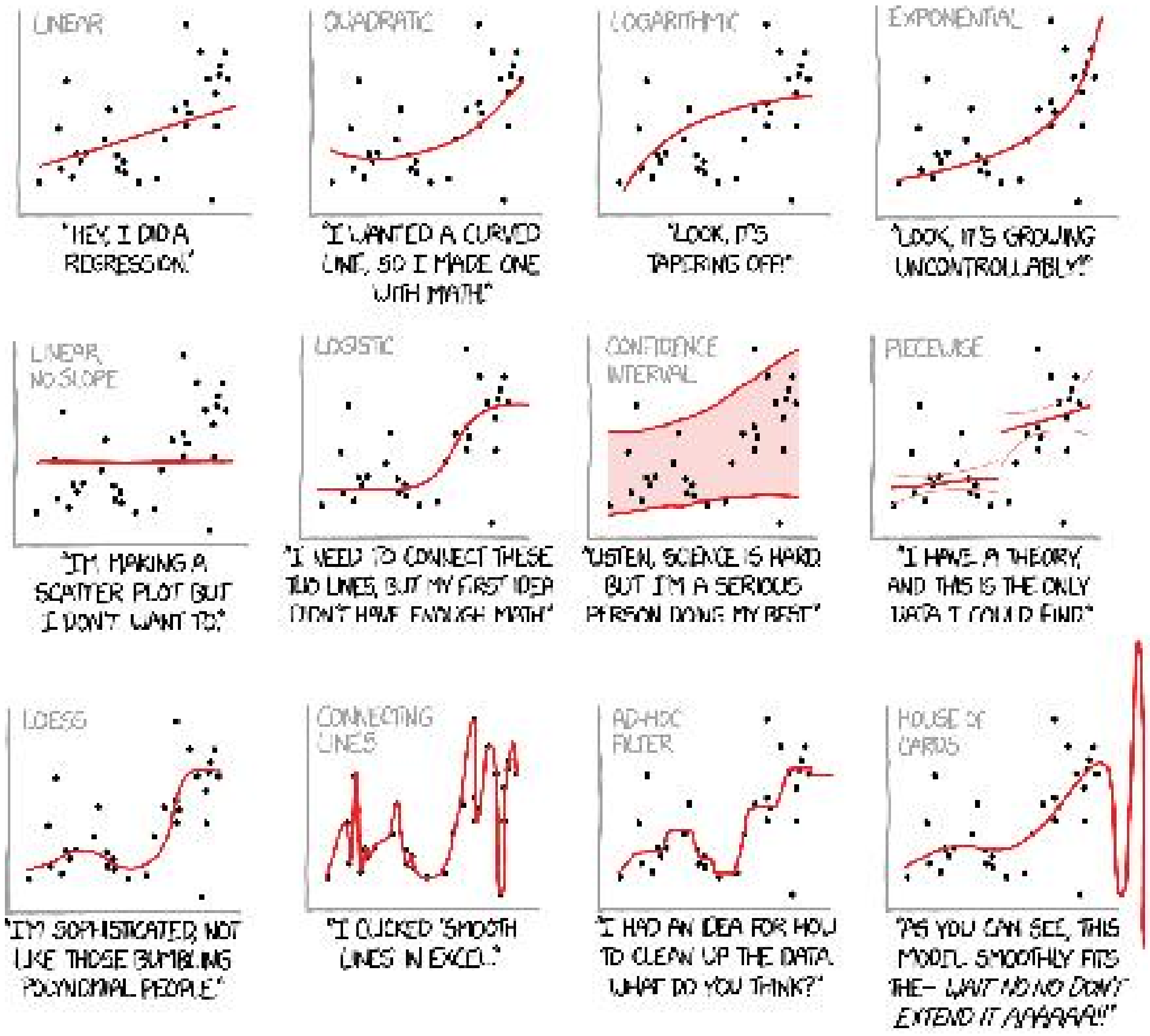


Generative Modelling

Visualization of wind deficits on individual WTs and at the the farm level via Conditional Variational Autoencoders (CVAEs)

Data is not Enough

Data-driven Modeling
The Pitfalls



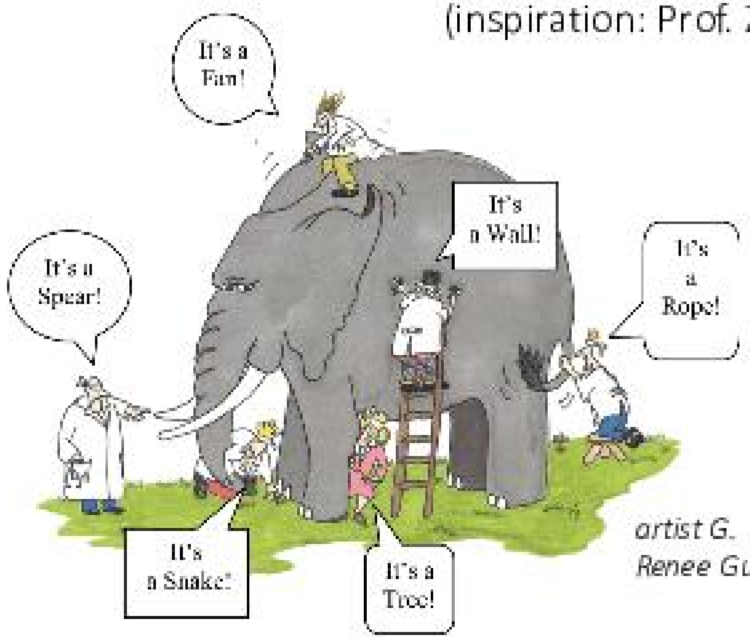
interpretability

Explainability
Physics
connotation

adaptivity
extrapolation

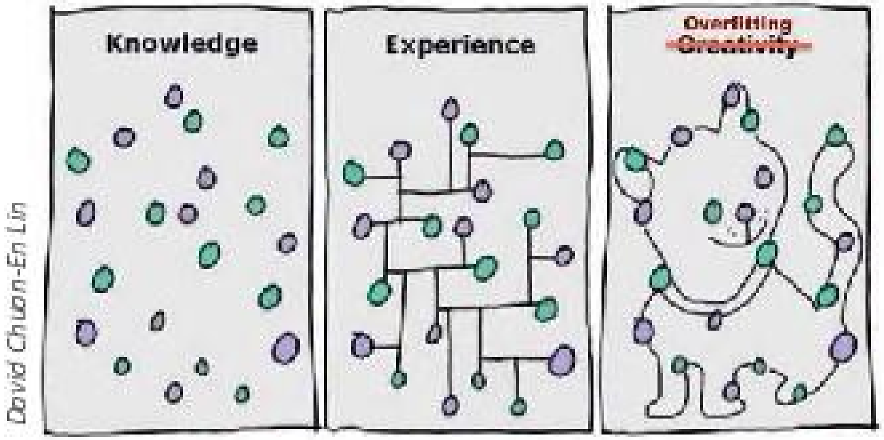
generalization: 6 blind men & an elephant

(inspiration: Prof. Zhilu Lai)



artist G. Renee Guzlas

How to capitalize on knowledge (inductive bias)?



https://www.explainxkcd.com/wiki/index.php/2048:_Curve-Fitting

Data is not enough

"as-designed", or DTP

Physics-based representations

Hybrid Models
- Advanced SHM/twinning tasks Detection, localization, quantification, prognosis
- Used on the fly for diagnostics & control
- Are eXplainable/Interpreatable

Interactive, closed loop DTs

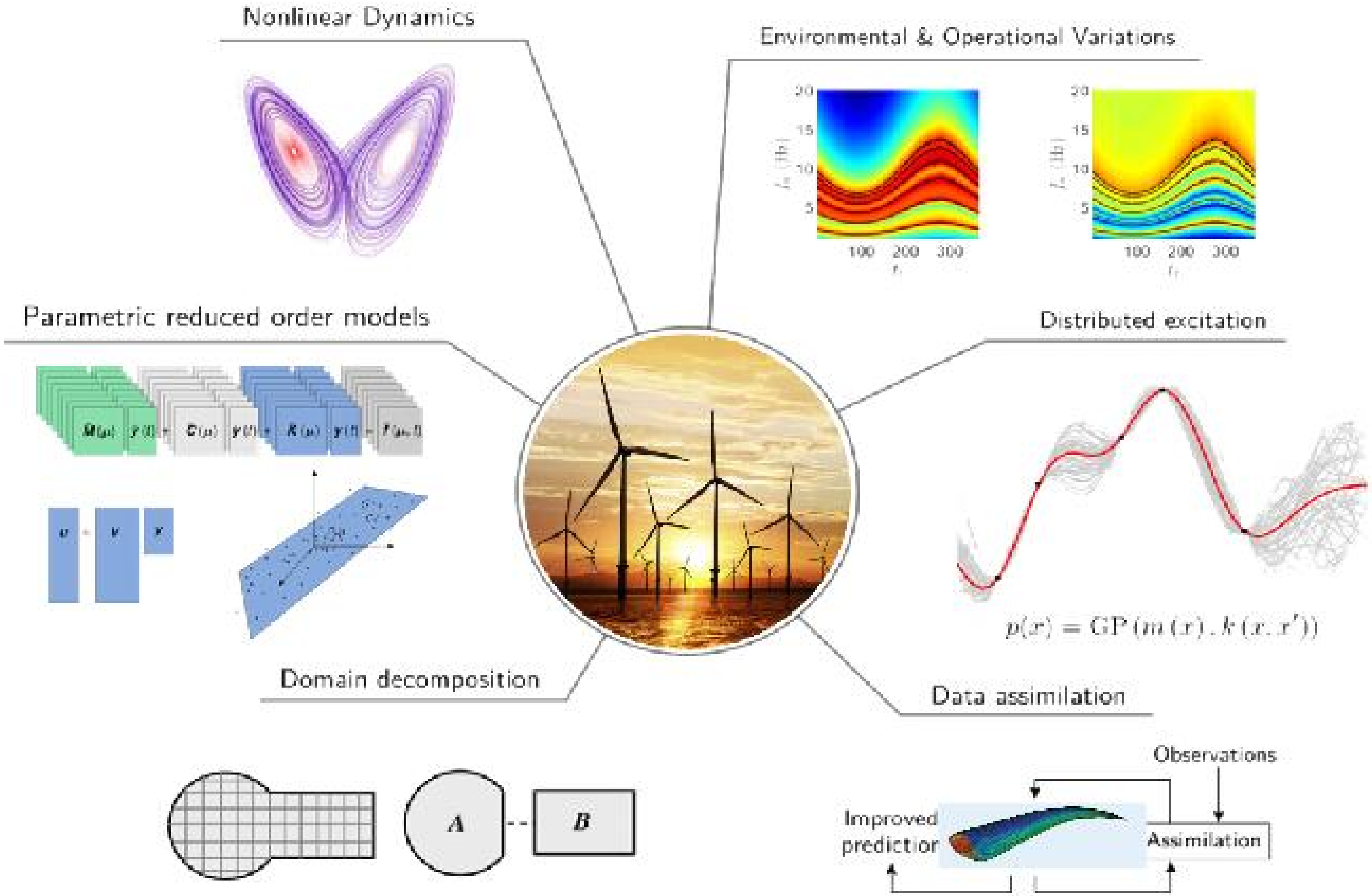
Purely data-driven representations

"as-is", diagnosis of a DTI

Hybrid

Simulation Paths

Hybrid Modeling – Fusing Physics with Data, feasibly in Real-Time



Main Use Cases

– Reduced Order Modeling for Virtualization & Twinning

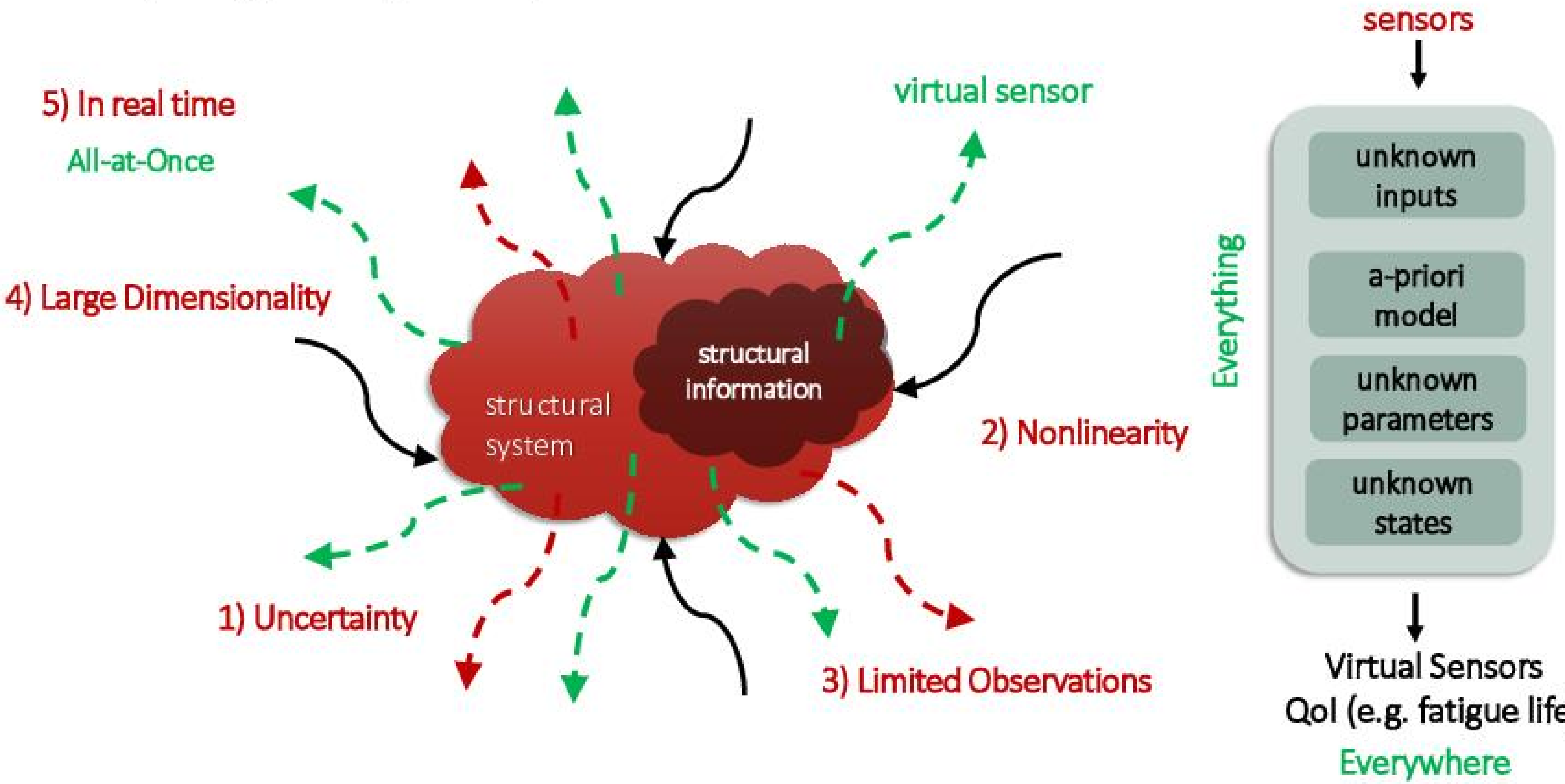
– Virtual sensing for real-time estimation & condition assessment

– Data driven indicators for diagnosis & prognosis

– Applications in Hybrid Simulation & Control

Virtual Sensing for RTDTs – Predictive Estimation at the Full Field

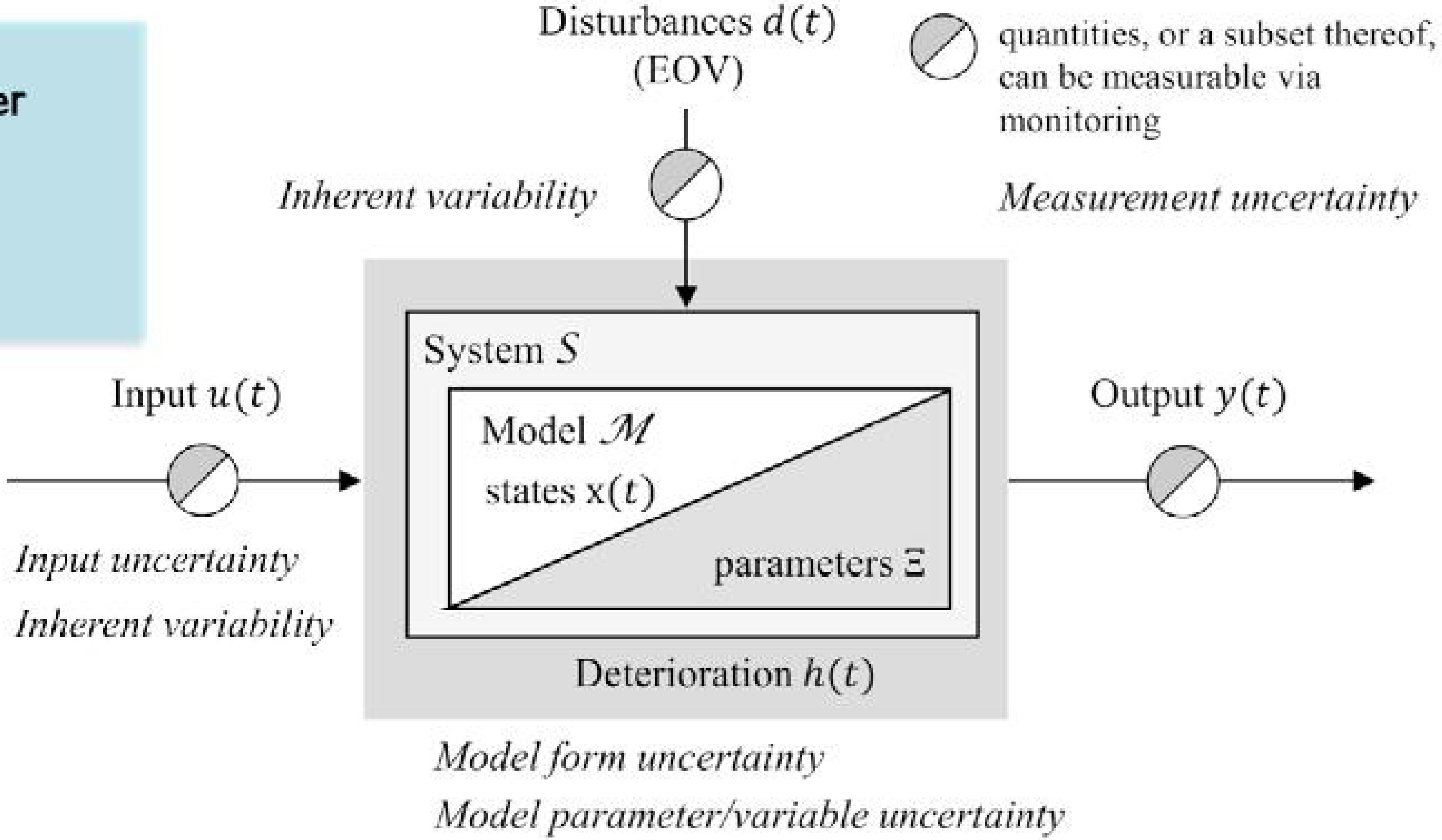
Everything / Everywhere / All-at-Once

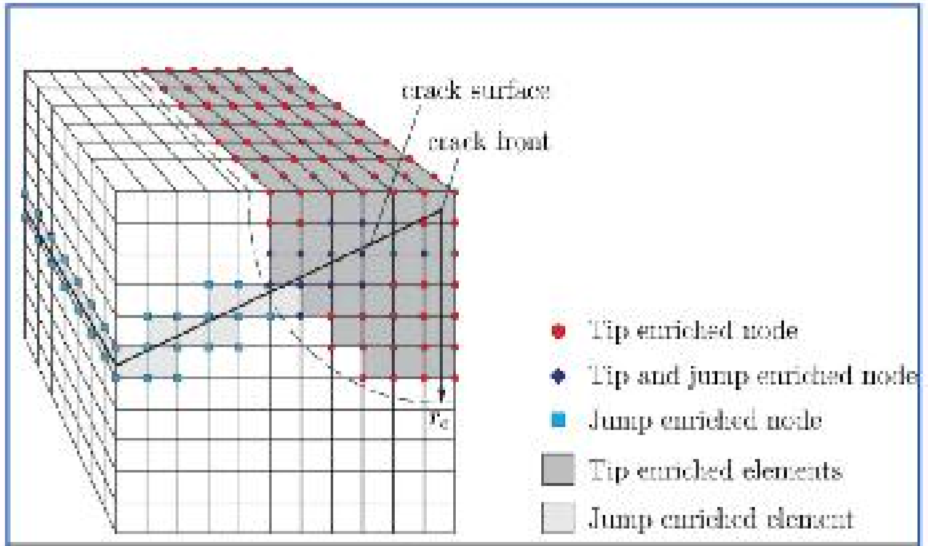


Input-state-parameter estimation of structural systems from limited output information
V.K.Dertimanis, E.N.Chatzi, S.Eftekhar Azam, C.Papadimitriou, MSSP (2019)

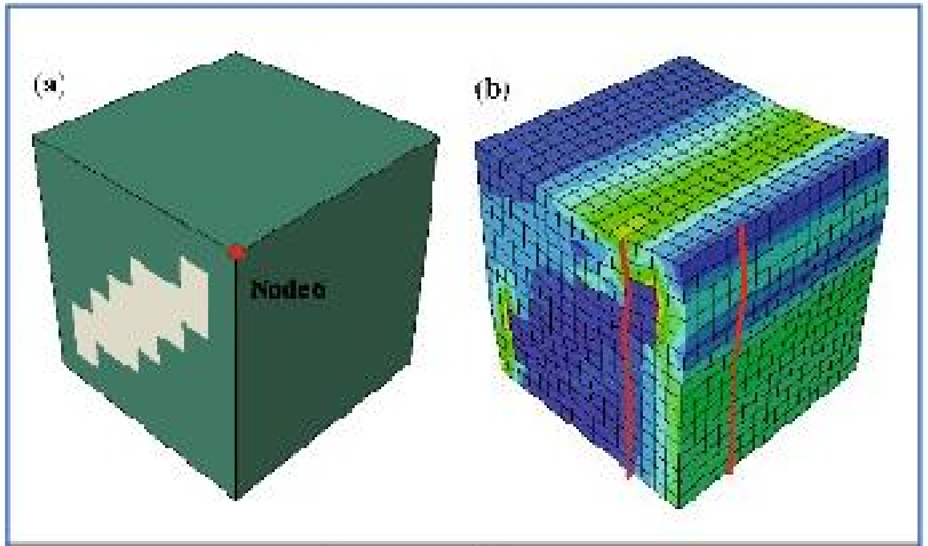
On the Consistent Classification and Treatment of Uncertainties in SHM

Paper

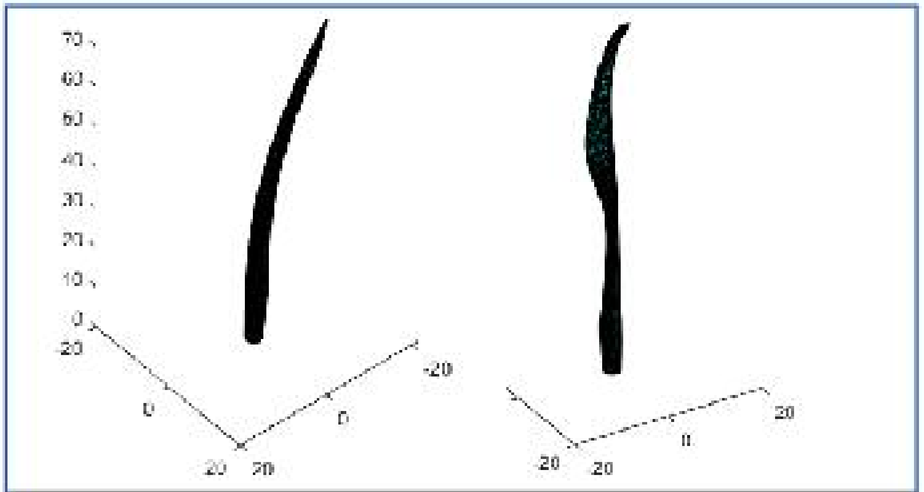




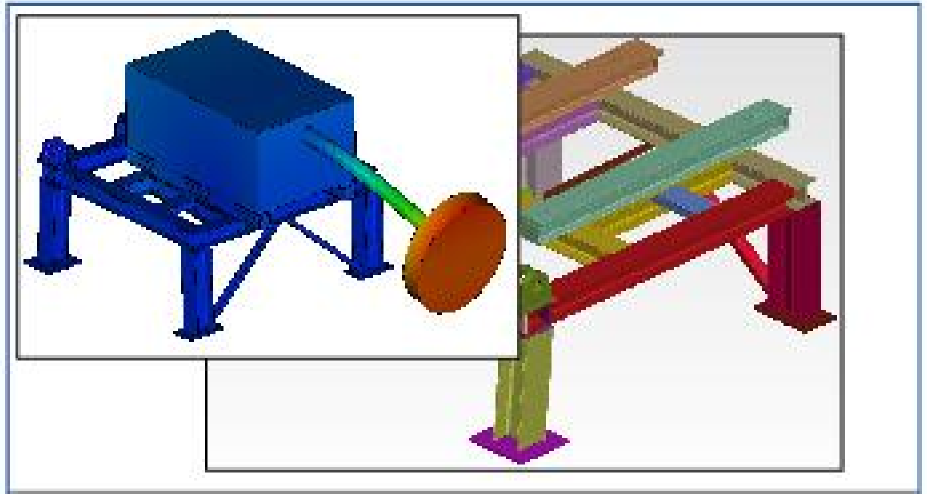
fracture



hysteresis/plasticity



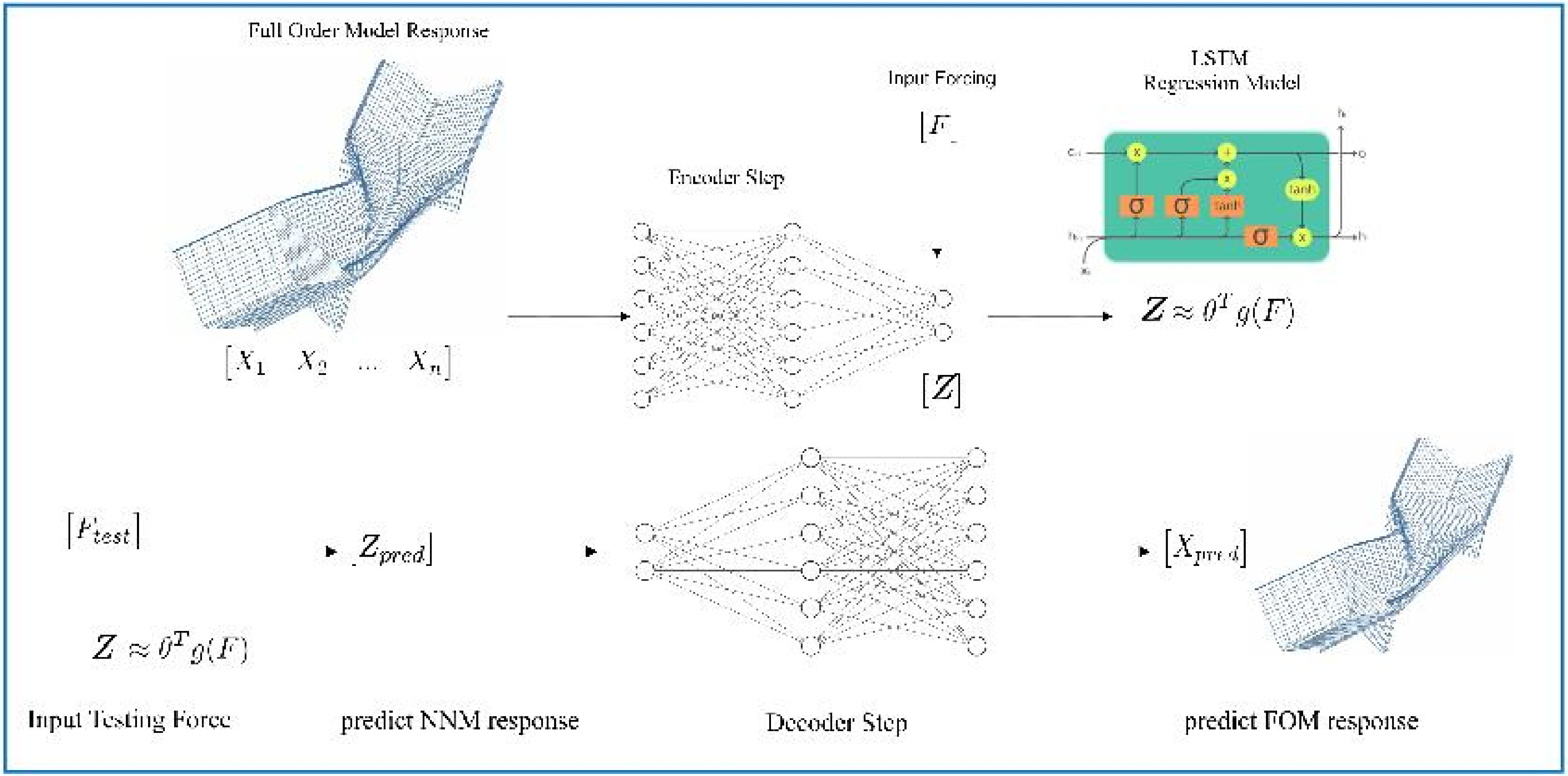
large deformations



substructuring

[real-time] Reduced Order Representations relying on First Principles

ML-driven ROMs for Nonlinear Dynamics



[Simpson, Dervilis, Chatzi (2021), On the use of Nonlinear Normal Modes for Nonlinear Reduced Order Modelling]

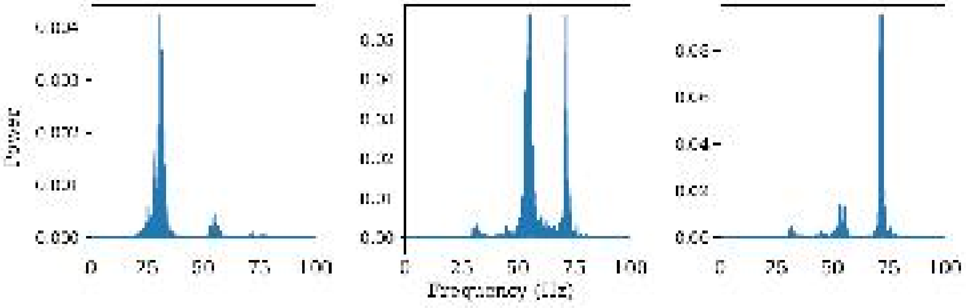
also check: Vlachas, P.R., Arampatzis, G., Uhler, C. et al. Multiscale simulations of complex systems by learning their effective dynamics. Nat Mach Intell 4, 359–366 (2022)

Applications on Nonlinear Systems

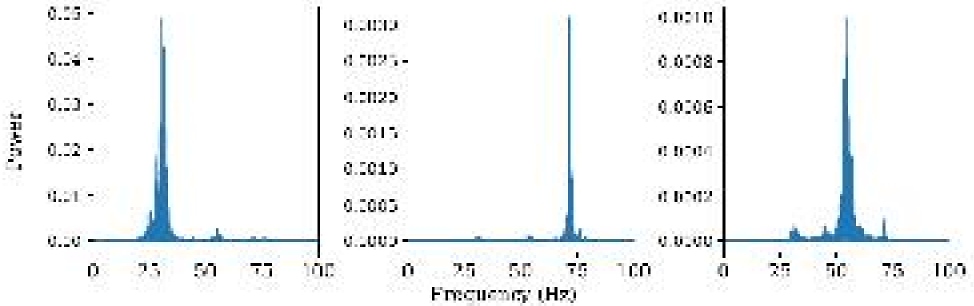
- Three-storey shear-frame structure from Los Alamos National Laboratory (LANL)

nonlinear bumper

PCA decomposition

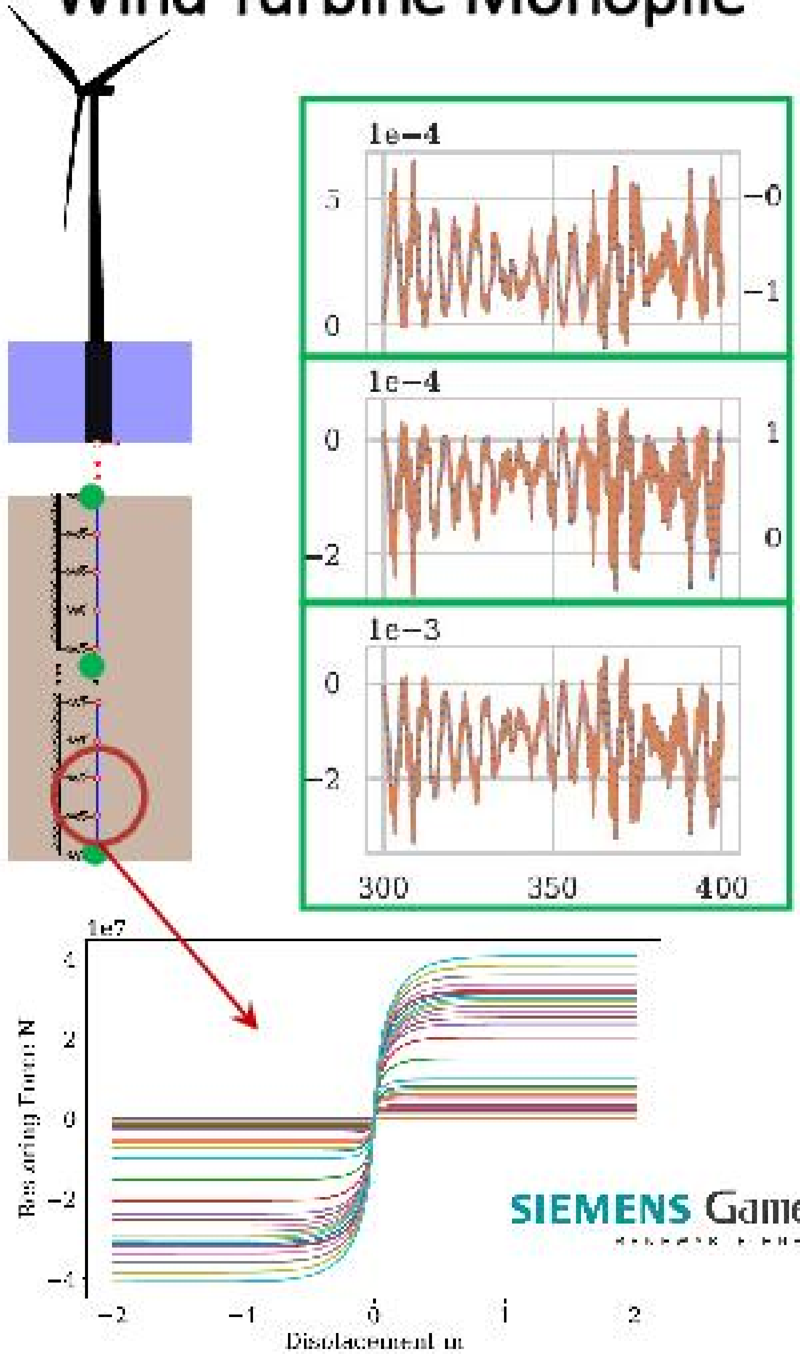


VAE decomposition



- What about **parametric dependence/uncertainty**?

Wind Turbine Monopile



SIEMENS Gamesa

Physics-based ML-boosted parametric ROMs

Example: Two-Story Frame with Hysteretic Links

Ground motion excitation

Parametric dependency:

round motion orientation & Spectro-temporal signal parameters

Bouc Wen Hysteretic model

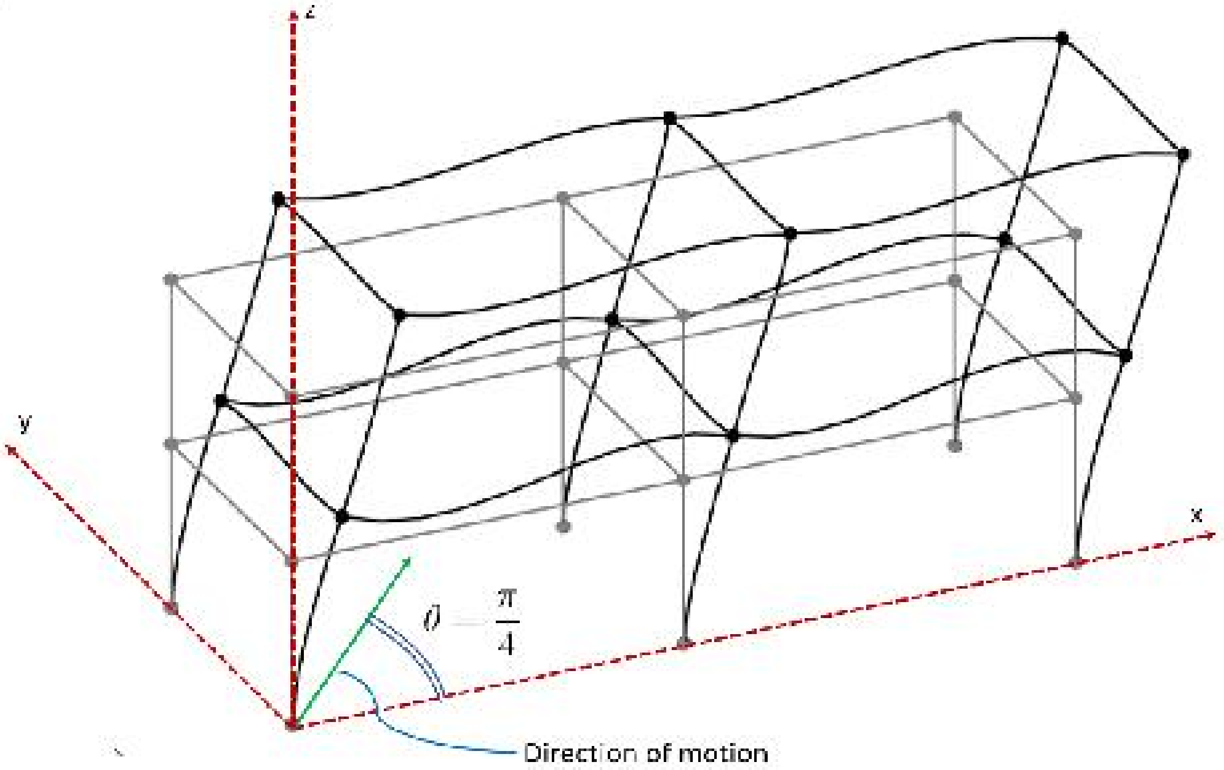
Total restoring force:

$$\mathbf{R} = \mathbf{R}_{linear} + \mathbf{R}_{hysteretic} = \alpha k \mathbf{u} + (1-\alpha) k \mathbf{z}$$

Pametric dependency: degradation/deterioration effects:

$$\dot{\mathbf{z}} = \frac{A\dot{\mathbf{u}} - \nu(t)(\beta|\dot{\mathbf{u}}|\mathbf{z}|\mathbf{z}|^{w-1} - \gamma\dot{\mathbf{u}}|\mathbf{z}|^{w})}{\eta(t)}$$

$$\nu(t) = 1.0 - \delta_\nu \epsilon(t), \quad \eta(t) = 1.0 + \delta_\eta \epsilon(t), \quad \epsilon(t) = \int_0^t \mathbf{z}\dot{\mathbf{u}}\delta t$$



Benchmark example featured in:

- Vlachas K. et al., *Journal of Sound and Vibration* 502 (2021): 116055.
- Simpson T. et al., *Journal of Engineering Mechanics* 147.10 (2021): 04021061.

[everything] Parametric Model Order Reduction

Problem statement

General a nonlinear, parametric, dynamical structural system:

$$\mathbf{M}(\mathbf{p})\ddot{\mathbf{u}}(t) + \mathbf{g}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}\right) = \mathbf{f}(t, \mathbf{p})$$

$$\mathbf{u}(t) \in \mathbb{R}^n, \mathbf{M}(\mathbf{p}) \in \mathbb{R}^{n \times n}, \mathbf{f}(t, \mathbf{p}) \in \mathbb{R}^n, \mathbf{g}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t)\right) \in \mathbb{R}^n$$

Parametric dependency on *k* parameters denoted by: $\mathbf{p} = [p_1, ..., p_k]^T \in \Omega \subset \mathbb{R}^k$

Relevant notation:

M is the system mass matrix ***u*** is the response time history

f is the vector of external loads ***g*** are the nonlinear, state-dependent internal forces

[everything] Parametric Model Order Reduction

Projection based reduction

*The **goal of parametric MOR is to generate a low-dimensional, equivalent system** such that the underlying physics along with the parametric dependencies of interest are further retained.*

$$\mathbf{M}_{\mathrm{r}}(\mathbf{p}_j)\ddot{\mathbf{u}}_{\mathrm{r}}(t) + \mathbf{g}_{\mathrm{r}}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}_j\right) = \mathbf{f}_{\mathrm{r}}(t, \mathbf{p}_j)$$

$$\mathbf{M}_{\mathrm{r}}(\mathbf{p}_j) \in \mathbb{R}^{r\times r}, \mathbf{g}_{\mathrm{r}}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}_j\right) \in \mathbb{R}^r, \mathbf{f}_{\mathrm{r}}(t, \mathbf{p}_j) \in \mathbb{R}^r$$

$$r \ll n$$

Galerkin Projection Basis

$$\mathbf{u}(t) = \mathbf{V}(\mathbf{p}_j)\mathbf{u}_{\mathrm{r}}(t) \qquad \mathbf{M}_{\mathrm{r}}(\mathbf{p}_j) = \mathbf{V}(\mathbf{p}_j)^T\mathbf{M}(\mathbf{p}_j)\mathbf{V}(\mathbf{p}_j)$$

$$\mathbf{f}_{\mathrm{r}}(\mathbf{p}_j) = \mathbf{V}(\mathbf{p}_j)^T\mathbf{f}(t, \mathbf{p}_j) \qquad \mathbf{g}_r(\mathbf{p}_j) = \mathbf{V}(\mathbf{p}_j)^T\mathbf{g}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}_j\right)$$

Vlachas et al. 2020, https://arxiv.org/abs/2003.07716

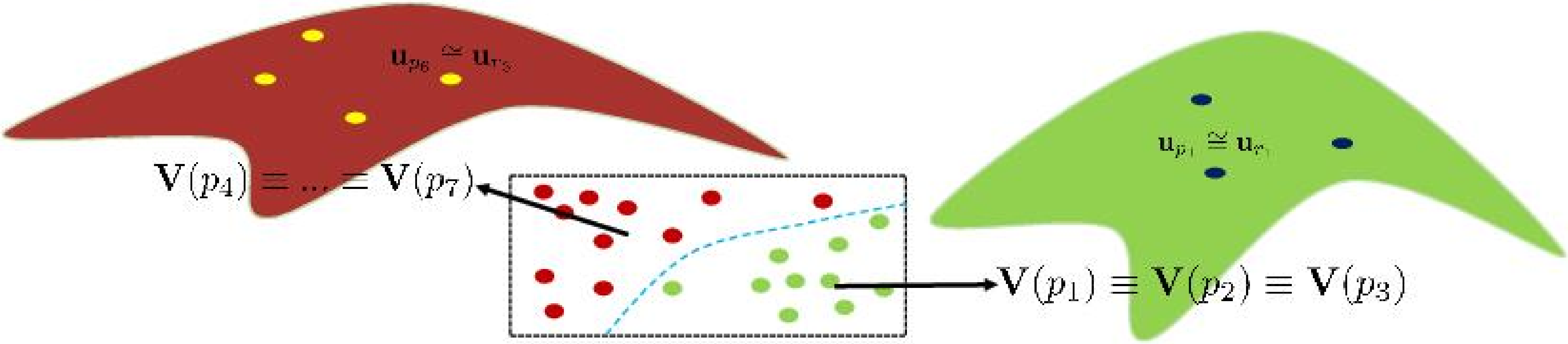
[everything] Parametric Reduced Order Modelling

Handling Nonlinear Behaviour

Addressing Nonlinearities:

- Localized phenomena dominate response due to nonlinear terms
- Solutions span substantially different subspaces

How to link basis functions to the parametric space? → Partitioning / Clustering strategies



Original parameter space

$$\mathbf{V}^{rg}_{global} = \left[\mathbf{V}^{rg}_{local,1}, \mathbf{V}^{rg}_{local,2}, \ldots, \mathbf{V}^{rg}_{local,N}\right] \longrightarrow \mathbf{V}^{rg}_{\text{local},i} = \mathbf{V}^{rg}_{\text{global}} \mathbf{\Xi}_i$$

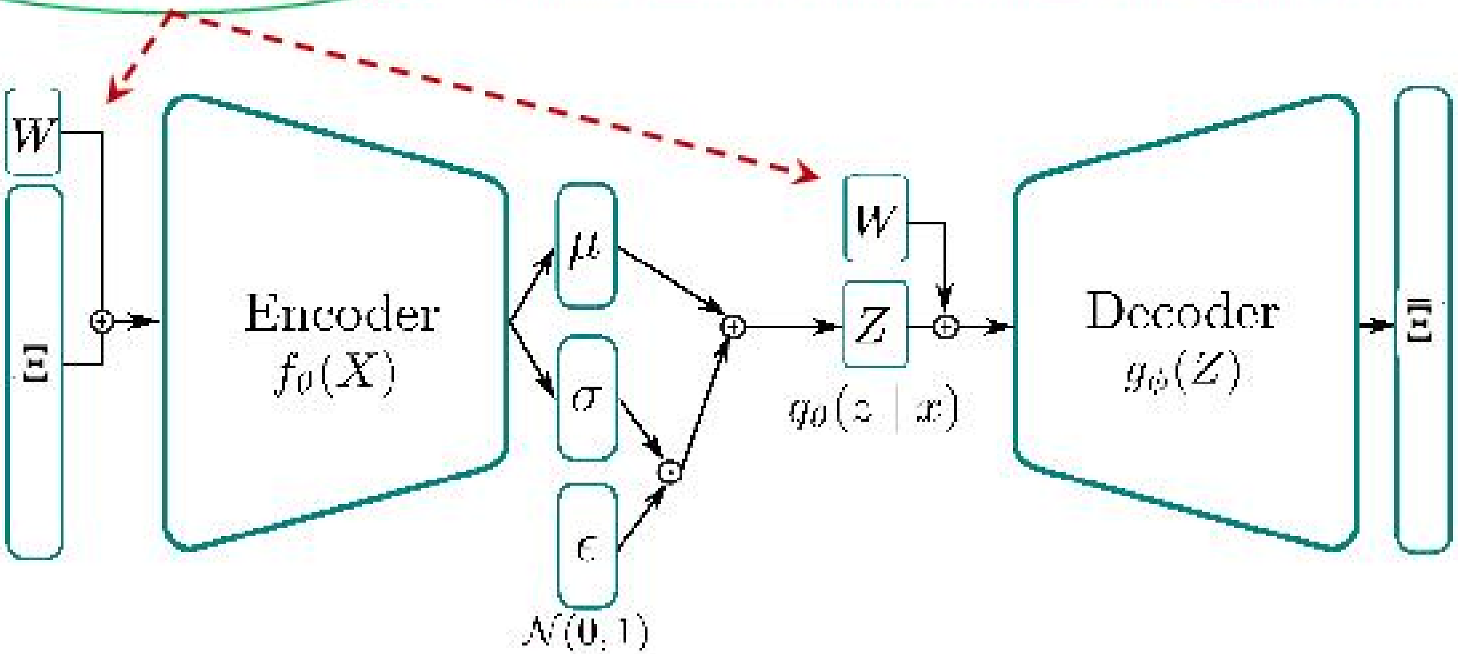
Vlachas et al. 2020, https://arxiv.org/abs/2003.07716

VAE-scheme for parametric ROM treatment

Conditional Variational AutoEncoder

In our case → *Explicitly* treat parametric *dependencies* → Conditional VAE

The *parametric dependencies* are *injected both at the input and at the latent space* during training



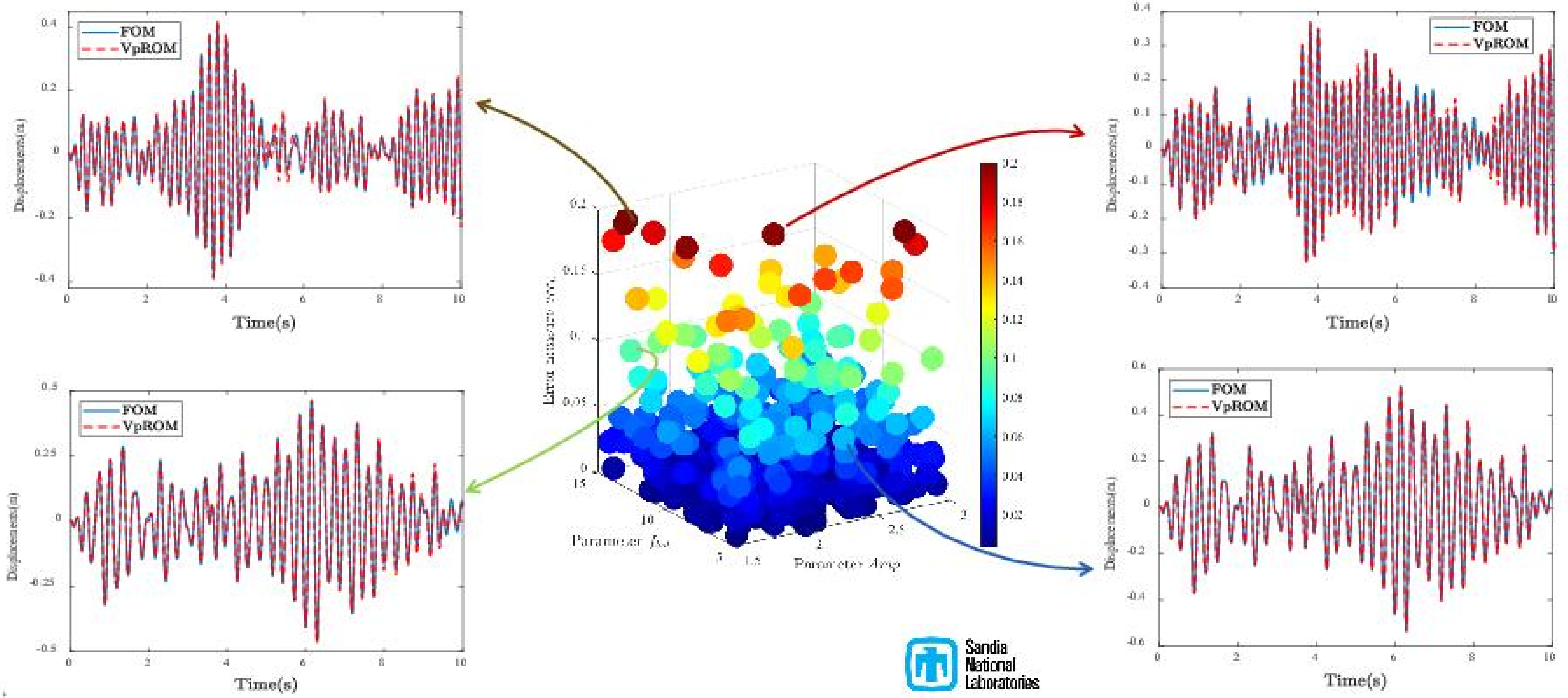
$$\mathbf{V}^{\mathrm{rg}}_{global} = \left[\mathbf{V}^{\mathrm{rg}}_{local,1}, \mathbf{V}^{\mathrm{rg}}_{local,2}; \dots; \mathbf{V}^{\mathrm{rg}}_{local,N}\right]$$

→

$$\mathbf{V}^{\mathrm{rg}}_{\mathrm{local},i} = \mathbf{V}^{\mathrm{rg}}_{\mathrm{global}} \boldsymbol{\Xi}_i$$

Framework Validation

Shear frame with hysteretic links

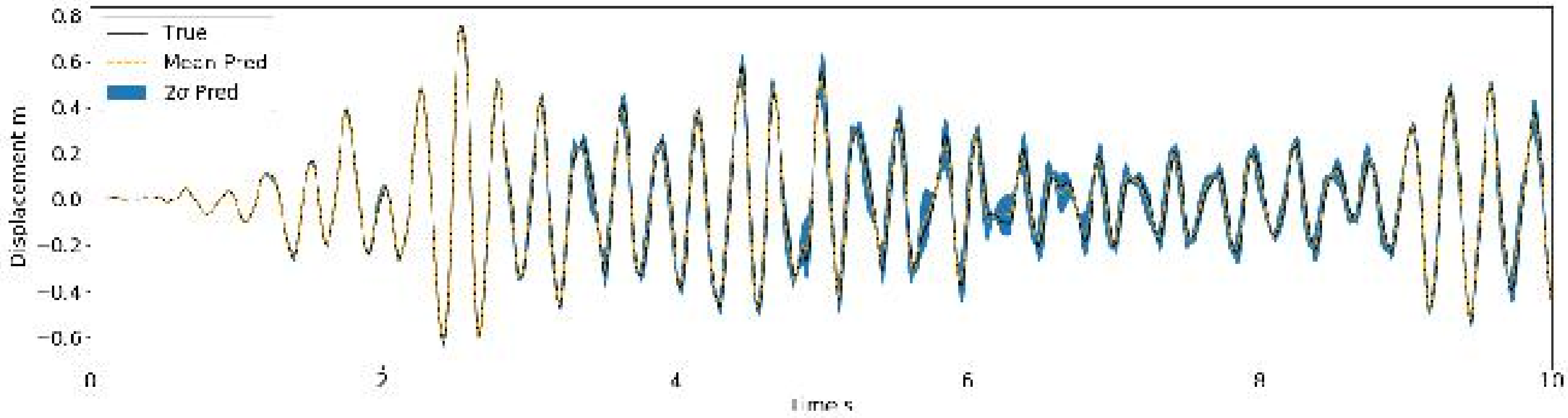


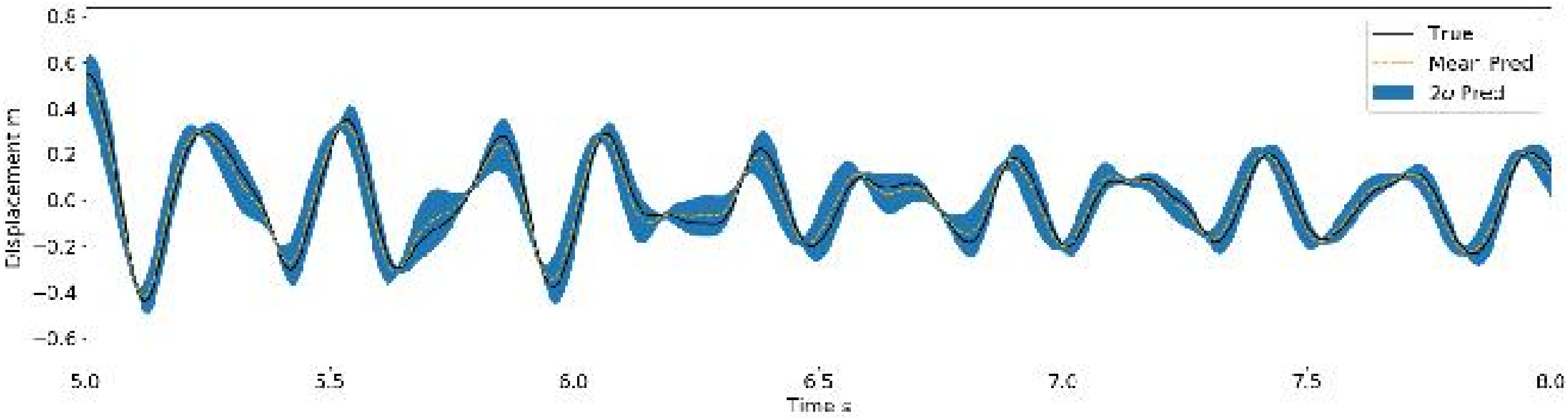
Sandia National Laboratories

Uncertainty Quantification

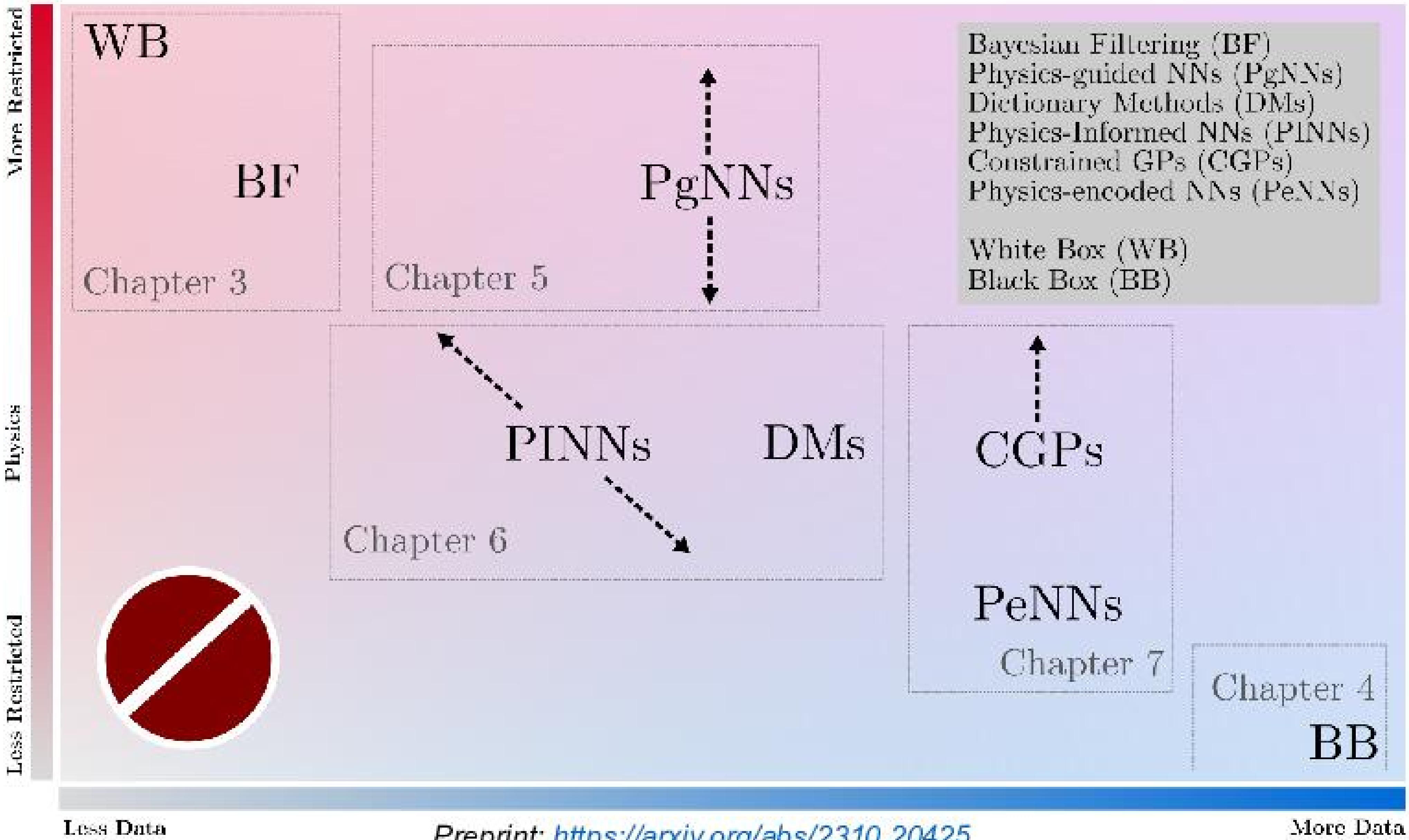
Confidence bounds produced via the generative VpROM model

Parametric ROM evaluated ***40 times using 40 VAE draws*** → Plot ***mean and SD*** at each time step





At the Nexus of Models & Data → Physics - enhanced Learning



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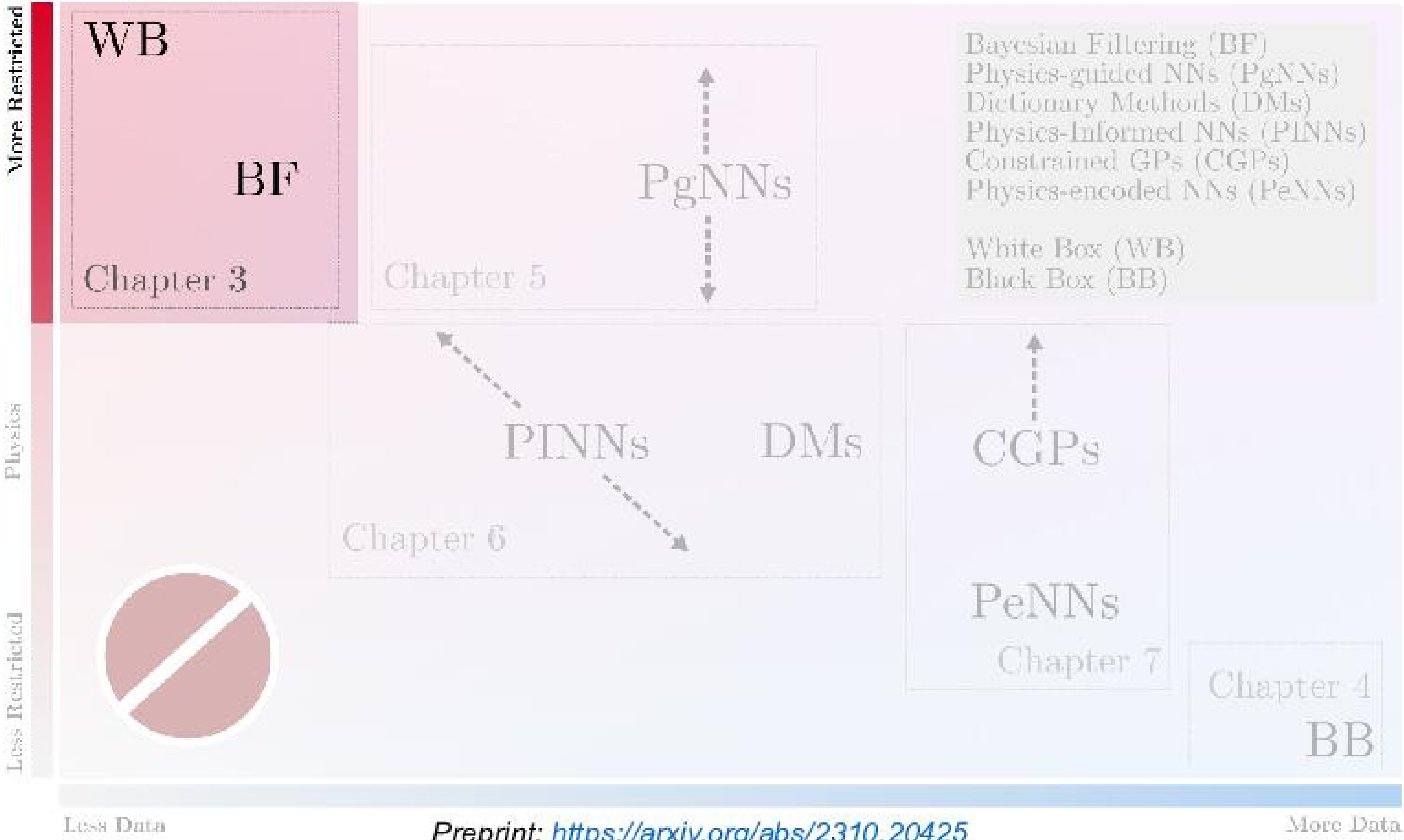
Preprint: https://arxiv.org/abs/2310.20425

At the Nexus of Models & Data → Physics - enhanced Learning



Preprint: https://arxiv.org/abs/2310.20425

White Box Hybrid Schemes: Bayesian Filtering

Consider the general dynamical system described by the following nonlinear continuous **state-space (process) equation**, which is transparent (the model form is fully available/accessible)

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t))$$

and the nonlinear **observation equation** at time $t = k\Delta t$

$$\mathbf{y}_{\mathbf{m}}(t) = \mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{v}(t))$$

or in discrete form:

$$\mathbf{x}_{k+1} = \mathbf{F}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k)$$

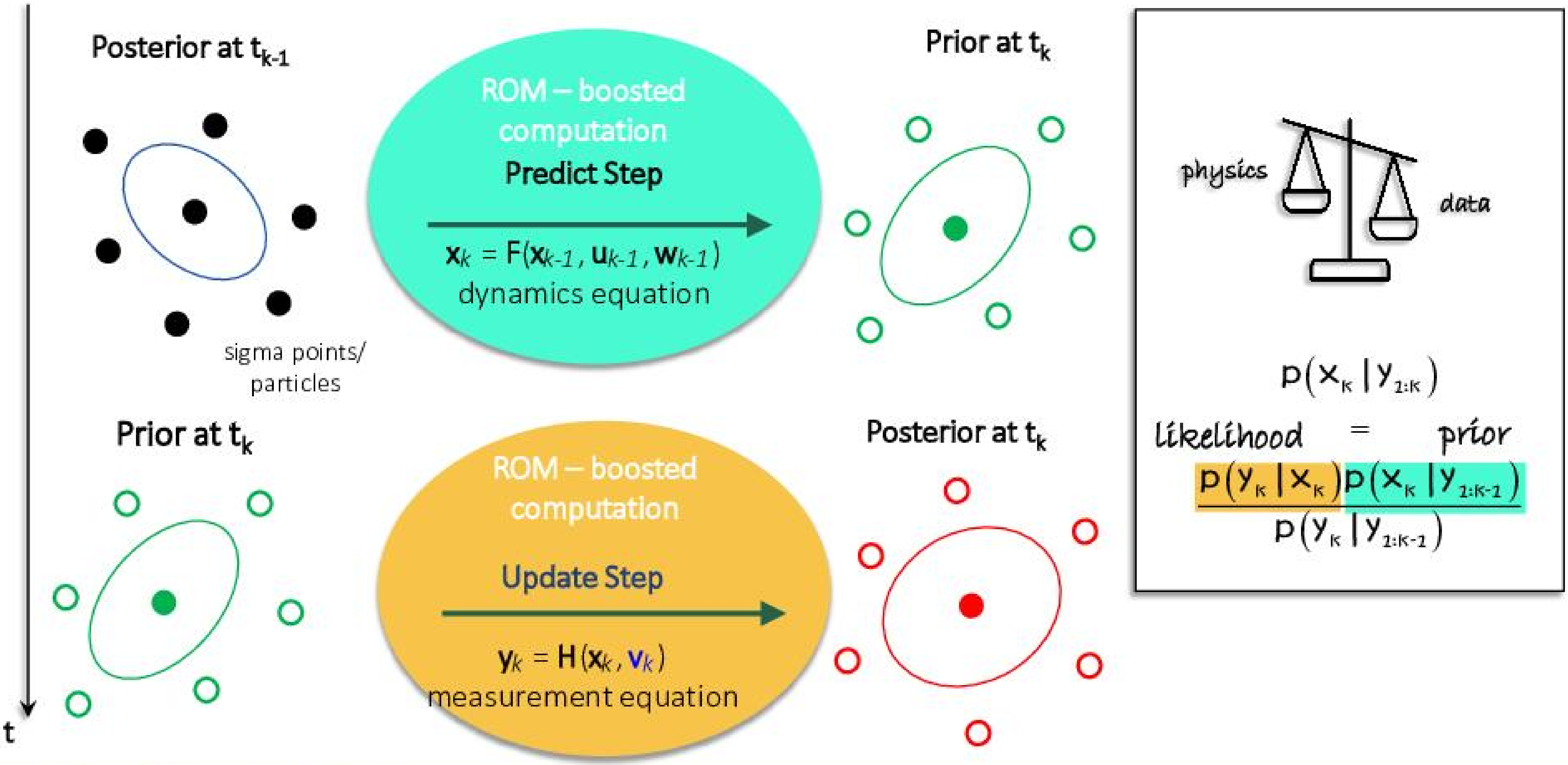
$$\mathbf{y}_k = \mathbf{H}(\mathbf{x}_k, \mathbf{v}_k)$$

model (possibly ROM)

data

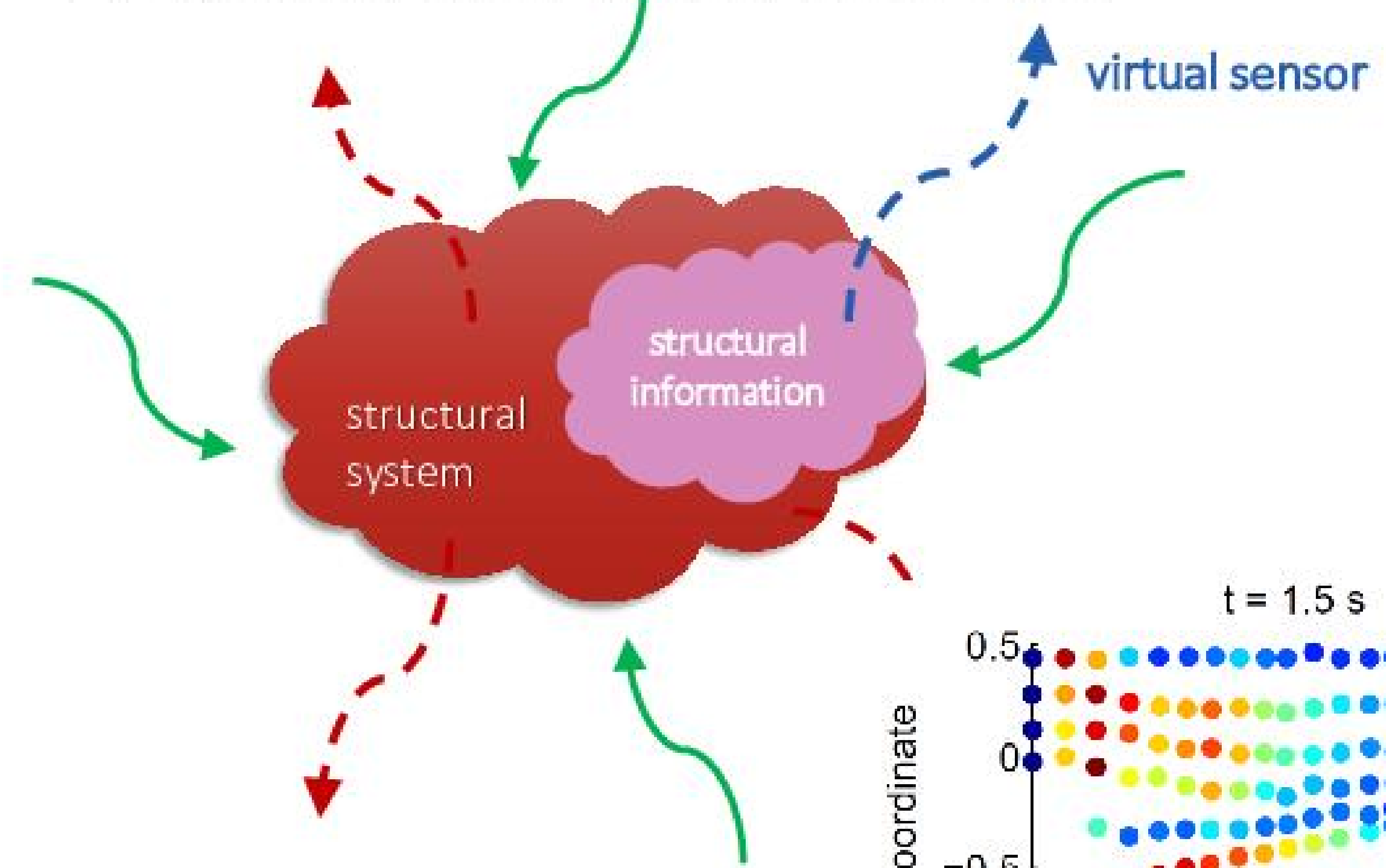
uncertainty

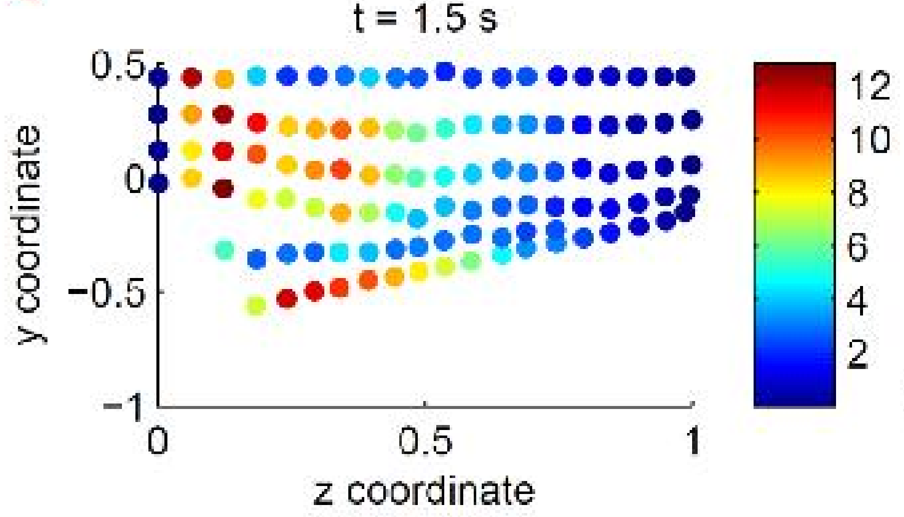
where $\mathbf{x}_k$ is assumed to be a random variable, $\mathbf{w}_k$ is the process noise vector with covariance matrix $\mathbf{Q}_k$, $\mathbf{v}_k$ is the observation noise vector with covariance matrix $\mathbf{R}_k$, and functions **F, H** can be nonlinear in nature



Nonlinear Bayesian Filtering

Virtual Sensing
Everything / Everywhere / All-at-Once





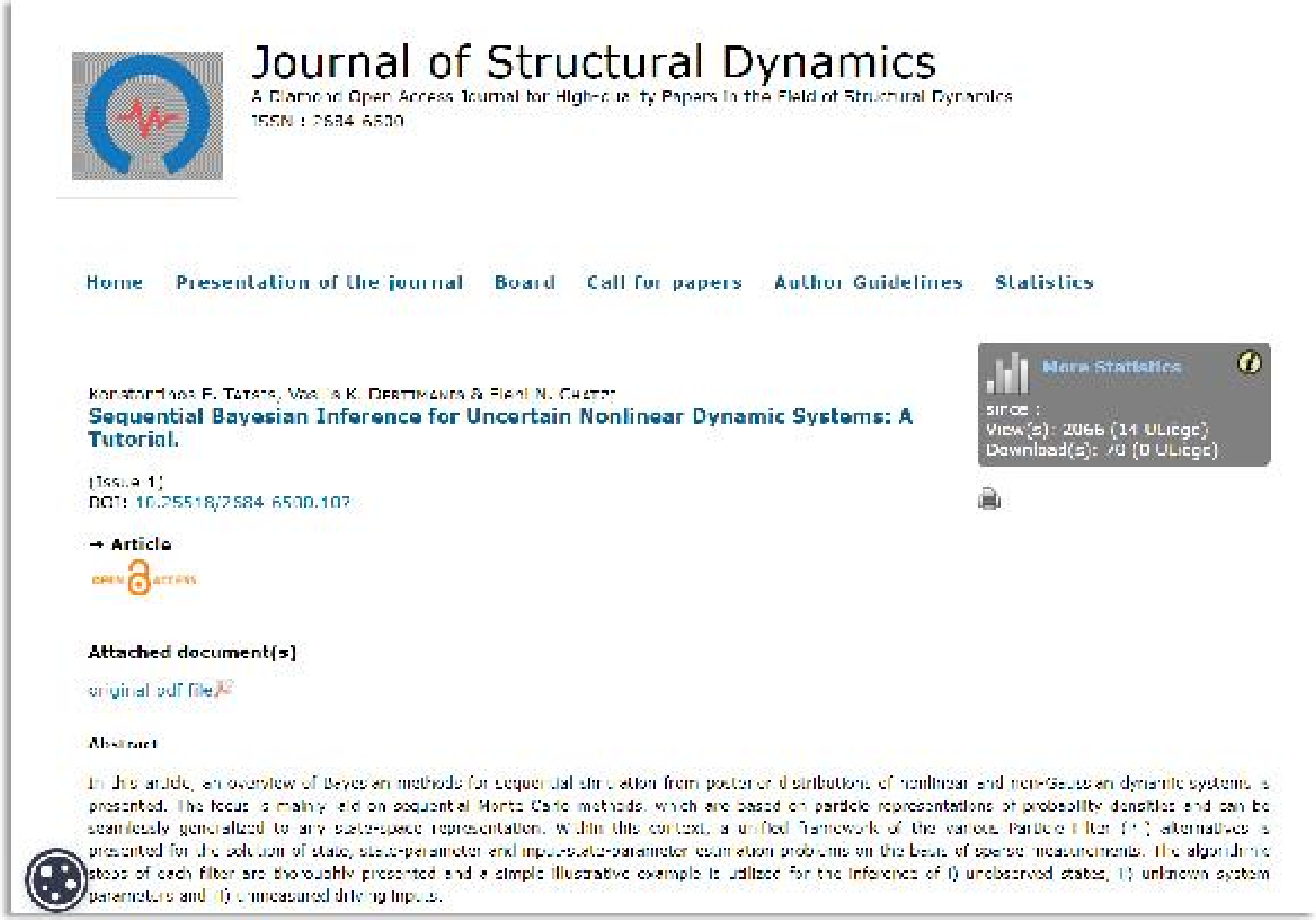
Developed schemes for

- State estimation
- Joint state-parameter estimation (collab with A. Smyth)
- Input-state estimation (collab with S. Eftekhar-Azam, Costas Papadimitriou)
- Joint input-state-parameter estimation
- Non-smooth Systems (collab with M. Chatzis)
- Systems with spatially distributed Inputs

Bayesian Filtering for Virtual Sensing
A Tutorial

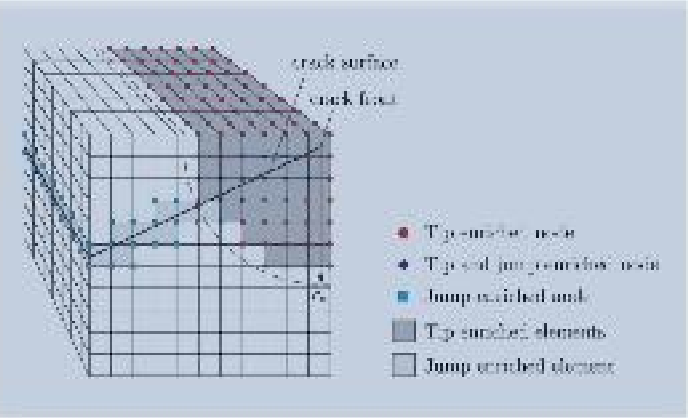
Paper

Code



Real-Time Vibration based Crack Detection using pROMs

Application: Crack Detection on Fuselage



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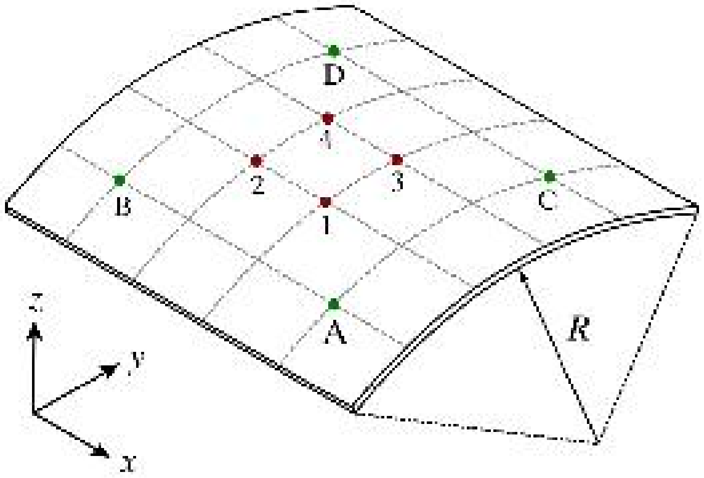
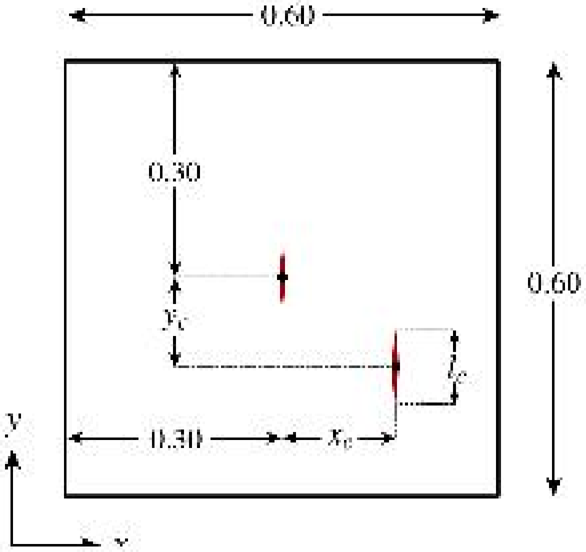
Tools:

- XFEM
- Reduced Order Modeling
- Hierarchical Bayesian Filtering





Computationally affordable physics-based Models

Parametric Model Order Reduction

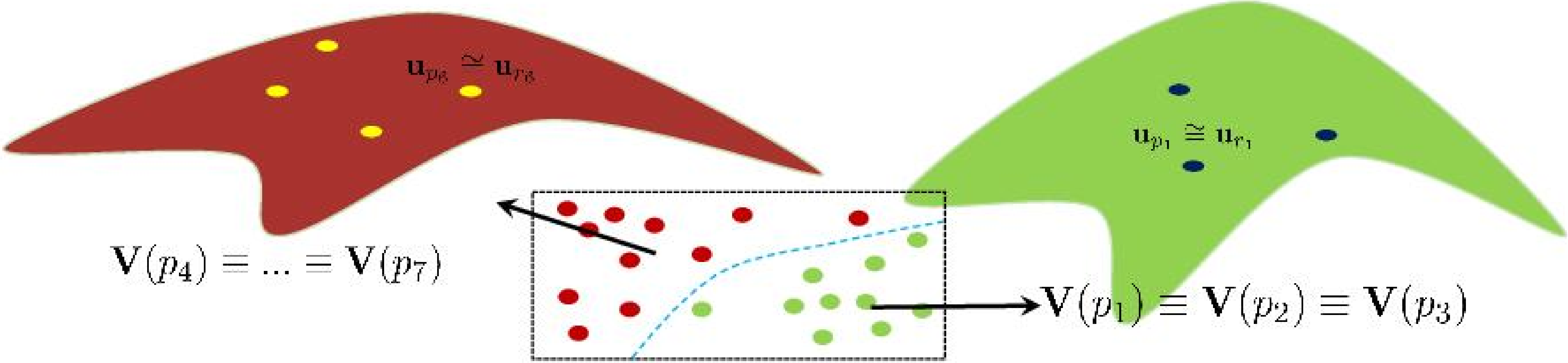
θ : Crack parameters $\rightarrow$ non-smooth base vectors

$$\mathbf{M}(\theta)\ddot{\mathbf{u}}(t)+\mathbf{C}(\theta)\dot{\mathbf{u}}(t)+\mathbf{K}(\theta)\mathbf{u}(t)=\mathbf{S}_p\mathbf{p}(t), \quad \mathbf{u}\in\mathbb{R}^n$$

$$\downarrow$$

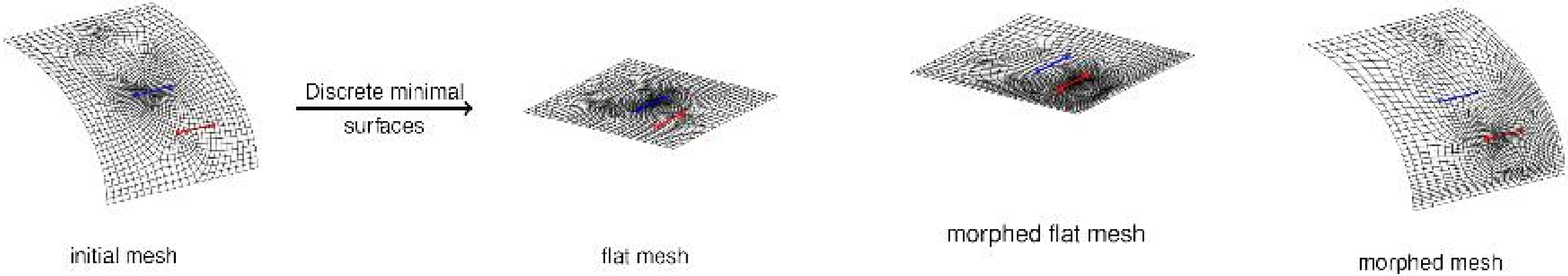
$$\tilde{\mathbf{M}}(\theta)\ddot{\mathbf{q}}(t)+\tilde{\mathbf{C}}(\theta)\dot{\mathbf{q}}(t)+\tilde{\mathbf{K}}(\theta)\mathbf{q}(t)=\mathbf{V}(\theta)^T\mathbf{S}_p\mathbf{p}(t)\ ,\mathbf{q}=\mathbf{V}(\theta)\mathbf{q}(t), \quad \mathbf{q}\in\mathbb{R}^k$$

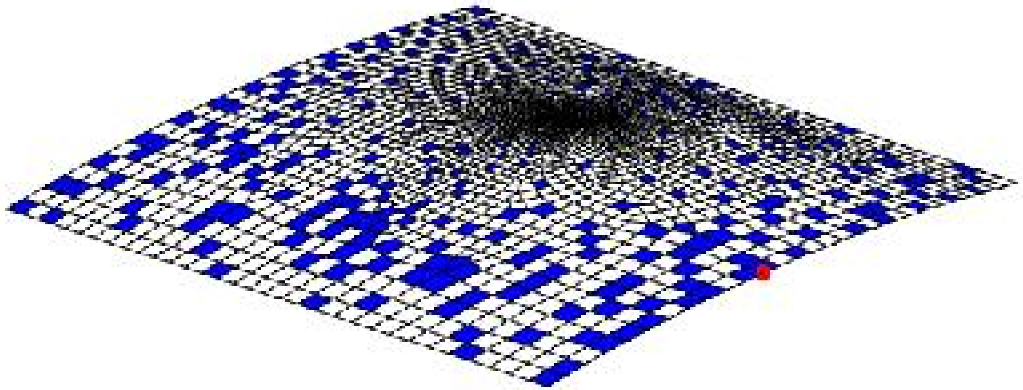
$\mathbf{V}(\theta)$ Reduced basis, extracted via clustering over regions of the parameter space

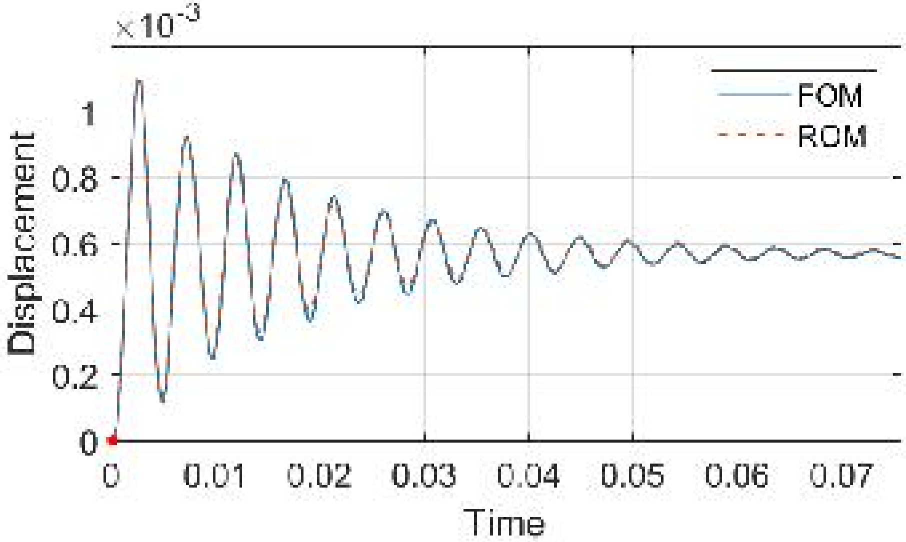


Computationally affordable physics-based Models

Parametric Model Order Reduction







Agathos, Tatsis, Chatzi et al. (2021)

Real-time state-input-parameter estimation

Hierarchical Bayesian Filtering approach

Input-State Estimation
The Augmented Kalman Filter (AKF)

$$\mathbf{x}_{k+1} = \mathbf{A}_d(\theta)\mathbf{x}_k + \mathbf{B}_d(\theta)\mathbf{p}_k + \mathbf{v}_k$$

$$\mathbf{y}_k = \mathbf{G}_d(\theta)\mathbf{x}_k + \mathbf{J}_d(\theta)\mathbf{p}_k + \mathbf{w}_k$$

$$\mathbf{x}_k^a = \begin{bmatrix} \mathbf{x}_k \\ \mathbf{p}_k \end{bmatrix}$$

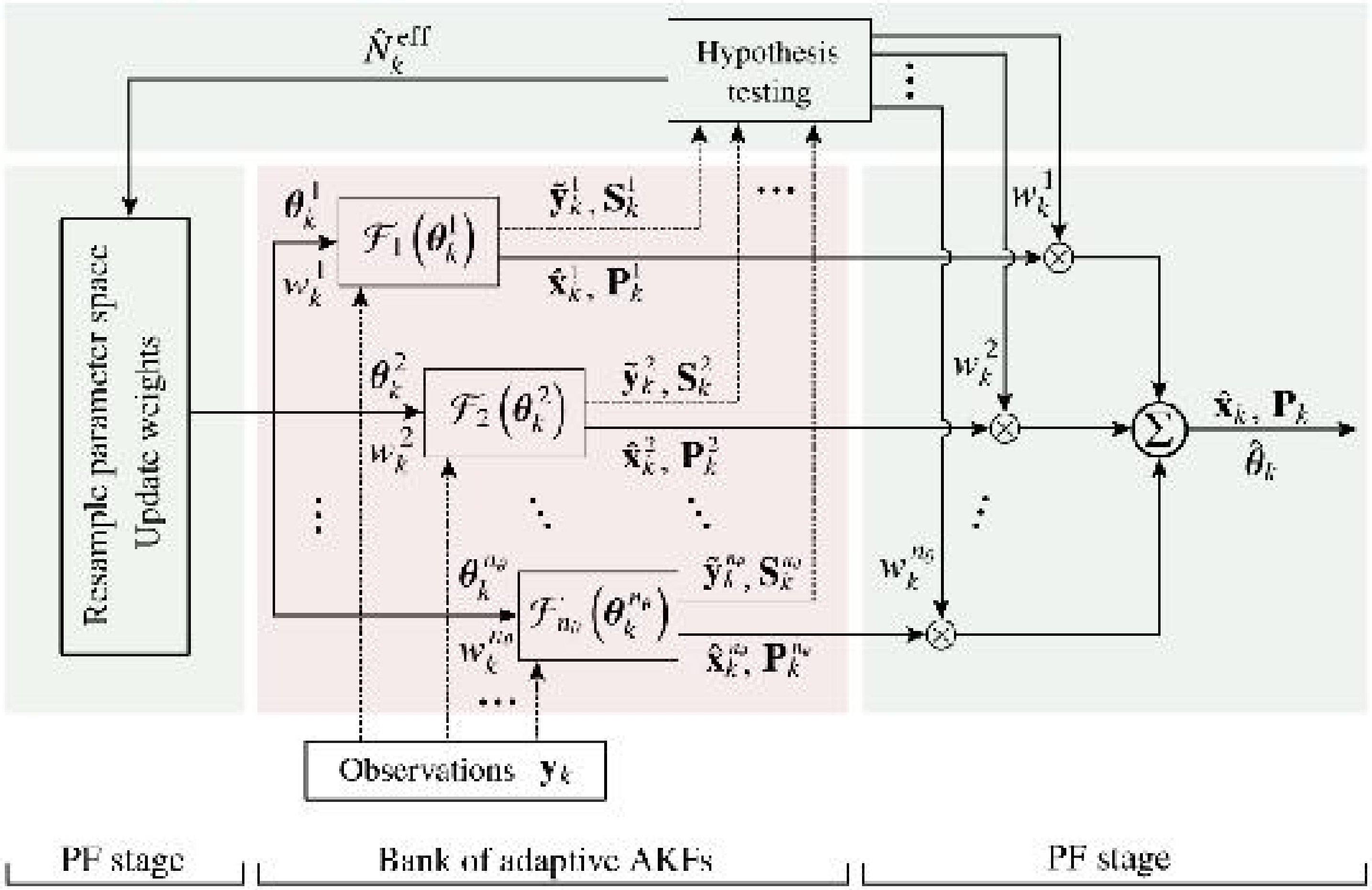
$$\mathbf{p}_{k+1} = \mathbf{p}_k + \mathbf{r}_k$$

or GP-LFM*

$$\mathbf{x}_k^a = \begin{bmatrix} \mathbf{A}_d(\theta) & \mathbf{B}_d(\theta) \\ \mathbf{0} & \mathrm{I} \end{bmatrix}\mathbf{x}_k^a + \mathbf{v}_k$$

$$\mathbf{y}_k = \begin{bmatrix} \mathbf{G}_d(\theta) & \mathbf{J}_d(\theta) \end{bmatrix}\mathbf{x}_k^a + \mathbf{w}_k$$

Joint input-state-parameter estimation



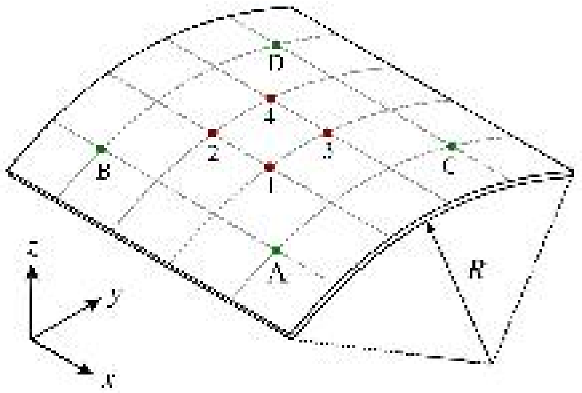
Tatsis, Chatzi et al. (2021, MSSP)

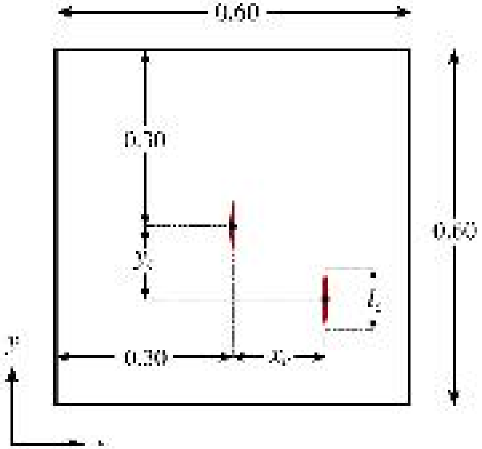
*Vettori, Di Lorenzo, Peeters, Chatzi (2023), Preprint: https://arxiv.org/pdf/2306.16302.pdf

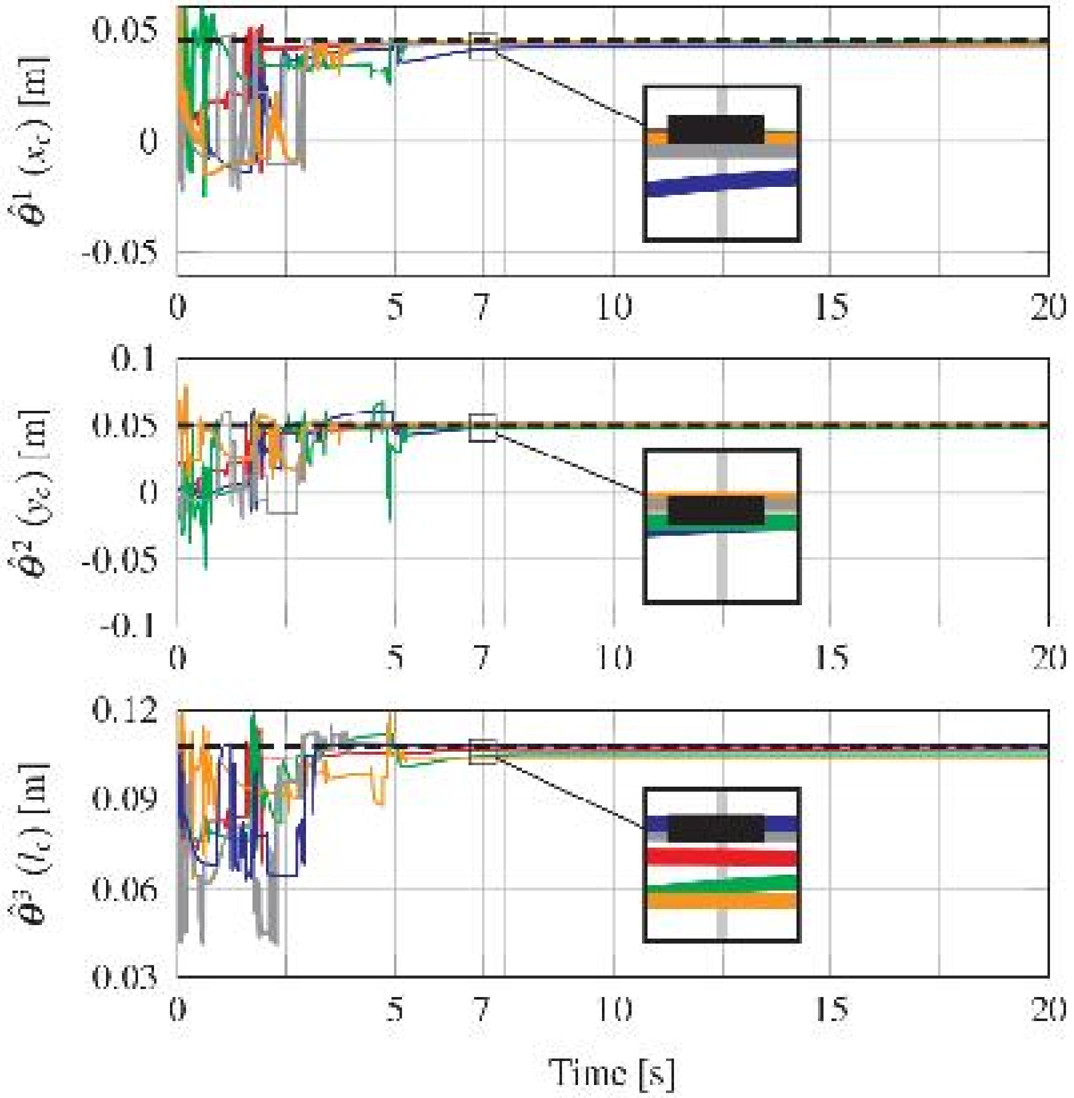
Inverse Formulation – Damage Detection

Results – Crack Localization

Parameter estimation results from five different runs depicted in red, blue, green orange and grey; actual values represented via black dashed lines



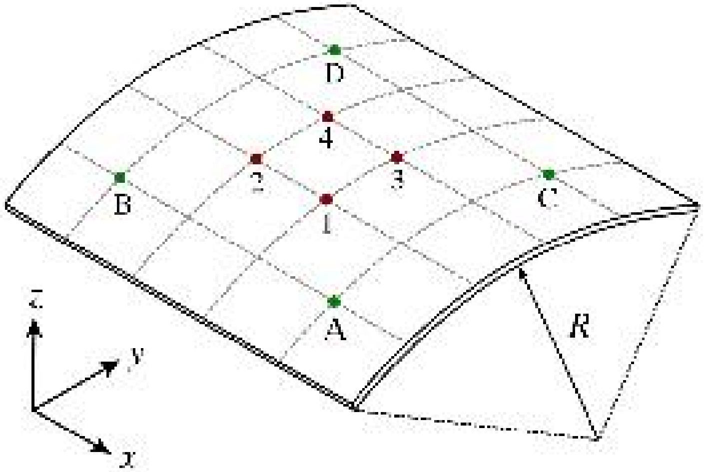




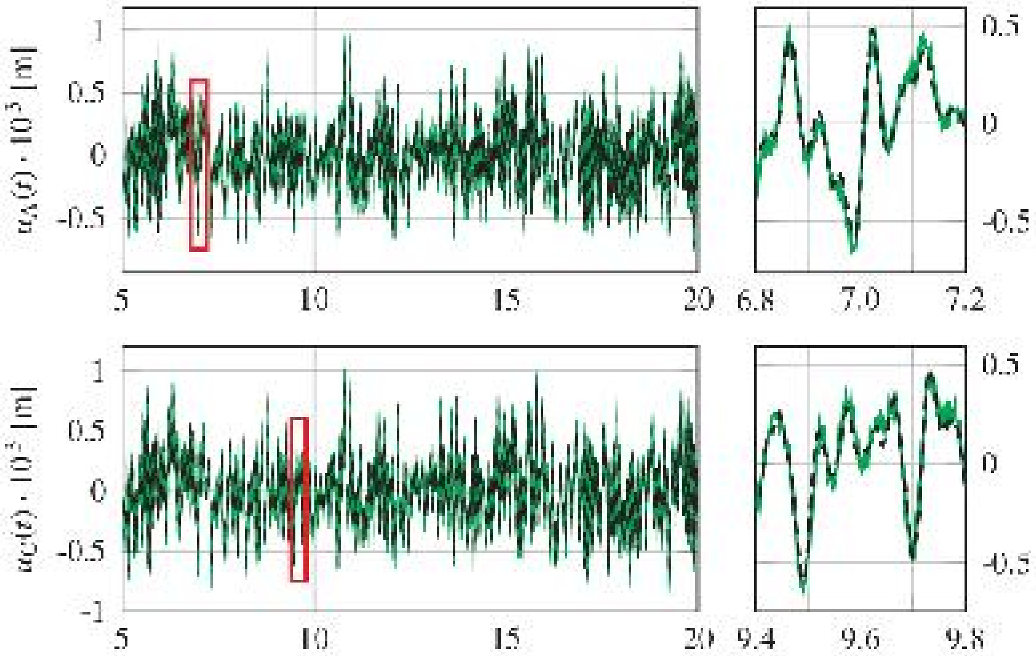
Tatsis, Chatzi et al. (2021, MSSP)

Inverse Formulation – Virtual Sensing

Results – Response Prediction in unmeasured locations



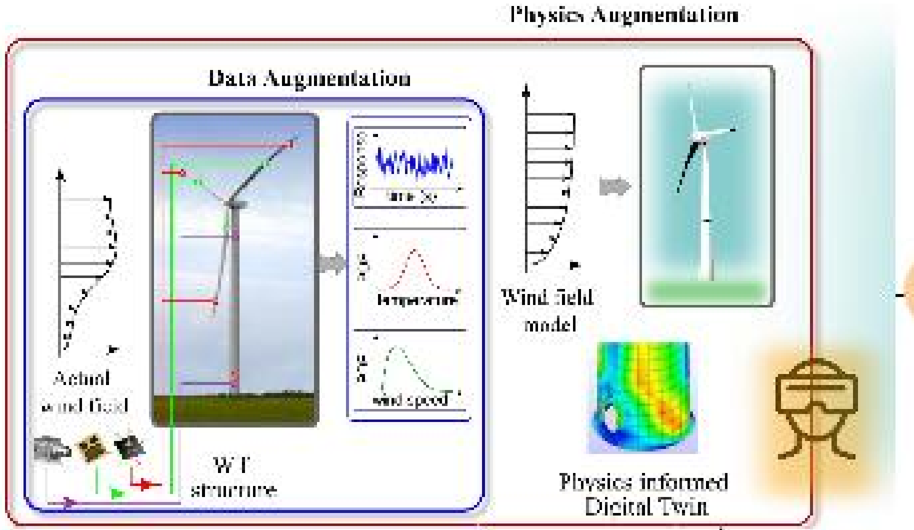
Response estimation at unmeasured points A, B, C and D; green line represents the actual noisy signal and black dashed line depicts the predicted response



	FOM1	FOM2	ROM
size	65,730	50	40
elements	10,402	10,402	258–270
number of clusters	–	–	8
maximum error %	–	0.0000	1.8279
mean error %	–	0.0000	0.6123
solution time (s)	71.5291	22.8316	0.1288
speedup	–	–	177.2420

Tatsis, Chatzi et al. (2021, MSSP)

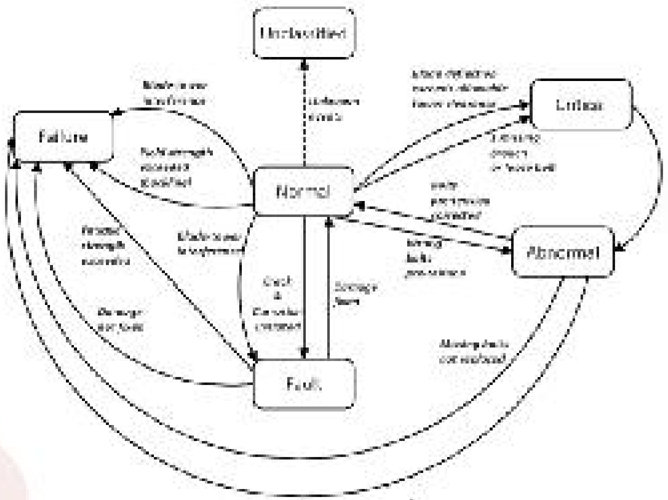
Application: Interactive Digital Twins - Closing the loop



API

Courtesy of Dr. Silvia Vettori – ETHZ/Siemens DIS

Object-Oriented Real-Time Decision-Tree



Courtesy of Dr. Silvia Vettori – ETHZ/Siemens DIS & SMM Chair

API

Cloud computing, storage & analytics

SIEMENS
Ingenuity for life

RTDT

irmos

Grey Box Modeling – Coping with Model Mismatch

Imprecisely known/modelled systems

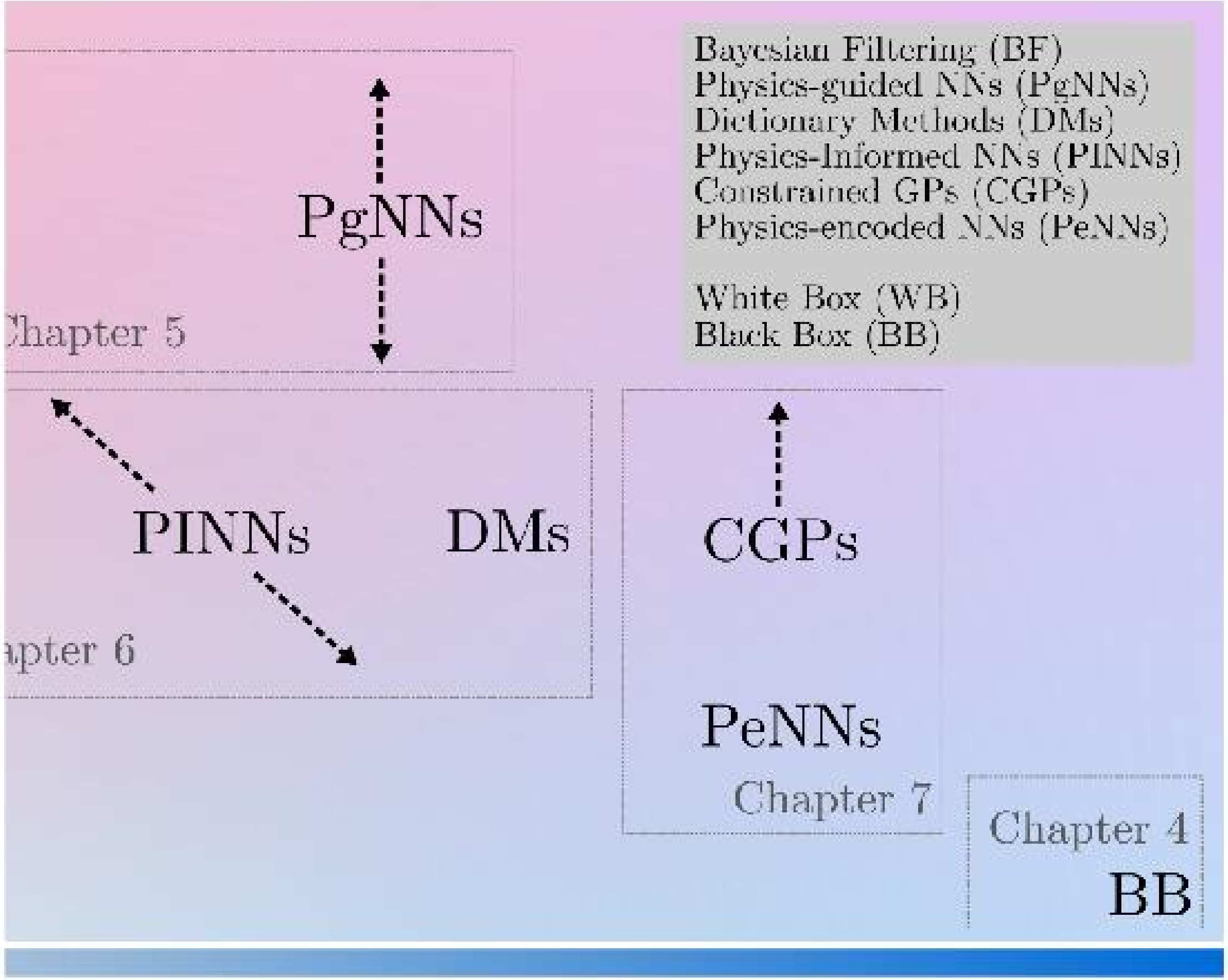
Complex structures & assemblies

Systems exhibiting nonlinearities

At the Nexus of Models & Data → Physics - enhanced Learning

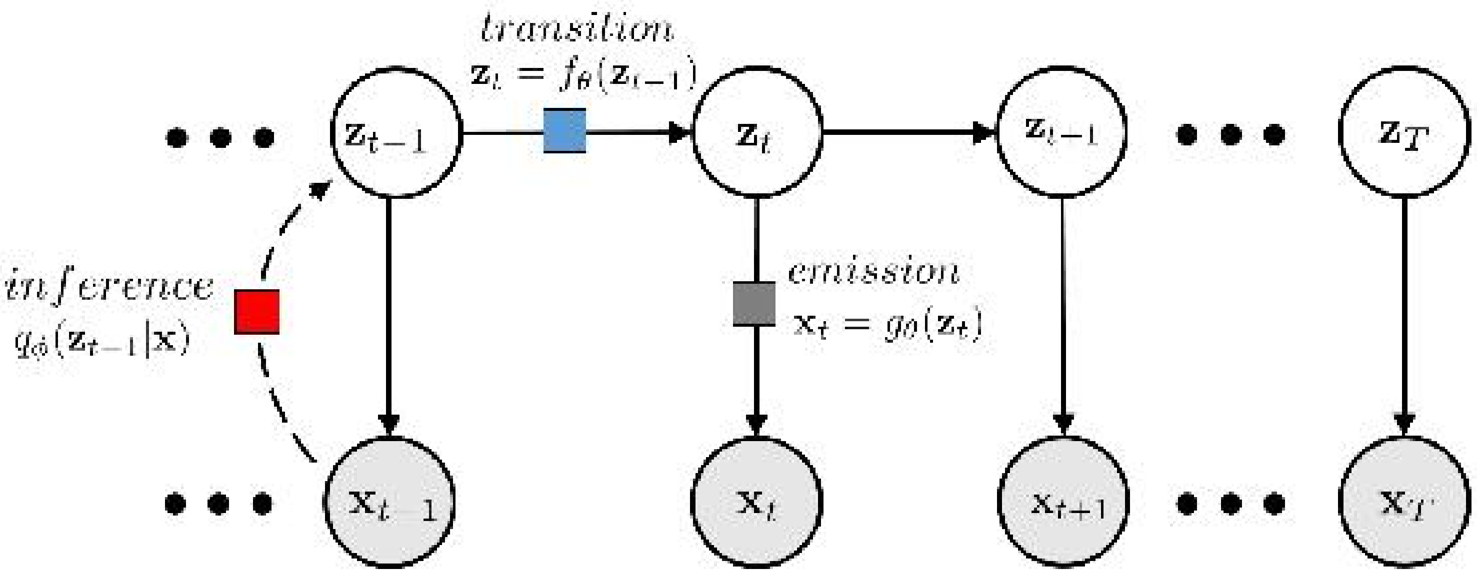
Paper

Code



Preprint: https://arxiv.org/abs/2310.20425

Black Box – The temporal VAE / DMM



$$\mathcal{L}(\theta, \phi; \mathbf{x}) = \mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})} \log \frac{p(\mathbf{x}, \mathbf{z})}{q_\phi(\mathbf{z}|\mathbf{x})}$$

↓

$$\mathcal{L}(\theta, \phi; \mathbf{x}) = \sum_{t=1}^{T} \mathcal{L}_{\text{reconstruction}} - \mathcal{L}_{\text{KL}}$$

$$= \sum_{t=1}^{T} \mathbb{E}_{q_\phi(\mathbf{z}_t|\mathbf{x})}[\log p_\theta(\mathbf{x}_t|\mathbf{z}_t)] - \mathbb{E}_{q_\phi(\mathbf{z}_{t-1}|\mathbf{x})}[\mathrm{KL}(q_\phi(\mathbf{z}_t|\mathbf{x}) || p_\theta(\mathbf{z}_t|\mathbf{z}_{t-1})]$$

Nonlinear State Space

$$\mathbf{z}_t = f(\mathbf{z}_{t-1}, \mathbf{u}_{t-1}) + w_t, \quad (transition)$$
$$\mathbf{x}_t = g(\mathbf{z}_t) + v_t, \quad (observation)$$

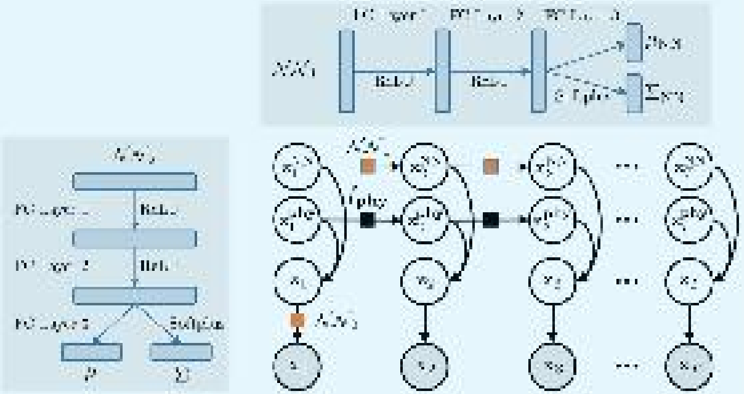
- The latent space has no physical connotation
- The model largely depends on the quality of inference model q.
- poor generalization ability

Goals

- Capture physically disentangled latent representation
- Generalize to behavior lying beyond the training dataset

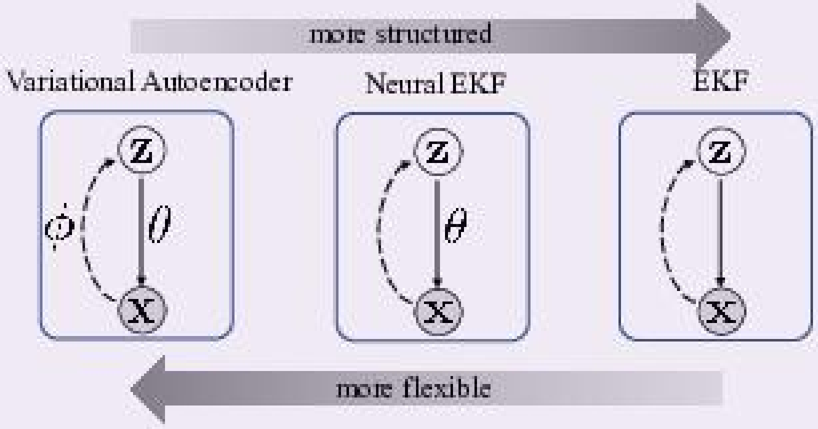
Physics-enhanced Learning
Developed Methods

Physics-guided Schemes



Physics-guided Deep Markov Models

Physics-encoded Schemes

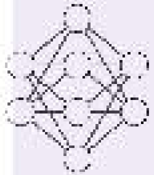


Neural Extended KFs

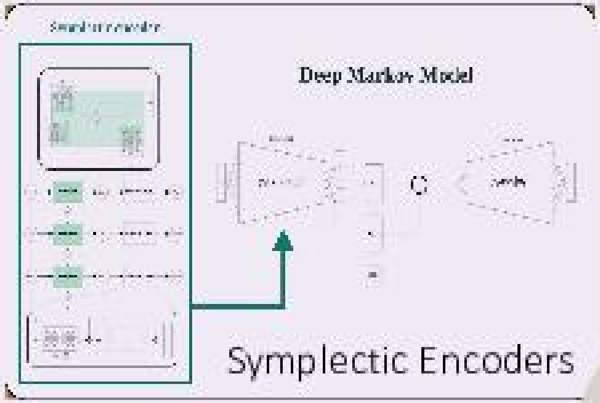
$$\frac{d\mathbf{h}(t)}{dt} = f(\mathbf{h}(t), t, \boldsymbol{\theta})$$

$$\downarrow$$

$$\frac{d\mathbf{h}(t)}{dt} = f_{phy}(\mathbf{h}(t), t) + NN(\mathbf{h}(t), t, \boldsymbol{\theta})$$

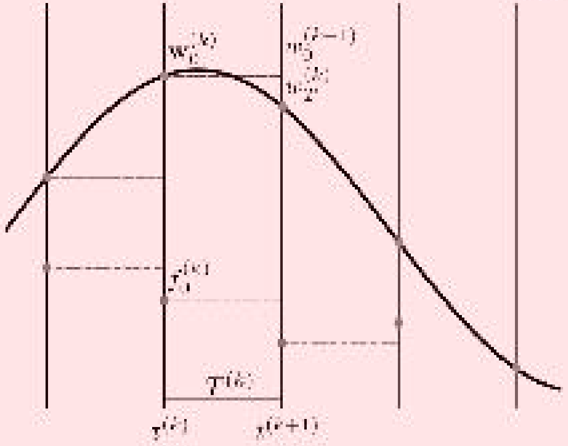


Physics-Informed Neural ODEs



Symplectic Encoders

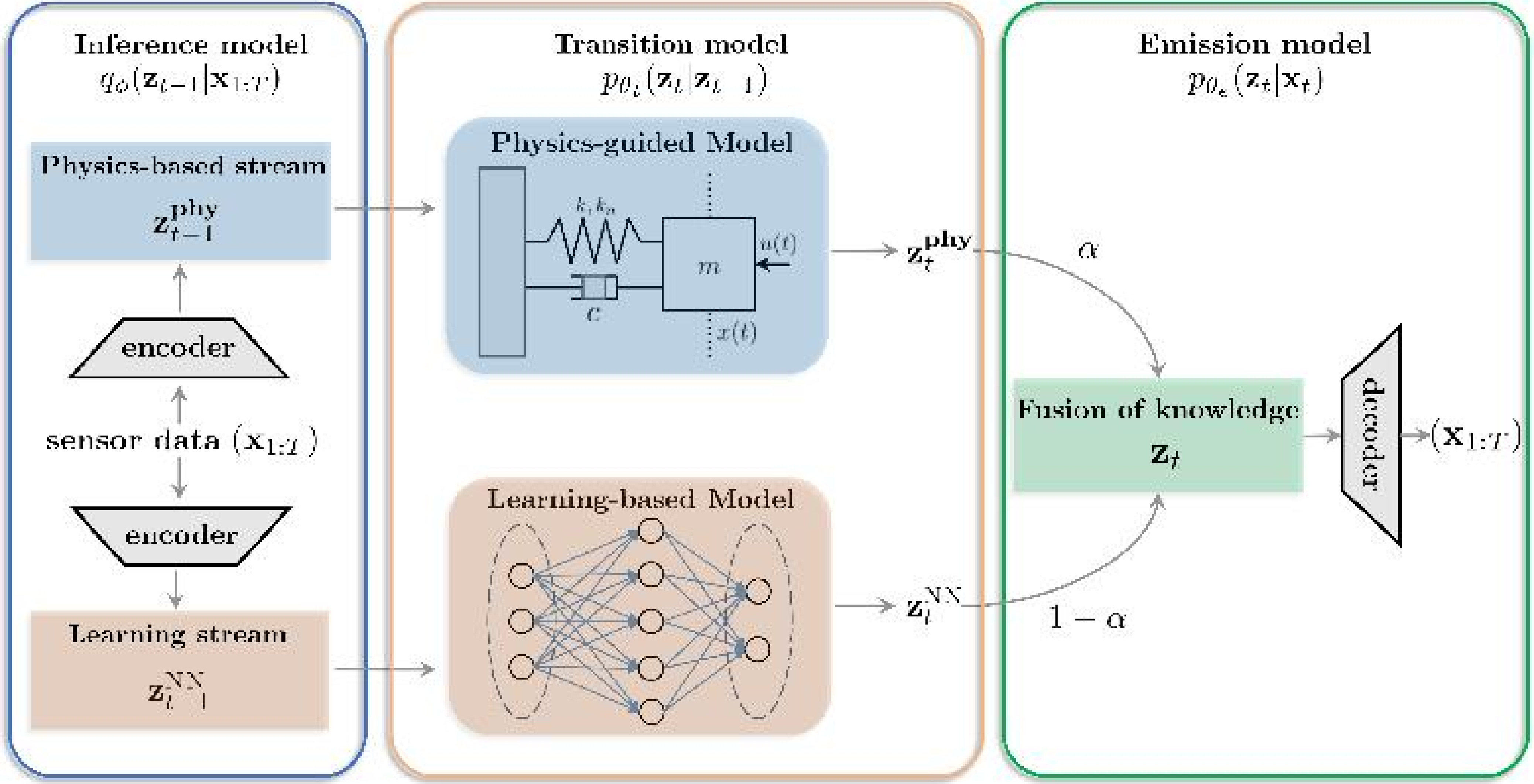
Physics-Informed



1-step ahead PINN predictors

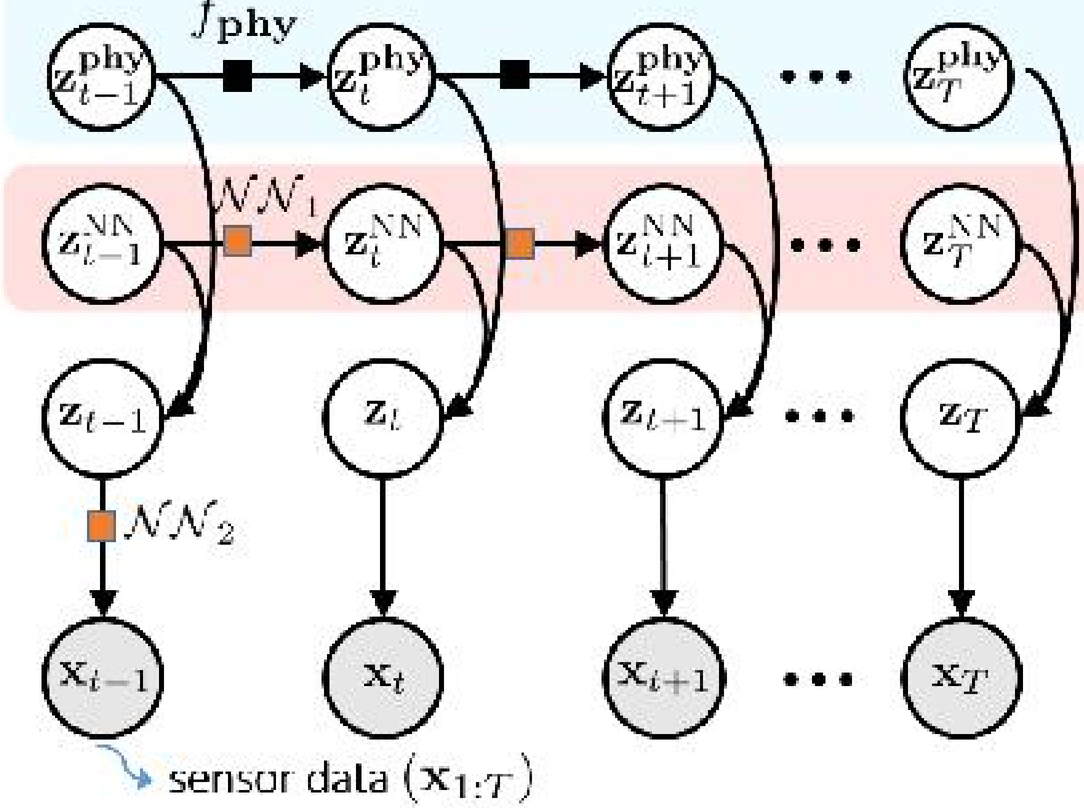
Lai, Z., Mylonas C., Nagarajaiah, S., Chatzi, E., JSV, 2021
Wei Liu, Zhilu Lai, Eleni Chatzi, MSSP, 2022 | Bacsa, K., Lai, Z., Liu, W., Todd, M., Chatzi, E., (nature) Science Reports (2023)

Physics-Guided Deep Markov Models for Learning Dynamics



Zhilu Lai, Wei Liu, Kiran Bacsa, Eleni Chatzi, Physics-guided Deep Markov Models (PgDMMs), 2022, MSSP

Physics as a bias in the NN architecture



$$\mathbf{z}_t^{\text{phy}} \sim \mathcal{N}(\mu_{\text{phy}}(\mathbf{z}_{t-1}^{\mathbf{phy}}), \mathbf{\Sigma}_{\mathbf{phy}}(\mathbf{z}_{t-1}^{\text{phy}}))$$

$$\mathbf{z}_t^{\text{NN}} \sim \mathcal{N}(\mu_{\text{NN}}(\mathbf{z}_{t-1}^{\text{NN}}), \mathbf{\Sigma}_{\text{NN}}(\mathbf{z}_{t-1}^{\text{NN}}))$$

$$\mu(\mathbf{z}_t) = \alpha\mu_{\mathbf{phy}}(\mathbf{z}_{t-1}^{\text{phy}}) + (1-\alpha)\mu_{\text{NN}}(\mathbf{z}_{t-1}^{\text{NN}}),$$

$$\mathbf{\Sigma}(\mathbf{z}_t) = \alpha^2\mathbf{\Sigma}_{\mathbf{phy}}(\mathbf{z}_{t-1}^{\mathbf{phy}}) + (1-\alpha)^2\mathbf{\Sigma}_{\text{NN}}(\mathbf{z}_{t-1}^{\text{NN}}),$$

$$\mu_{\mathbf{phy}}(\mathbf{z}_{t-1}^{\mathbf{phy}}) = f_{\mathbf{phy}}(\mathbf{z}_{t-1}^{\mathbf{phy}}),$$

$$\mathbf{\Sigma}_{\mathbf{phy}}(\mathbf{z}_{t-1}^{\mathbf{phy}}) = \mathcal{NN}_0(\mathbf{z}_{t-1}^{\mathbf{phy}}),$$

$$[\mu_{\text{NN}}(\mathbf{z}_{t-1}^{\text{NN}}), \mathbf{\Sigma}_{\text{NN}}(\mathbf{z}_{t-1}^{\text{NN}})] = \mathcal{NN}_1(\mathbf{z}_{t-1}^{\text{NN}}).$$

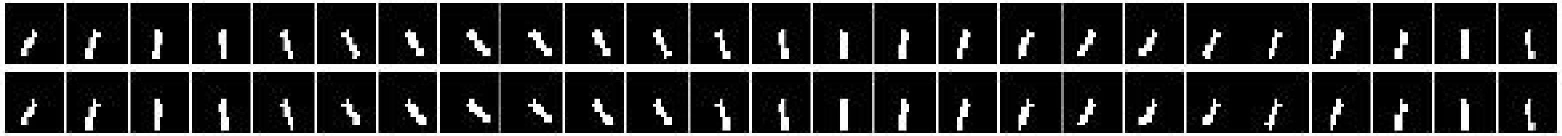
$$\log p(\mathbf{x}) \geq \mathcal{L}_{\mathbf{phy}}(\theta, \phi; \mathbf{x})$$

$$:= \sum_{t=1}^{T} \mathbb{E}_{q_\phi} \log p(\mathbf{x}_t|\mathbf{z}_t) - \sum_{t=1}^{T} \mathbb{E}_{q_\phi^{\mathbf{phy}}(\mathbf{z}_{t-1}^{\mathbf{phy}}|\mathbf{x}_{1:T})}[\text{KL}(q_\phi^{\mathbf{phy}}(\mathbf{z}_t^{\mathbf{phy}}|\mathbf{z}_{t-1}^{\mathbf{phy}}, \mathbf{x}_{1:T})||p_\theta(\mathbf{z}_t^{\mathbf{phy}}|\mathbf{z}_{t-1}^{\mathbf{phy}}))]$$

$$- \sum_{t=1}^{T} \mathbb{E}_{q_\phi^{\text{NN}}(\mathbf{z}_{t-1}^{\text{NN}}|\mathbf{x}_{1:T})}[\text{KL}(q_\phi^{\text{NN}}(\mathbf{z}_t^{\text{NN}}|\mathbf{z}_{t-1}^{\text{NN}}, \mathbf{x}_{1:T})||p_\theta(\mathbf{z}_t^{\text{NN}}|\mathbf{z}_{t-1}^{\text{NN}}))]$$

[Wei Liu, et al. 2022, MSSP, Pg-DMM]

Verification – Pendulum (testing results)



Reconstructed images:
ground-truth (top), PgDMM (bottom)

© Victor Televca

physics-based modeling

$$ml^2\ddot{\theta}(t) = -\mu\dot{\theta}(t) - mgl\sin\theta(t)$$

linearization

$$ml^2\ddot{\theta}(t) = -\mu\dot{\theta}(t) - mgl\theta(t)$$

$$\mathbf{z} = [\theta,\ \dot{\theta}]^T \qquad \dot{\mathbf{z}} = \mathbf{A}_c\mathbf{z}$$

$$\mathbf{A} = e^{\mathbf{A}_c\Delta t}$$

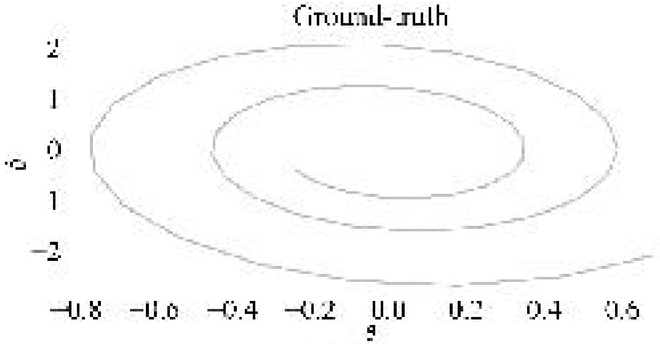
PgDMM approximation

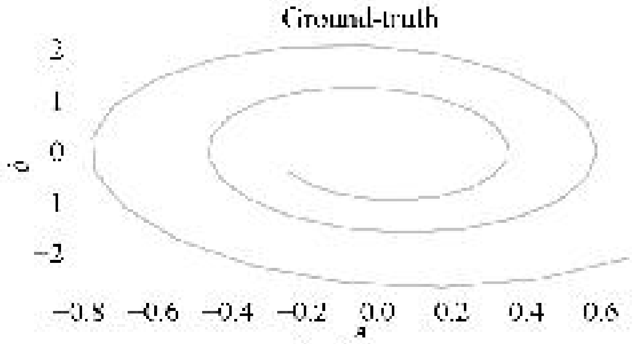
$$\mathbf{z}_t^{\text{phy}} \sim \mathcal{N}(\mathbf{A}\mathbf{z}_{t-1}^{\text{phy}}, \Sigma_{\text{phy}}(\mathbf{z}_{t-1}^{\text{phy}}))$$

$$\mathbf{z}_t^{\text{NN}} \sim \mathcal{N}(\mu_{\text{NN}}(\mathbf{z}_{t-1}^{\text{NN}}), \mathbf{\Sigma}_{\text{NN}}(\mathbf{z}_{t-1}^{\text{NN}}))$$

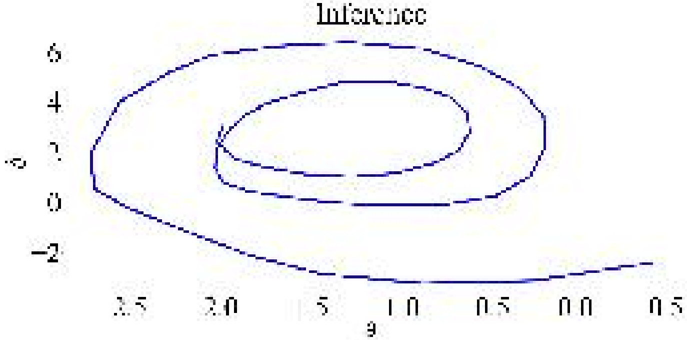
$$\mathbf{z}_t = \alpha\mathbf{z}_t^{\text{phy}} + (1-\alpha)\mathbf{z}_t^{\text{NN}}$$

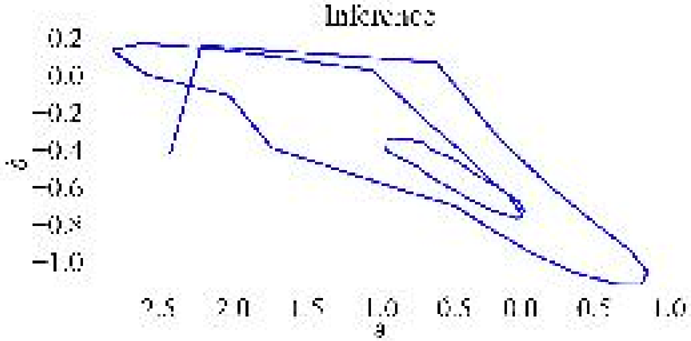
$$\mathbf{x}_t \sim \text{Bernoulli}(\mathcal{NN}_2(\mathbf{z}_t))$$





Ground-truth latent space





Inferred latent space:
PgDMM (left), DMM (right)

Drive-by Monitoring

On Board Monitoring

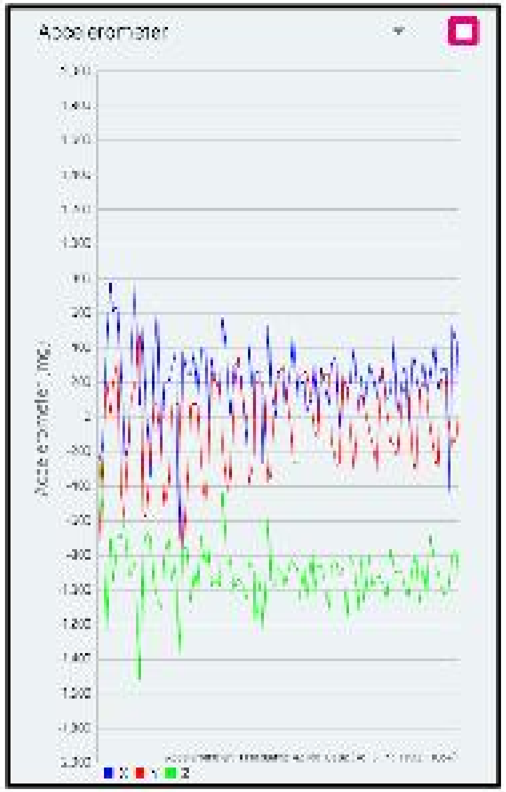
SBB CFF FFS

Online Monitoring & Decision aid

STEVAL-STWINKT1

[https://www.st.com/en/evaluation-tools/steval-stwinkt1.html]

- Wireless connection (Wireless BLE4.2 &Wi-Fi)
- 3-axis accelerometer + 3-axis Gyro
- Humidity and temperature sensor
- Pressure Sensor

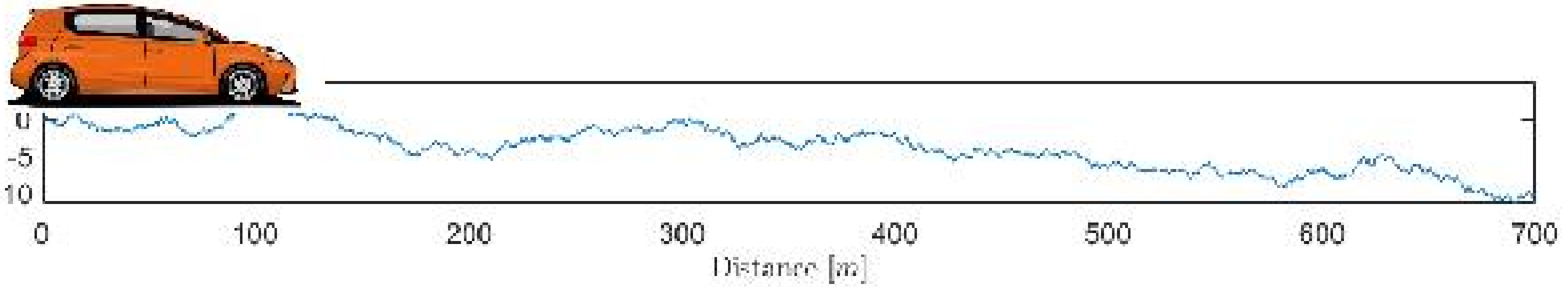


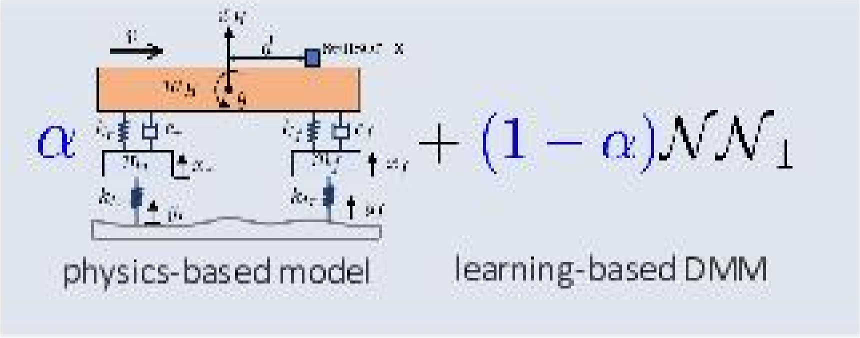
Future Resilient Systems
Singapore ETH Center

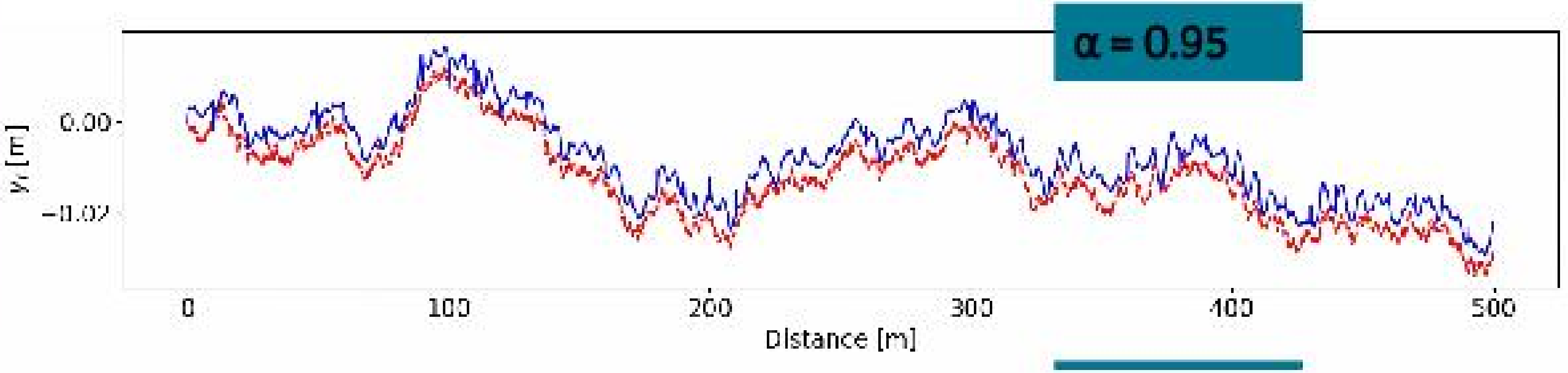
Swiss Federal Railways

Application: Infrastructure Assessment

Mobile Sensing





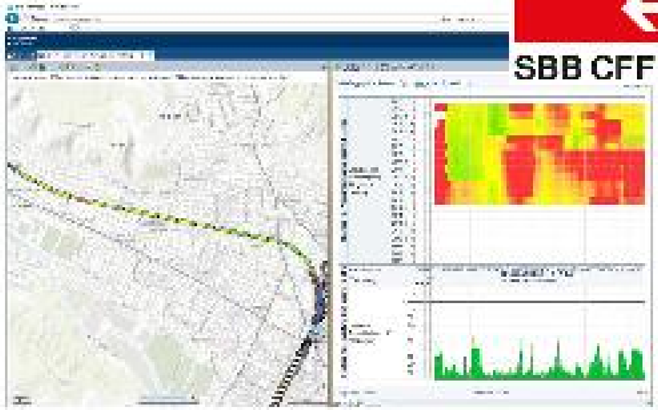


For smooth ride on Ganga Expressway, UPEIDA to deploy AI in collaboration with ETH Zurich, RTDT

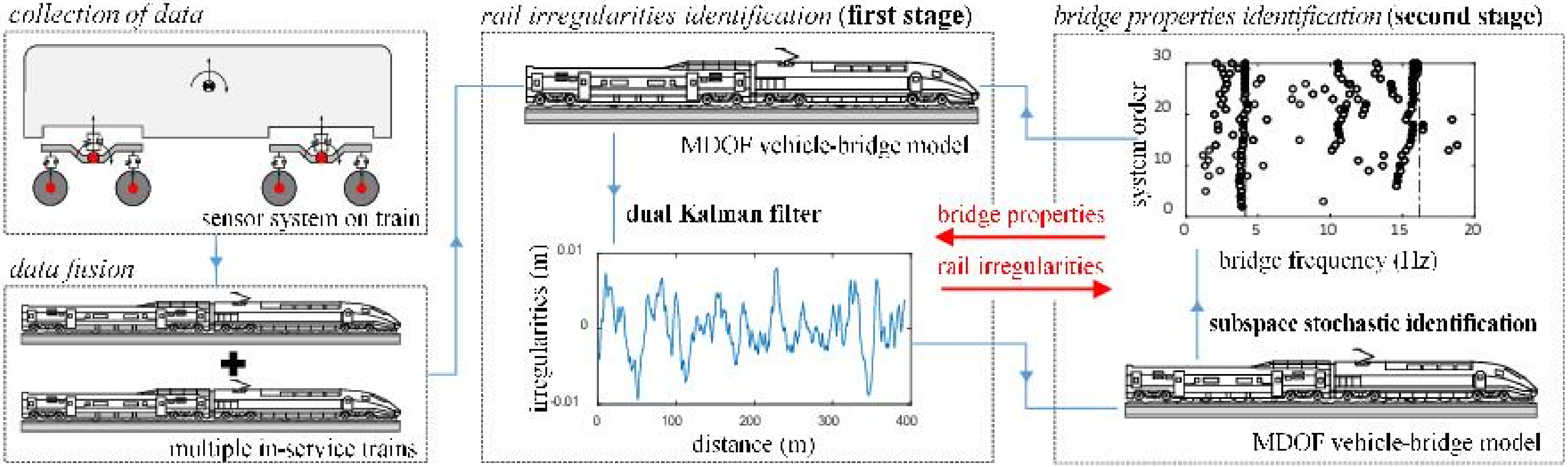
A team from UPEIDA, concessionaires and independent engineers visited Zurich, Switzerland and Germany from 24-29 March 2024, to study national road infrastructure/ motorways, new technologies, mew material and consultant/ contractors approach to build motorways/ highways/ structures/ road safety and traffic control system with a view to implement them in Ganga Expressway.

ETH zürich

Application: On Board Monitoring for Transport Infrastructure



SBB CFF FFS

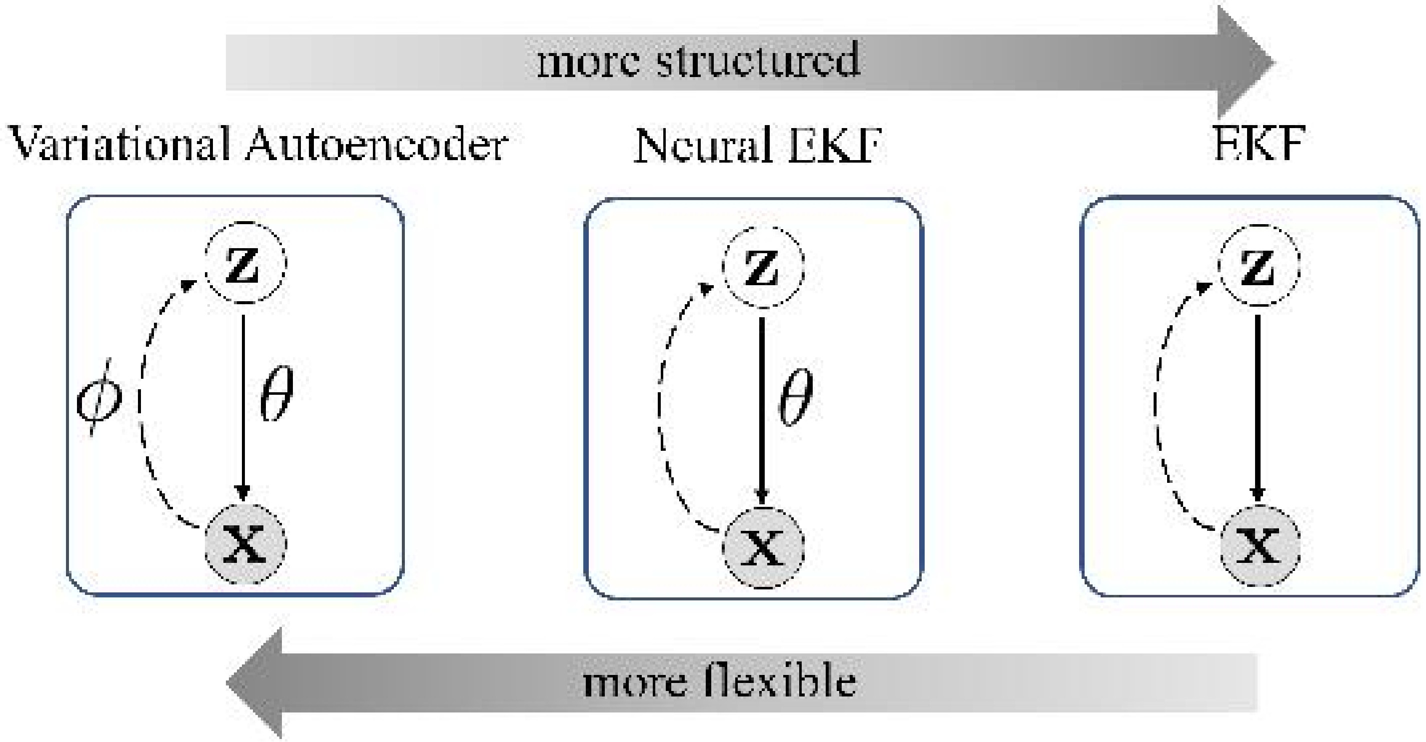


Neural EKF

Comparison with VAE

Paper

Code



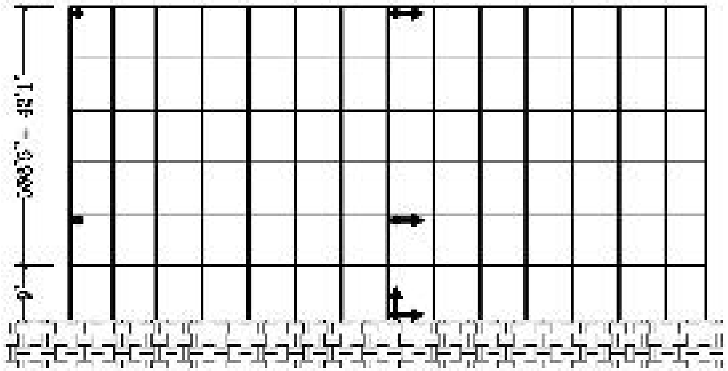
- The VAE employs an inference network parametrized by φ, which is an extra set of parameters independent of dynamics model parameters ϑ.
- The objective function evidence lower bound (ELBO) largely depends on the quality of the inference model and only weakly on the transition dynamics model, often rendering the training of the dynamics model insufficient and the learned model unsuitable for predictive purposes.
- Conducting inference by means of a Neural EKF only depends on the dynamics model of parameters ϑ and does not require additional inference model parameters φ to be trained, thus guaranteeing accurate dynamics models for prediction.

Applications: Virtual Sensing & Damage Detection

*Experimental benchmark by our group, available here

Seismic Response Prediction, Neural EKF

6-story hotel building, San Bernardino



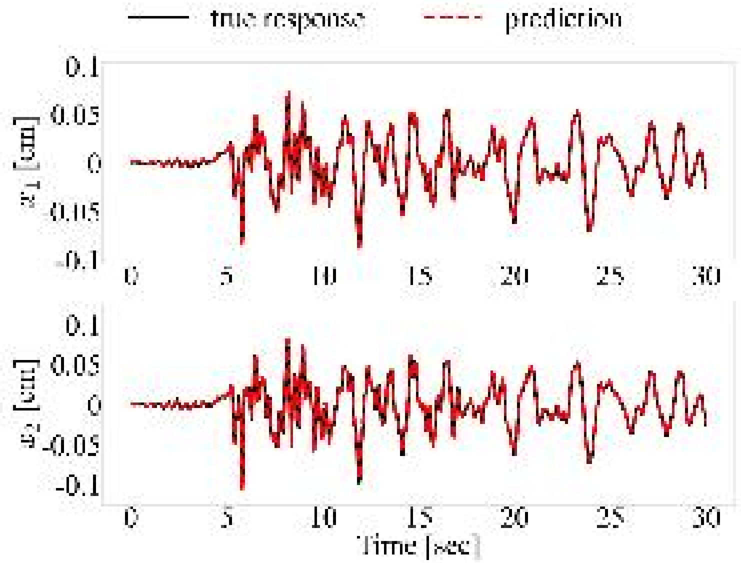
Output (response):

3^{rd} floor response

Roof response

Input:

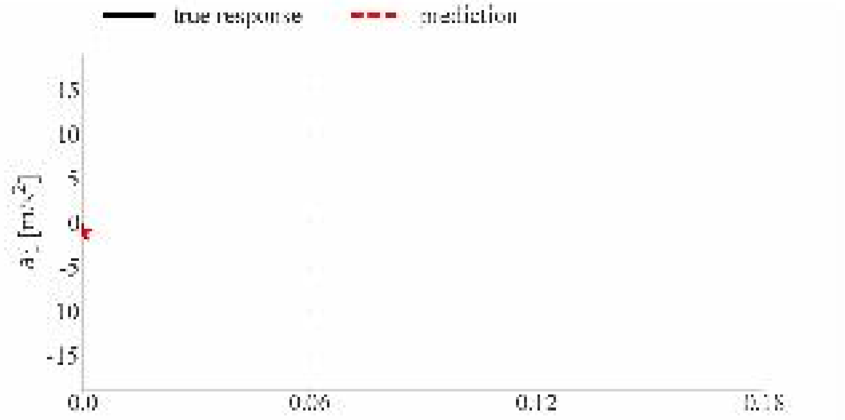
Motion measured at channel 1 (1^{st} floor)

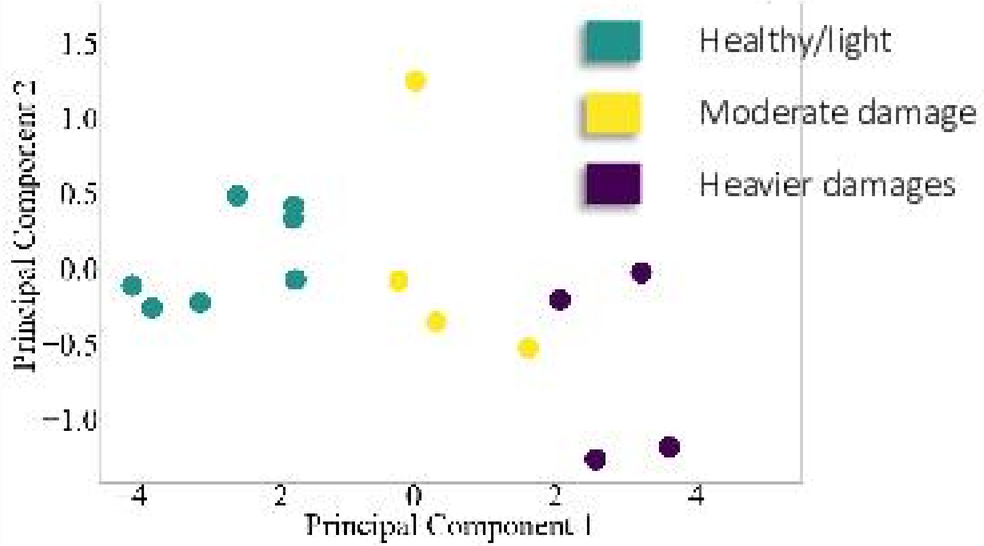


*Data iobtained from the Center for Engineering Strong Motion Data (CESMD) https://www.cesmd.org/

Damage Detection, Neural EKF

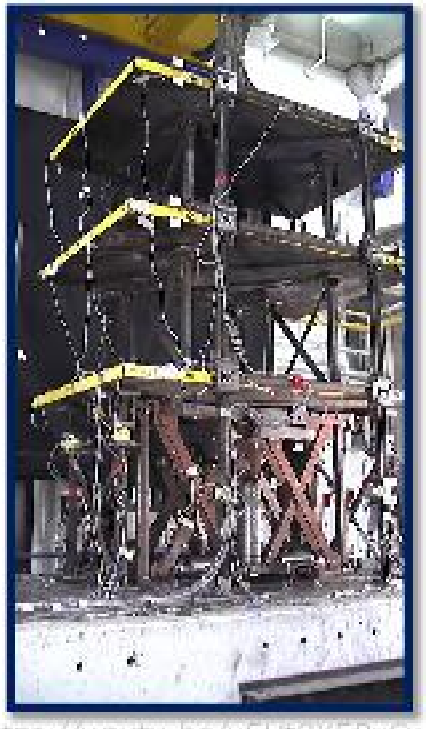
Wind Turbine blade in climatic chamber





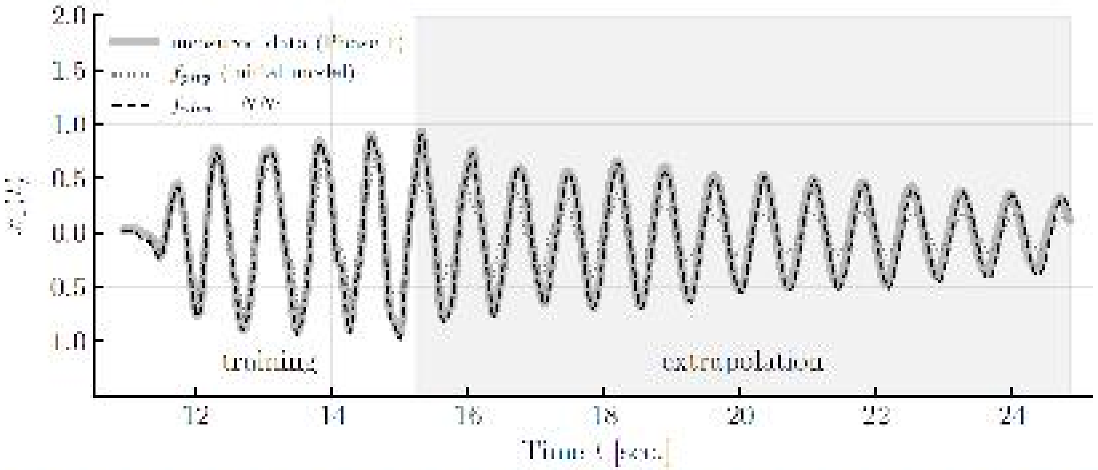
ETH zürich

Physics-informed Neural ODEs

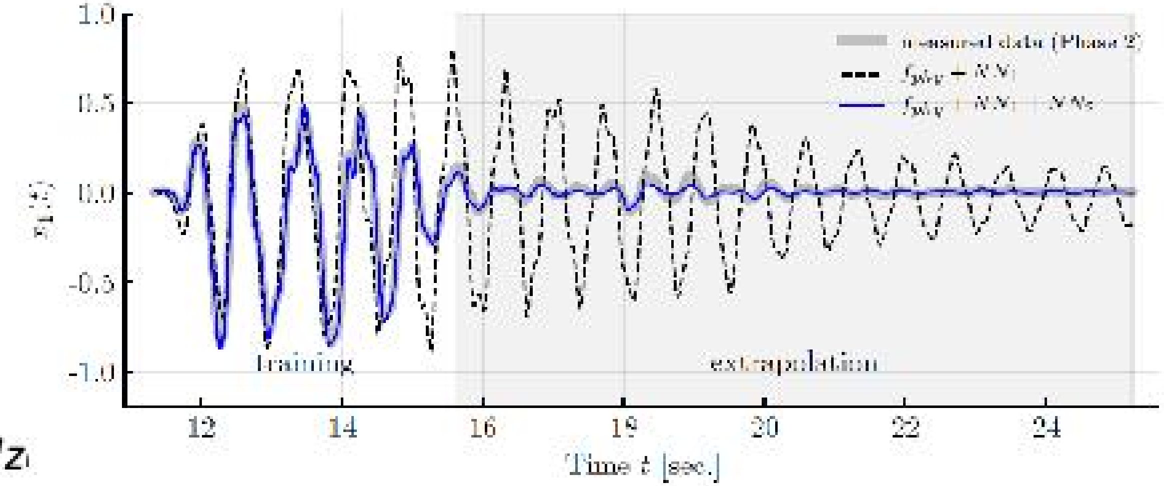


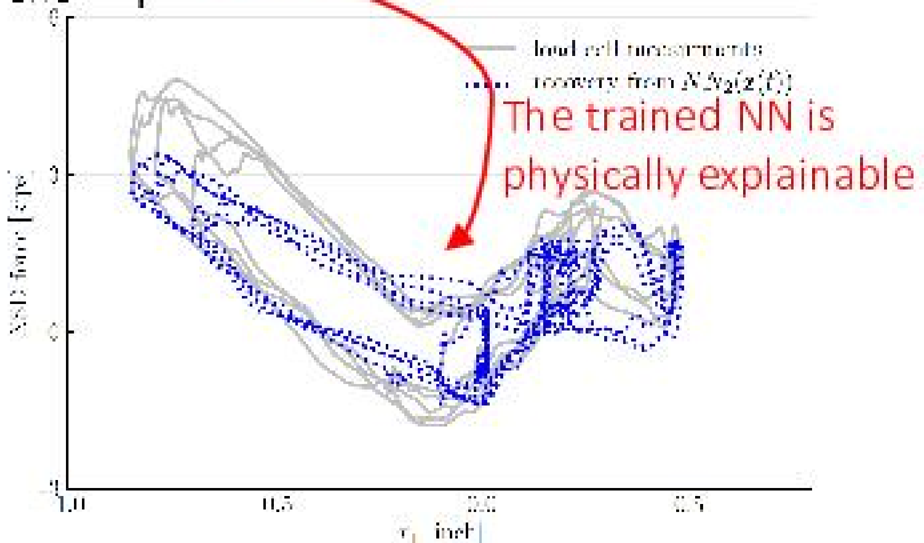
[https://youtu.be/yEHt8Y5PvCw]

$$\frac{d\mathbf{z}(t)}{dt} = f_{phy}(\mathbf{z}(t)) + \begin{bmatrix} \mathbf{0} \\ NN_1(\mathbf{z}(t)) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ -\mathbf{1}\ddot{x}_g(t) \end{bmatrix} \quad \text{(Phase 1)}$$



$$\frac{d\mathbf{z}(t)}{dt} = f_{phy}(\mathbf{z}(t)) + \begin{bmatrix} \mathbf{0}_{3\times 1} \\ NN_1(\mathbf{z}(t)) \end{bmatrix} - \begin{bmatrix} \mathbf{0}_{3\times 1} \\ NN_2(\mathbf{z}(t)) \\ \mathbf{0}_{2\times 1} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times 1} \\ -\mathbf{1}\ddot{x}_g(t) \end{bmatrix} \quad \text{(Phase 2)}$$



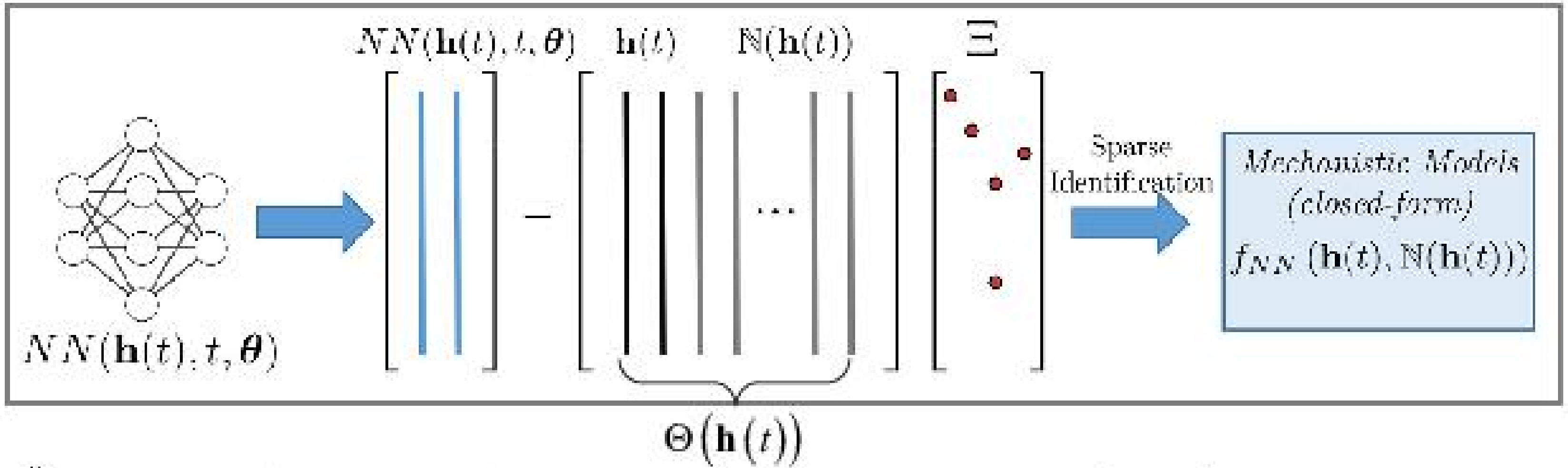


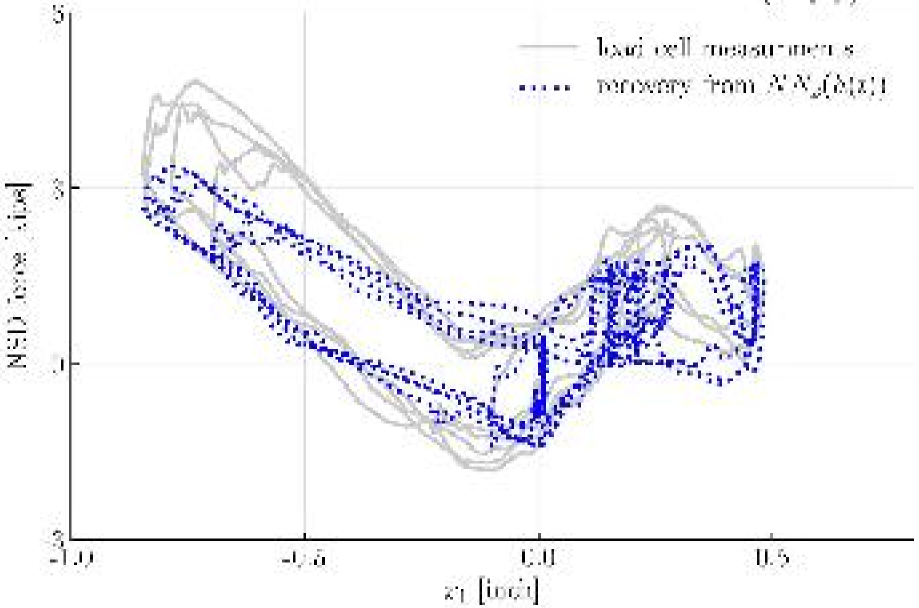
Venturing into discovery

Physics-informed Neural ODEs

Lai, Mylonas, Nagarajaiah, Chatzi, JSV, (2021)

Equation Discovery – Dictionary learning





$$NN_2(h(t)) = 415x_1(t) - 310x_2(t) - 391x_1^2 + 100x_2^2 - 199x_1^3 + 83x_2^3$$

$$NN(\mathbf{h}(t), t, \boldsymbol{\theta}) = \Theta(\mathbf{h}(t))\Xi$$

$$\Theta(\mathbf{h}(t)) = \begin{bmatrix} \mathbf{h}^T(t_1) & \mathbb{N}(\mathbf{h}^T(t_1)) \\ \mathbf{h}^T(t_2) & \mathbb{N}(\mathbf{h}^T(t_2)) \\ \vdots & \vdots \\ \mathbf{h}^T(t_N) & \mathbb{N}(\mathbf{h}^T(t_N)) \end{bmatrix}$$

Material characterization as a discovery problem

Measurable

Strain/Displacement → **Constitutive Law**: Symbolic Expression → Stress distribution in the whole medium

Non-Measurable

Requires expensive testing to fit experimentally

Deep Learning ↓

Discover a model that predicts symbolic expressions that describe how different materials behave from displacement data

Discovery via formal grammars

Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

Representing mathematical expressions as trees

Example: Neo-Hookean Material

$$W = 0.5\left(\overline{I}_1 - 3\right) + 1.5\left(J - 1\right)^2$$

Symbolic regression first translates the mathematical expressions to structured representations (S-expressions) and then learns to predict expressions from data

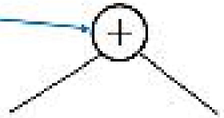
Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

Representing mathematical expressions as trees

ETH AI CENTER

Example: Neo-Hookean

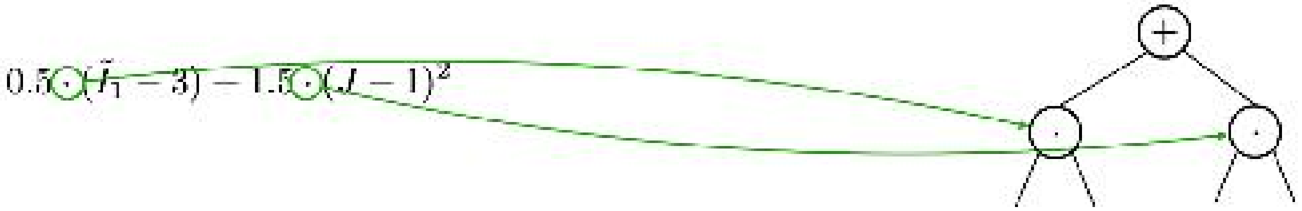
$$0.5 \cdot (\tilde{I}_1 - 3) + 1.5 \cdot (J - 1)^2$$



Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

Representing mathematical expressions as trees

Example: Neo-Hookean

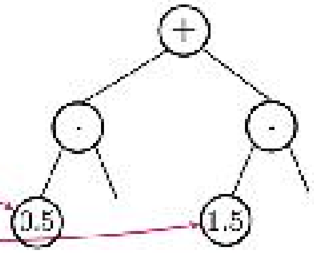


Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

Representing mathematical expressions as trees

Example: Neo-Hookean

$$0.5 \cdot (\bar{I}_1 - 3) + 1.5 \cdot (J - 1)^2$$



Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

Representing mathematical expressions as trees

Example: Neo-Hookean

Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

Representing mathematical expressions as trees

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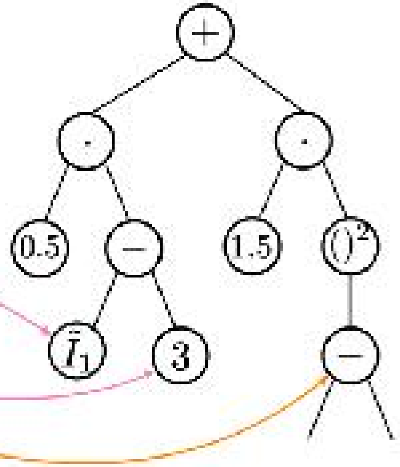
Example: Neo-Hookean

$$0.5 \cdot (\bar{I}_1 - 3) + 1.5 \cdot (J - 1)^2$$

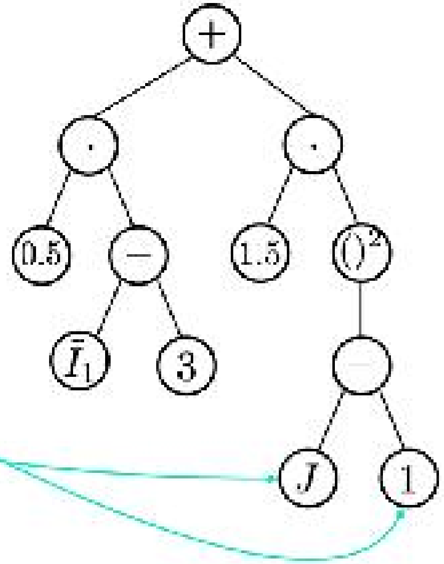


Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

Representing mathematical expressions as trees

Example: Neo-Hookean

$$0.5 \cdot (\bar{I}_1 - 3) + 1.5 \cdot (J - 1)^2$$



Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

Formal Grammars

Definitions

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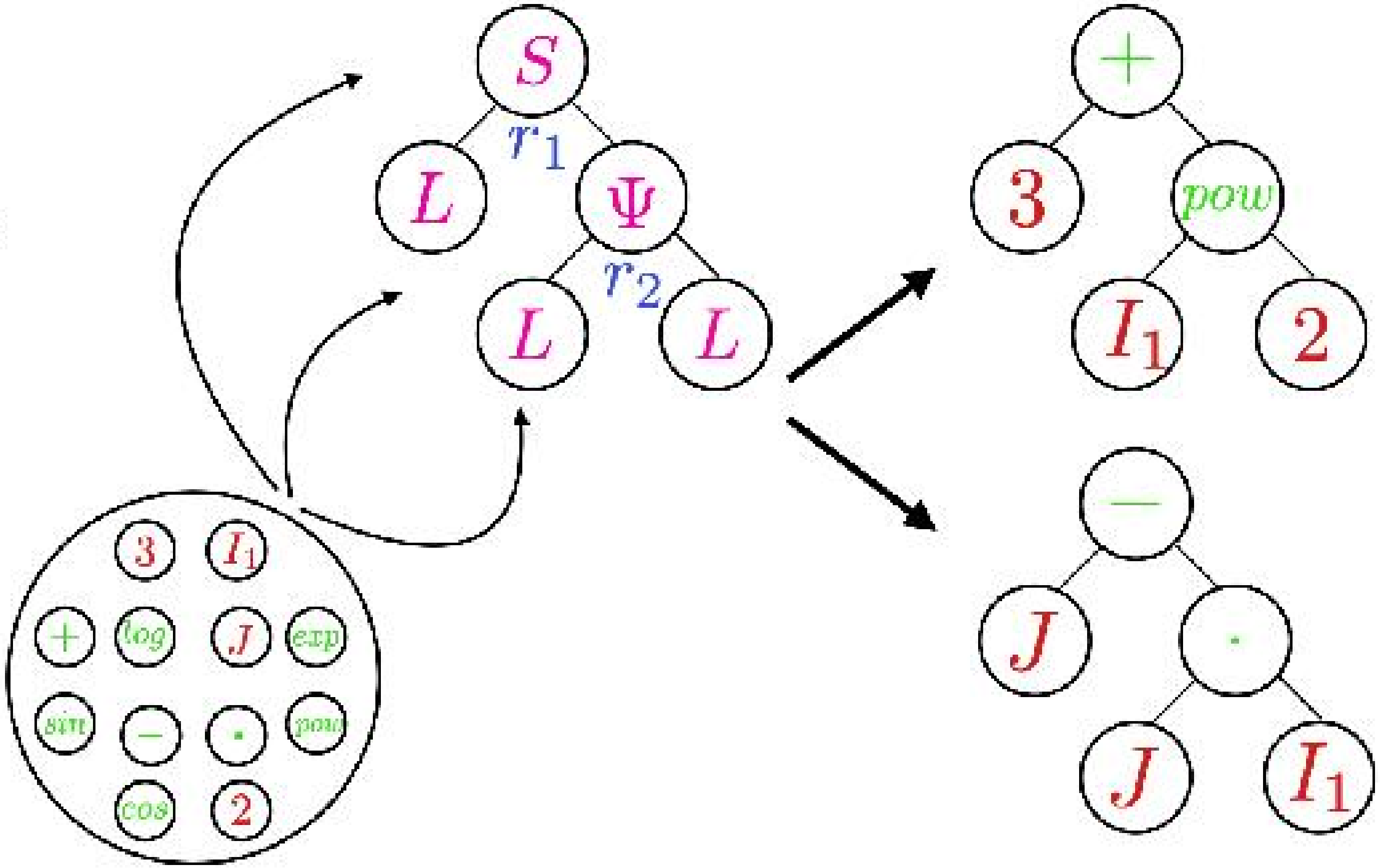
Formal Grammars are defined using:

- S: Starting Symbol
- Non-terminal symbols
- Terminal symbols
- Production rules



The non–terminals Ψ and L are used for recursive substitution of operations and literals (i.e. constants and variables).

Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

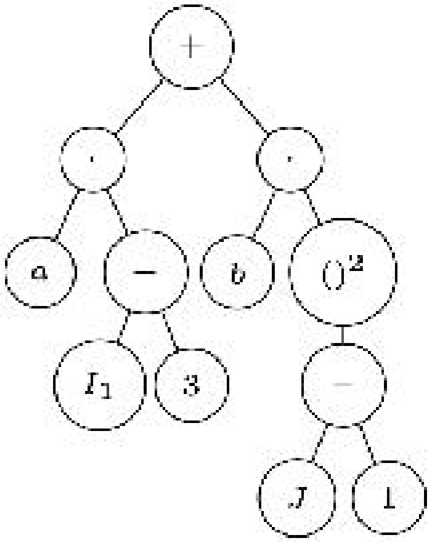
Formal Grammars

Syntax

Grammar Definition

$$
\begin{aligned}
S &= \{\, C \,\},\\
\Phi &= \{\, C, \Psi, L \},\\
\Sigma &= \{\, +, -, \cdot, (\cdot)^2, I_1, J, a, 1, b, 3 \,\},\\
R &= \{\, C \rightarrow \Psi + \Psi, (1)\\
&\qquad \Psi \rightarrow L \cdot \Psi, (2)\\
&\qquad \Psi \rightarrow (L - L), (3)\\
&\qquad \Psi \rightarrow (\Psi)^2, (4)\\
&\qquad \Psi \rightarrow L, (5)\\
&\qquad L \rightarrow a, (6)\\
&\qquad L \rightarrow 1, (7)\\
&\qquad L \rightarrow b, (8)\\
&\qquad L \rightarrow 3, (9)\\
&\qquad L \rightarrow I_1, (10)\\
&\qquad L \rightarrow J (11) \,\}.
\end{aligned}
$$

Example: Neo-Hookean



$a \cdot (I_1 - 3) + b \cdot (J - 1)^2$

Derivation

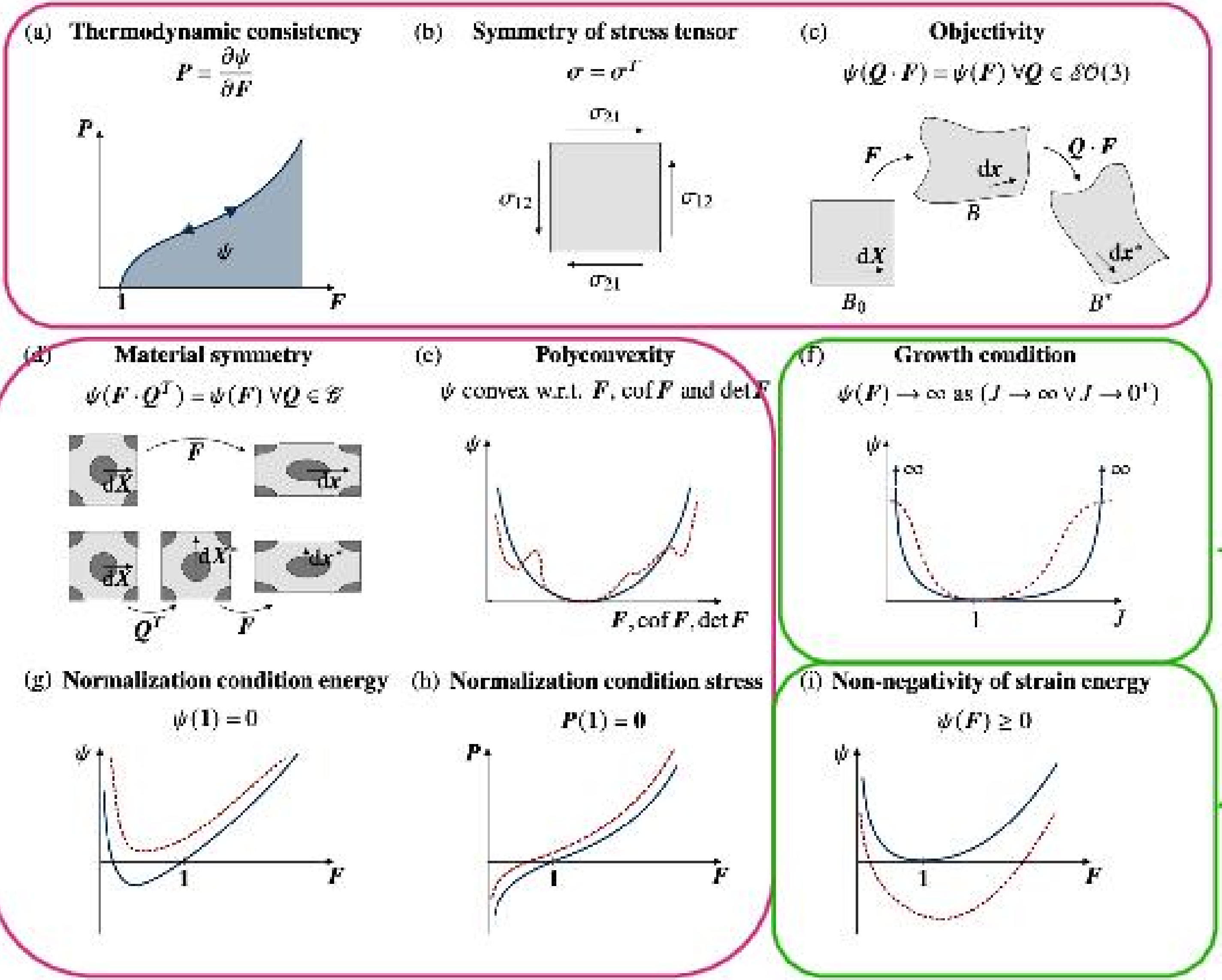
$$
\begin{aligned}
C &\xrightarrow{(1)} \Psi + \Psi,\\
&\xrightarrow{(2)} L \cdot \Psi + L \cdot \Psi,\\
&\xrightarrow{(3)} L \cdot (L - L) + L \cdot (L - L),\\
&\xrightarrow{(6)} a \cdot (L - L) + L \cdot (L - L),\\
&\xrightarrow{(10)} a \cdot (I_1 - L) + L \cdot (L - L),\\
&\xrightarrow{(9)} a \cdot (I_1 - 3) + L \cdot (L - L),\\
&\xrightarrow{(8)} a \cdot (I_1 - 3) + b \cdot (L - L),\\
&\xrightarrow{(4)} a \cdot (I_1 - 3) + b \cdot (L - L)^2,\\
&\xrightarrow{(11)} a \cdot (I_1 - 3) + b \cdot (J - L)^2,\\
&\xrightarrow{(7)} a \cdot (I_1 - 3) + b \cdot (J - 1)^2.
\end{aligned}
$$

Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

Formal Grammars

Semantics

Intrinsic



Perform symbolic calculations for the derived expressions to check if the extrinsic constraints hold

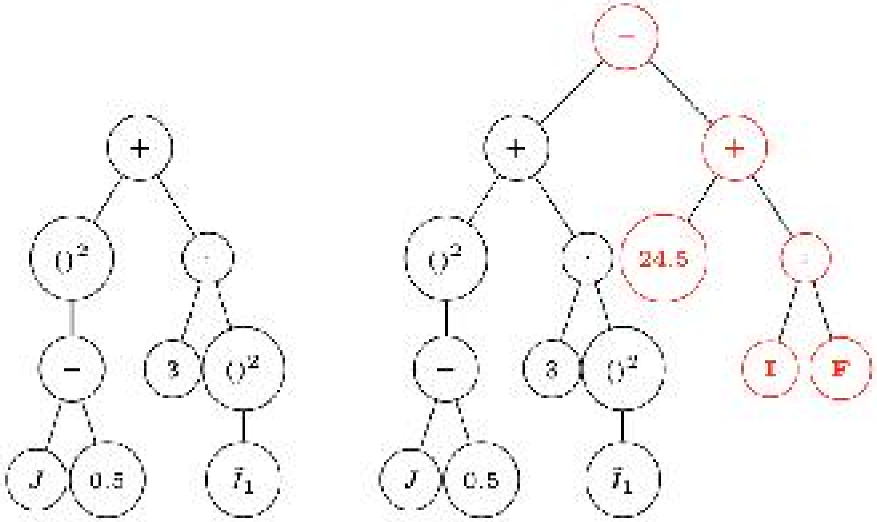
Extrinsic

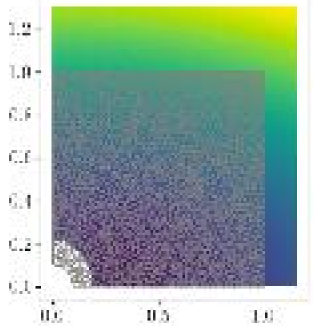
Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

The Language of Hyperelastic Materials

Define a language that derives syntactically valid mathematical expressions whose semantics satisfy physics and more specifically the constraints of constitutive laws.

$$
\begin{aligned}
&S = \{\, C \,\},\\
&\Sigma = \{+, -, /, \cdot, (\cdot)^2, (\cdot)^3, -\log, \exp, \tilde{I}_1, \tilde{I}_2, J, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \ \frac{\partial()}{\partial \mathbf{F}}|_{\mathbf{F}=\mathbf{I}} : (\mathbf{F}-\mathbf{I}), \, |_{\mathbf{F}=\mathbf{I}}\},\\
&R = \{C \rightarrow \Psi + \Psi^0 + P^c, (\mathbf{1})\\
&\qquad \Psi \rightarrow \Psi^1 + \Psi^2 + \Psi^3 + \Psi^4, (\mathbf{2})\\
&\qquad \Psi^0 \rightarrow -\Psi|_{\mathbf{F}=\mathbf{I}}, (\mathbf{3})\\
&\qquad \Psi^1 \rightarrow L\,(\mathbf{4}) \mid U(\mathbf{5}) \mid Y(\mathbf{6}) \mid T(\mathbf{7}),\\
&\qquad \Psi^2 \rightarrow L(\mathbf{8}) \mid U(\mathbf{9}) \mid Y(\mathbf{10}) \mid T(\mathbf{11}),\\
&\qquad \Psi^3 \rightarrow L(\mathbf{12}) \mid U(\mathbf{13}) \mid Y(\mathbf{14}) \mid T(\mathbf{15}),\\
&\qquad \Psi^4 \rightarrow L(\mathbf{16}) \mid U(\mathbf{17}) \mid Y(\mathbf{18}) \mid T(\mathbf{19}),\\
&\qquad P^c \rightarrow -\frac{\partial \Psi}{\partial \mathbf{F}}|_{\mathbf{F}=\mathbf{I}} : (\mathbf{F}-\mathbf{I}), (\mathbf{20})\\
&\qquad T \rightarrow (Y)^2(\mathbf{21}) \mid (Y)^3(\mathbf{22}) \mid \exp(Y)(\mathbf{23}) \mid -\log(Y)(\mathbf{24}),\\
&\qquad Y \rightarrow (V+O)\,(\mathbf{25}) \mid (V-O)(\mathbf{26}) \mid (V/O)(\mathbf{27}) \mid (V \cdot O)(\mathbf{28}) \mid (V)^2(\mathbf{29}) \mid (V)^3(\mathbf{30})\\
&\qquad U \rightarrow -\log(L)(\mathbf{31}) \mid \exp(L)(\mathbf{32})\\
&\qquad L \rightarrow V(\mathbf{33}) \mid O(\mathbf{34}),\\
&\qquad V \rightarrow \tilde{I}_1(\mathbf{35}) \mid \tilde{I}_2^{3/2}(\mathbf{36}) \mid J(\mathbf{37}),\\
&\qquad O \rightarrow P.P, (\mathbf{38})\\
&\qquad P \rightarrow D(\mathbf{39}) \mid DP(\mathbf{40}),\\
&\qquad D \rightarrow 0(\mathbf{41}) \mid 1(\mathbf{42}) \mid 2(\mathbf{43}) \mid 3(\mathbf{44}) \mid 4(\mathbf{45}) \mid 5(\mathbf{46}) \mid 6(\mathbf{47}) \mid 7(\mathbf{48}) \mid 8(\mathbf{49}) \mid 9(\mathbf{50})\ \},\\
&\Phi = \{\, C, \Psi, \Psi^0, \Psi^1, \Psi^2, \Psi^3, \Psi^4, P^c, T, L, U, Y, O, D, P \,\}
\end{aligned}
$$





Reject expressions that do not satisfy extrinsic constraints.

Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

Constructing a virtual library of Hyperelastic constitutive laws

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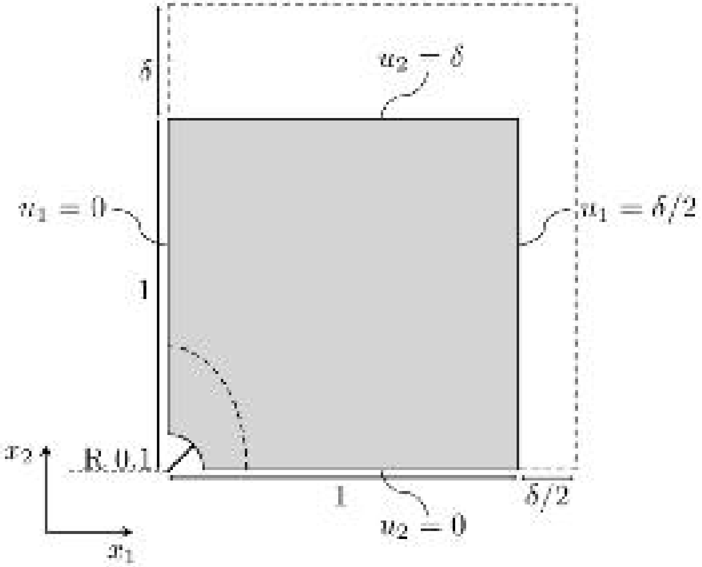
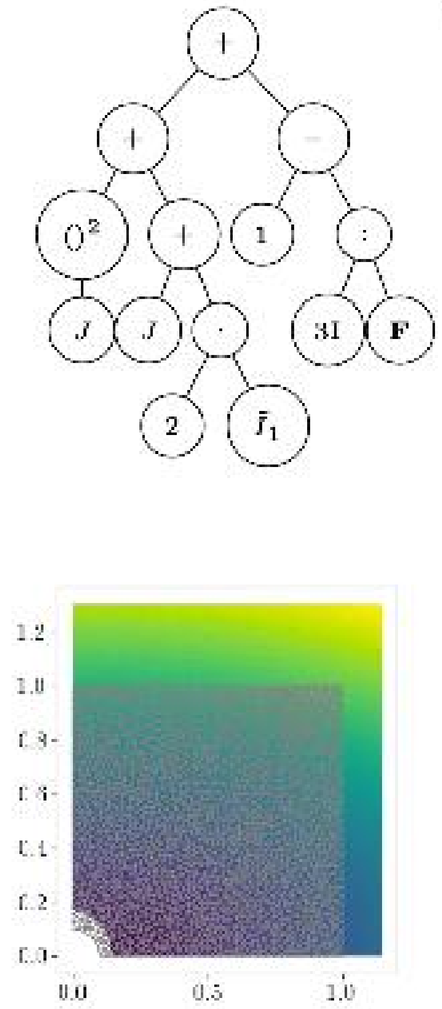
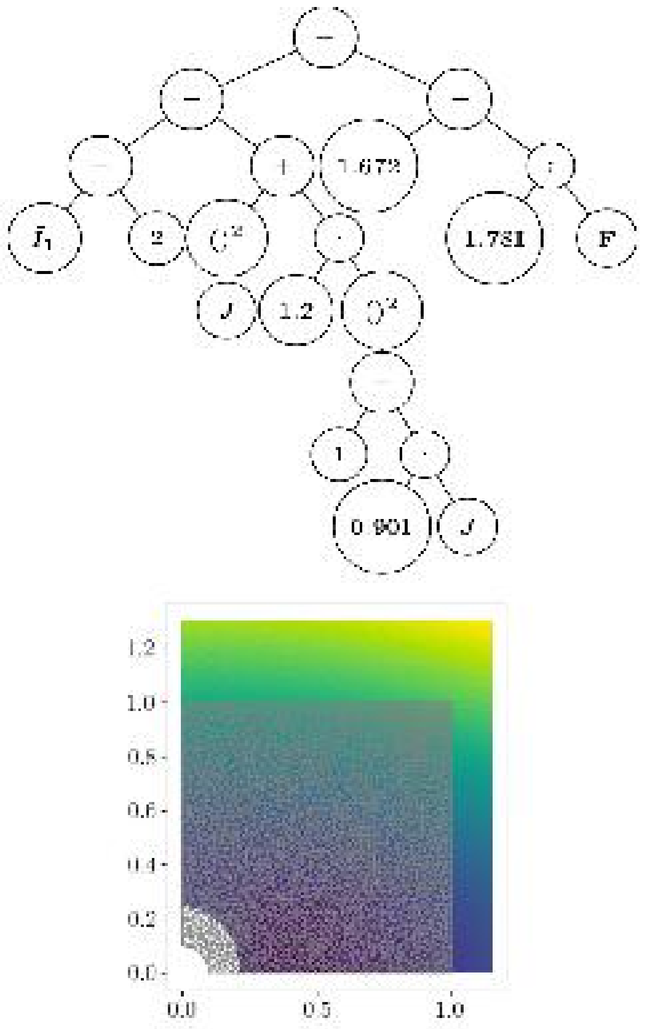


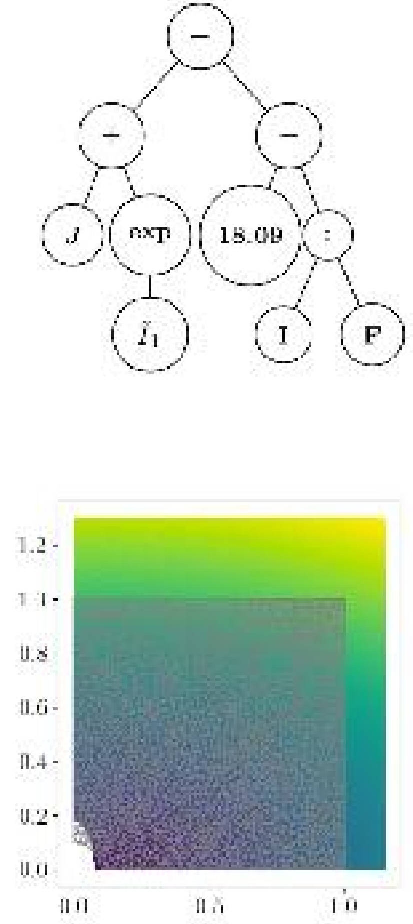
Solve the boundary values problem for different constitutive law models



$2 \cdot \tilde{I}_1 + J + J^2 + 1.0 - 3\mathbf{I} : \mathbf{F}$



$2 \cdot \tilde{I}_1 + J^2 + 1.232 \cdot (1.0 - 0.901 \cdot J)^2 - 1.672 - 1.78\mathbf{I} : \mathbf{F}$



$J + exp(\tilde{I}_1) - 18.09 - \mathbf{I} : \mathbf{F}$

Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

Data-driven discovery of constitutive laws: Overview

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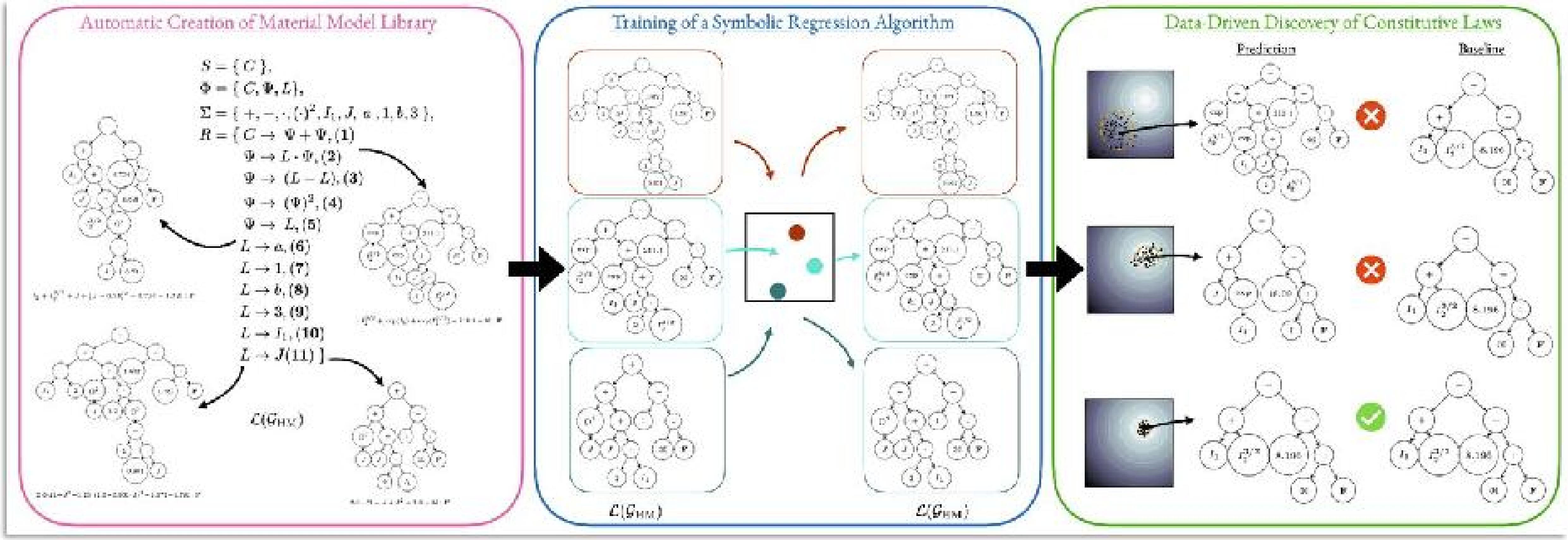

Kissas, Mishra, Chatzi, DeLorenzis 2024, https://arxiv.org/pdf/2402.04263.pdf

Training of a Symbolic Regression Algorithm

Recursive Tree Variational Autoencoders

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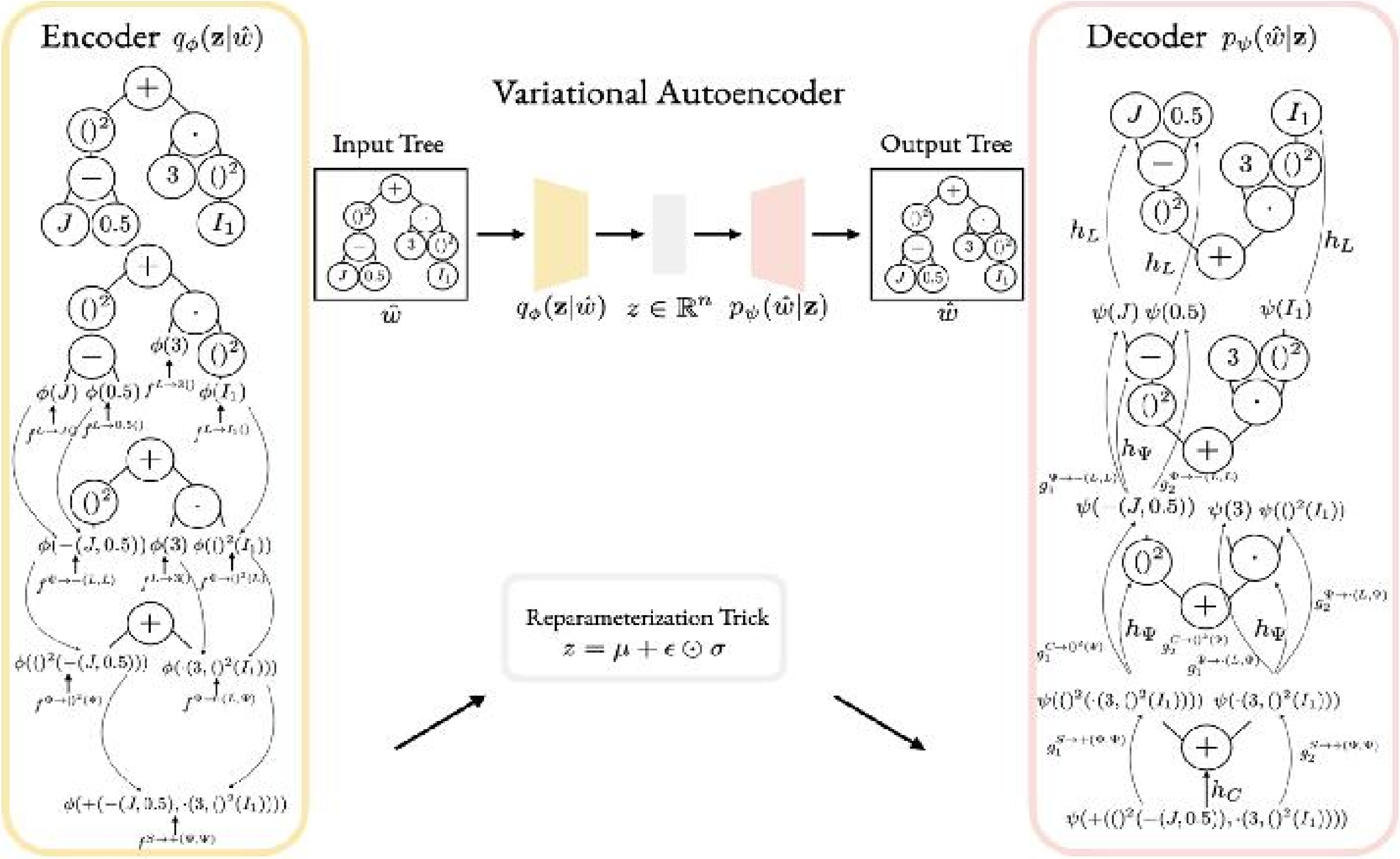

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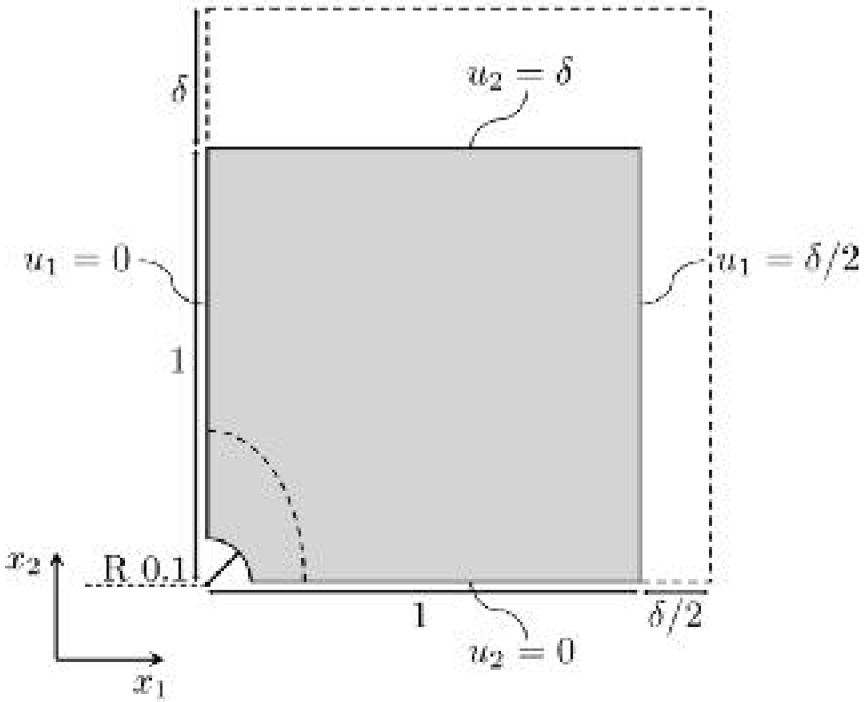
Application Data-driven discovery of constitutive laws
DIC Data on Plate under Biaxial Tension



Generate displacement measurements for different models and $\delta = 0.3$

Neo-Hookean

$$W = 0.5 \cdot (\tilde{I}_1 - 3) + 1.5 \cdot (J - 1)^2;$$

Ishihara

$$W = 0.5 \cdot (\tilde{I}_1 - 3) + (\tilde{I}_2 - 3) + (\tilde{I}_1 - 3)^2 + 1.5 \cdot (J - 1)^2;$$

Haines-Wilson

$$W = 0.5 \cdot (\tilde{I}_1 - 3) + (\tilde{I}_2 - 3) + 0.7 \cdot (\tilde{I}_1 - 3) \cdot (\tilde{I}_2 - 3) + 0.2 \cdot (\tilde{I}_1 - 3)^3 + 1.5 \cdot (J - 1)^2;$$

Gent-Thomas

$$W = 0.5 \cdot (\tilde{I}_1 - 3) - \log(\tilde{I}_2/3) + 1.5 \cdot (J - 1)^2.$$

None of these models are contained in the library

Data-driven discovery of constitutive laws

Results

Model	Discovered Expression	Relative $\mathcal{L}^2$ Error
NH, $\sigma^* = 0$	$\tilde{W} = 0.5 \cdot (I_1 - 3) + 1.5 \cdot (J - 1)^2$	0
NH, $\sigma^* = 10^{-4}$	$\tilde{W} = 0.5 \cdot (I_1 - 3) + 1.5 \cdot (J - 1)^2$	0
NH, $\sigma^* = 10^{-3}$	$\tilde{W} = 0.49 \cdot (I_1 - 3) + 1.5 \cdot (J - 1)^2$	0.012

Relative error between the predicted strain energy and the baseline for different models

Model	Discovered Expression	Relative $\mathcal{L}^2$ Error
IS, $\sigma^* = 0$	$\tilde{W} = \tilde{I}_1 - 3 + 0.5 \cdot (\tilde{I}_2 - 3) + (\tilde{I}_1 - 3)^2 + 1.5 \cdot (J - 1)^2$	0.011
IS, $\sigma^* = 10^{-4}$	$\tilde{W} = \tilde{I}_1 - 3 + 0.5 \cdot (\tilde{I}_2 - 3) + (\tilde{I}_1 - 3)^2 + 1.5 \cdot (J - 1)^2$	0.011
IS, $\sigma^* = 10^{-3}$	$W = (\tilde{I}_1 - 3) \cdot (\tilde{I}_2 - 3) + 1.5 \cdot (\tilde{I}_1 - 3) + 1.5 \cdot (J - 1)^2$	0.014
HW, $\sigma^* = 0$	$\tilde{W} = 1.5 \cdot (\tilde{I}_2 - 3) + (\tilde{I}_1 - 3)^2 + 1.5 \cdot (-1 + J)^2$	0.01
HW, $\sigma^* = 10^{-4}$	$\tilde{W} = (\tilde{I}_1 - 3) \cdot (\tilde{I}_2 - 3) + 1.5 \cdot (\tilde{I}_2 - 3) + 1.5 \cdot (-1 + J)^2$	0.007
HW, $\sigma^* = 10^{-3}$	$\tilde{W} = (\tilde{I}_1 - 3) \cdot (\tilde{I}_2 - 3) + 1.5 \cdot (\tilde{I}_2 - 3) + 1.5 \cdot (-1 + J)^2$	0.007
GT, $\sigma^* = 0$	$\tilde{W} = 0.5 \cdot (\tilde{I}_1 - 3) + 0.3 \cdot (\tilde{I}_2 - 3) + 1.5 \cdot (J - 1)^2$	0.026
GT, $\sigma^* = 10^{-4}$	$W = 0.5 \cdot (\tilde{I}_1 - 3) + 0.3 \cdot (\tilde{I}_2 - 3) + 1.5 \cdot (J - 1)^2$	0.026
GT, $\sigma^* = 10^{-3}$	$\tilde{W} = 0.5 \cdot (\tilde{I}_1 - 3) + 0.2 \cdot (\tilde{I}_2 - 3) + 1.5 \cdot (J - 1)^2$	0.026

Parameterize the deformation gradient for different types of loading.

$$\mathbf{F}_{UT} = \begin{bmatrix} 1+\gamma & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{F}_{UC} = \begin{bmatrix} 1/(1+\gamma) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{F}_{SS} = \begin{bmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

$$\mathbf{F}_{BT} = \begin{bmatrix} 1+\gamma & 0 & 0 \\ 0 & 1+\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{F}_{BC} = \begin{bmatrix} 1/(1-\gamma) & 0 & 0 \\ 0 & 1/(1-\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}; \mathbf{F}_{PS} = \begin{bmatrix} 1+\gamma & 0 & 0 \\ 0 & 1/(1+\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Data-driven discovery of constitutive laws

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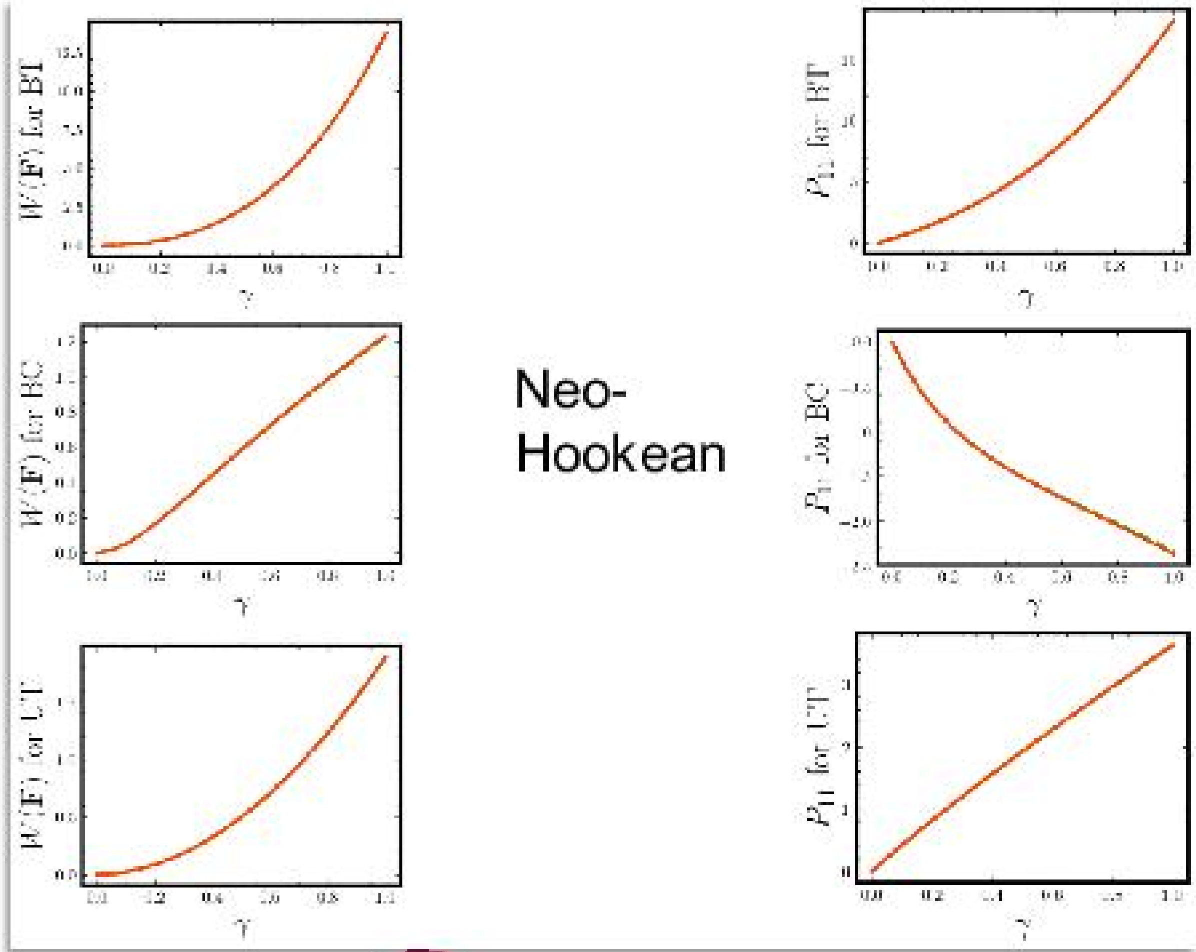
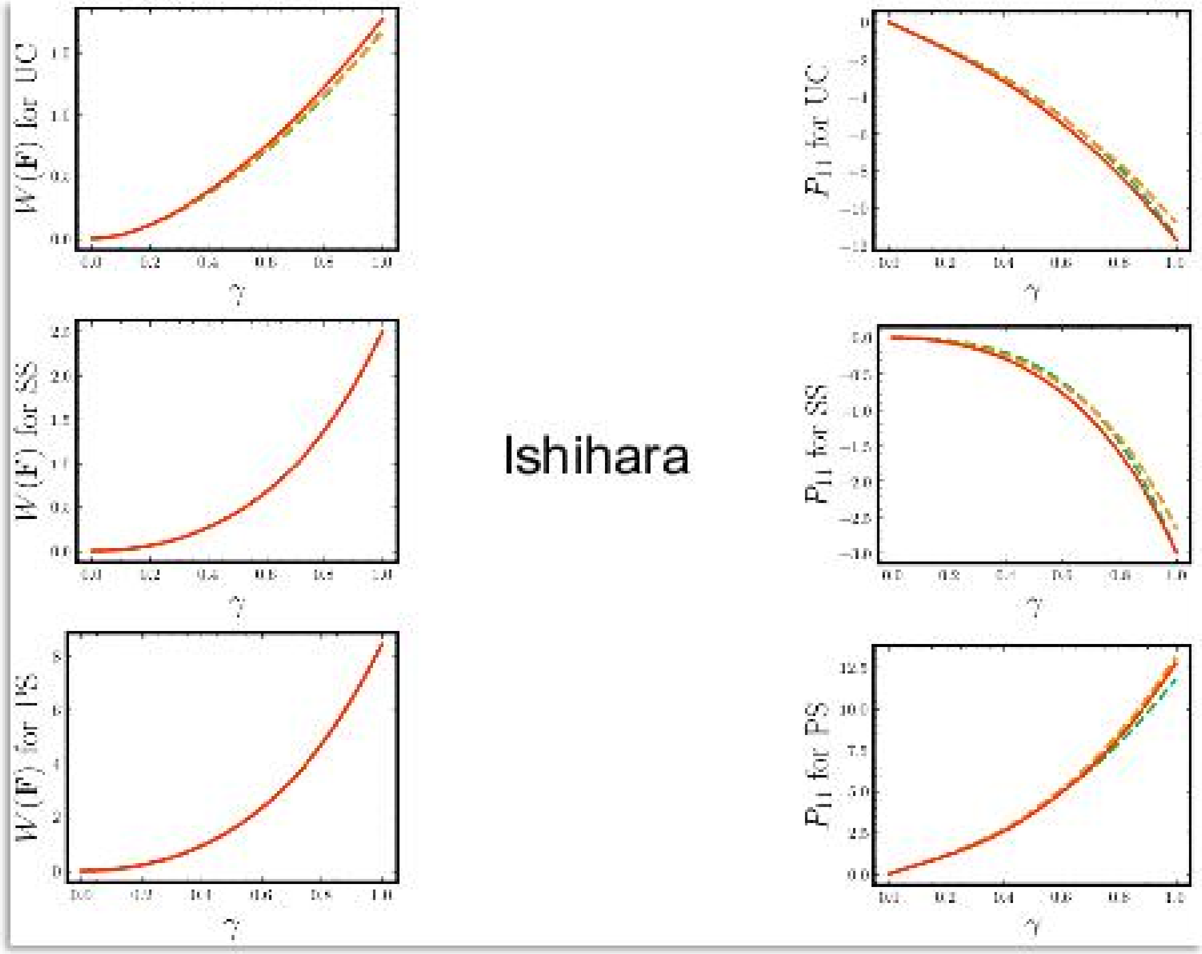
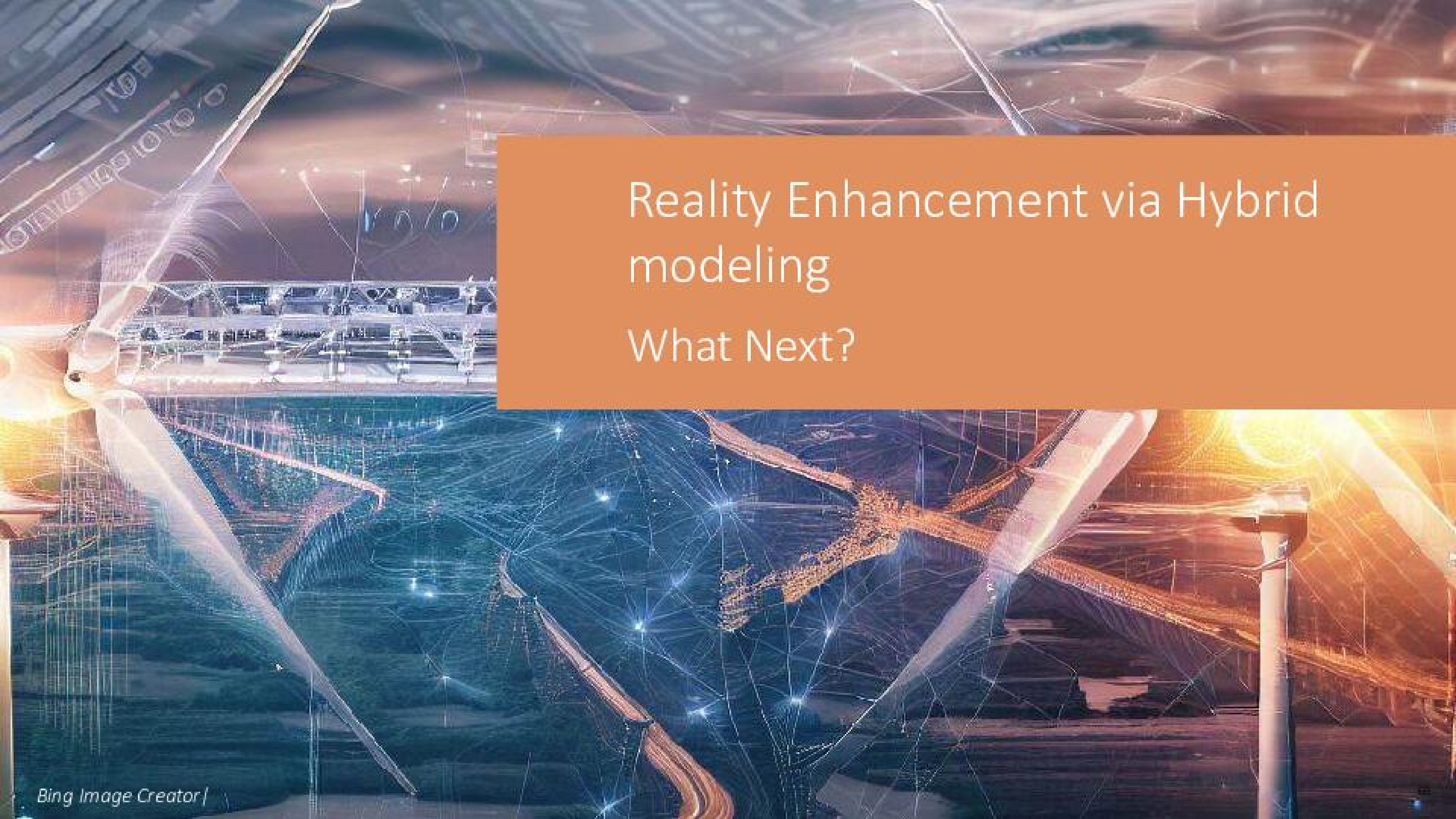
Results





Reality Enhancement via Hybrid modeling

What Next?

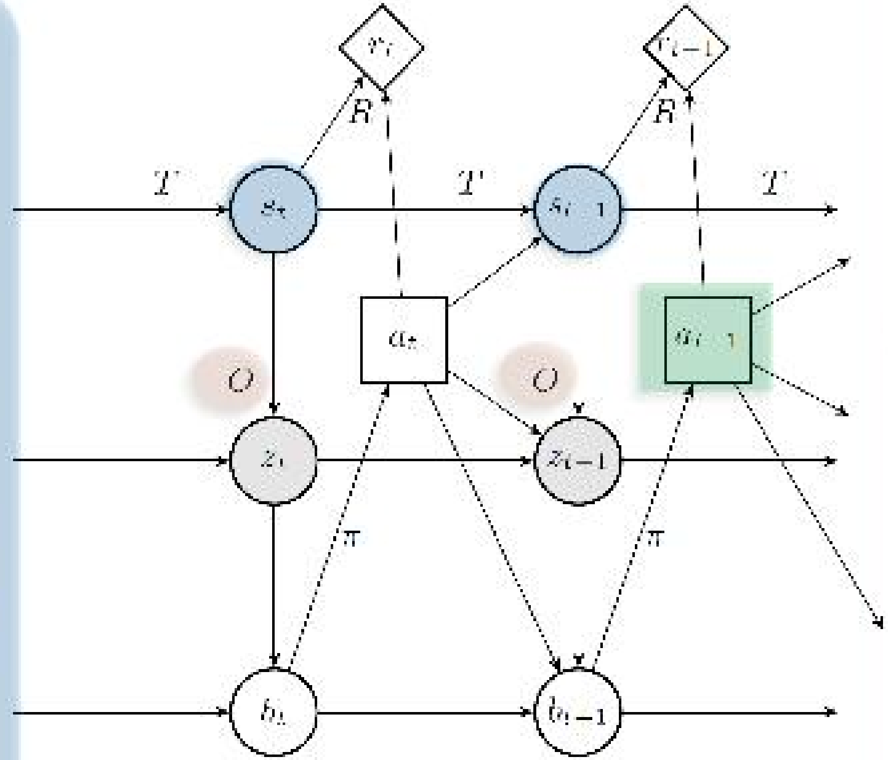


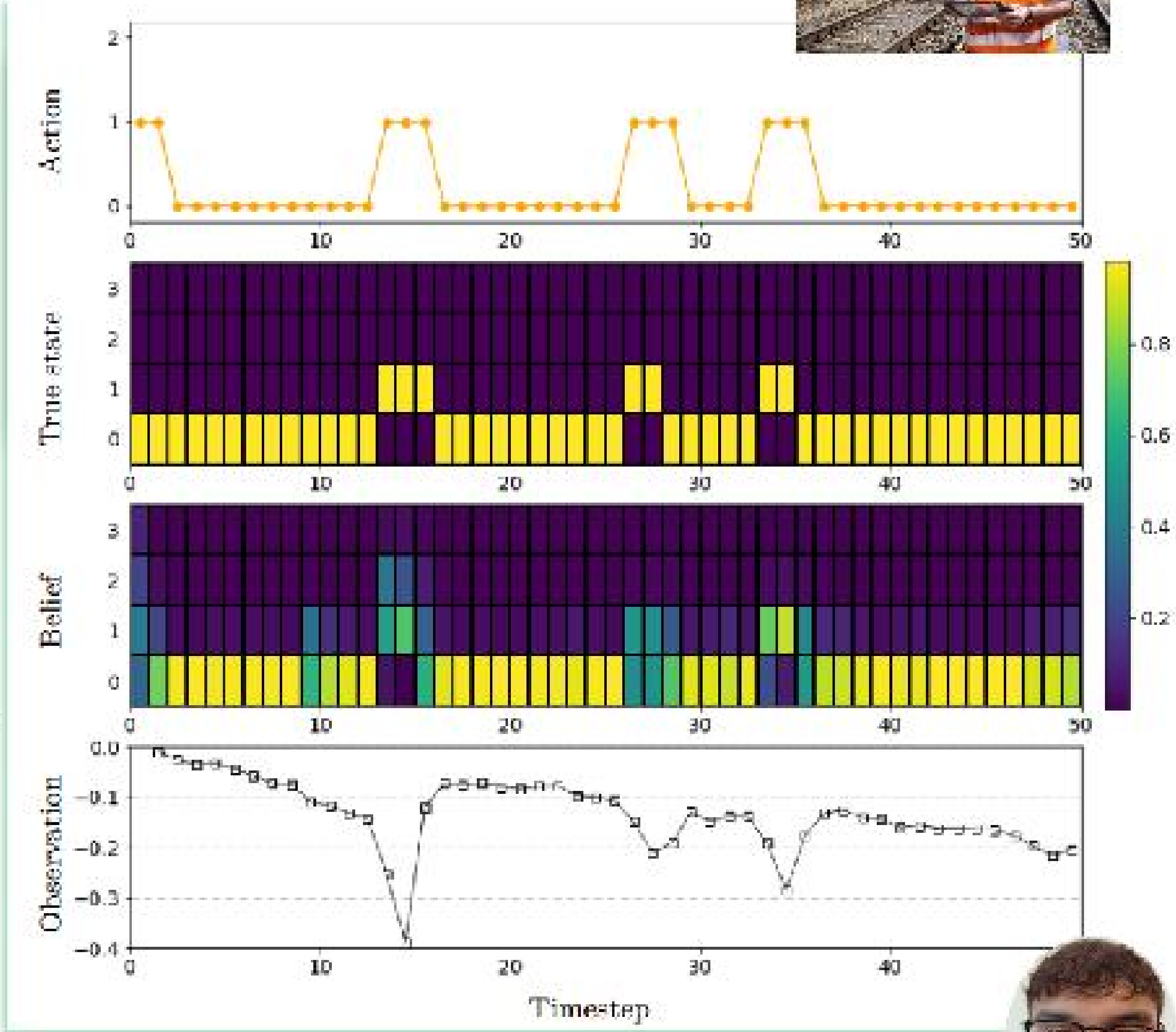
Bing Image Creator |

Spotlight: Mobile Sensing for Transport Infrastructure

Decision Support for Railway Assets

Optimizing Operation & Maintenance of critical assets





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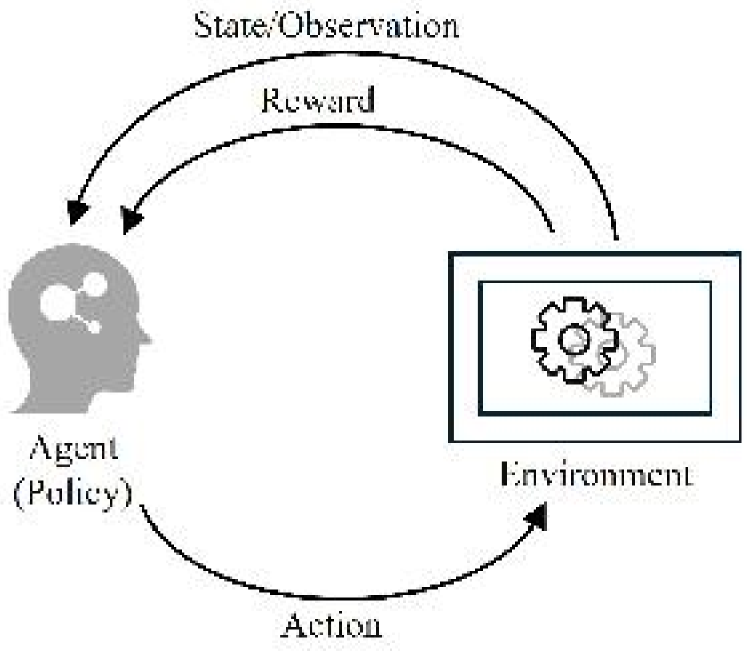
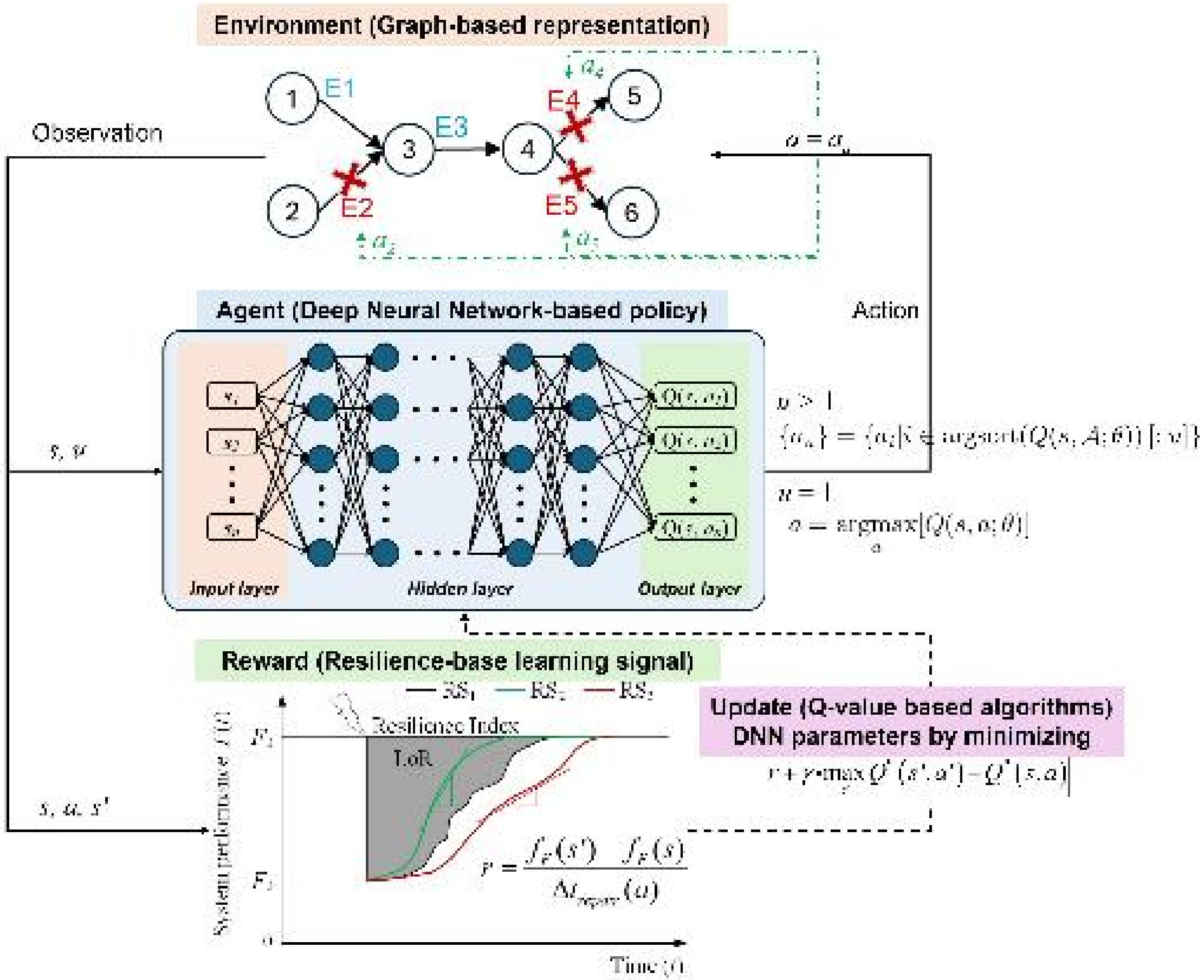
SBB CFF FFS

Arcieri, Hoelzl, Papakonstantinou, Straub, Chatzi, et al., Bridging POMDPs and Bayesian decision making for robust maintenance planning under model uncertainty: An application to railway systems, [LINK]

Optimizing Post-Disaster Recovery with DRL for Resilient Infrastructure

Harnessing the power of graph representations we can support decision making at network level

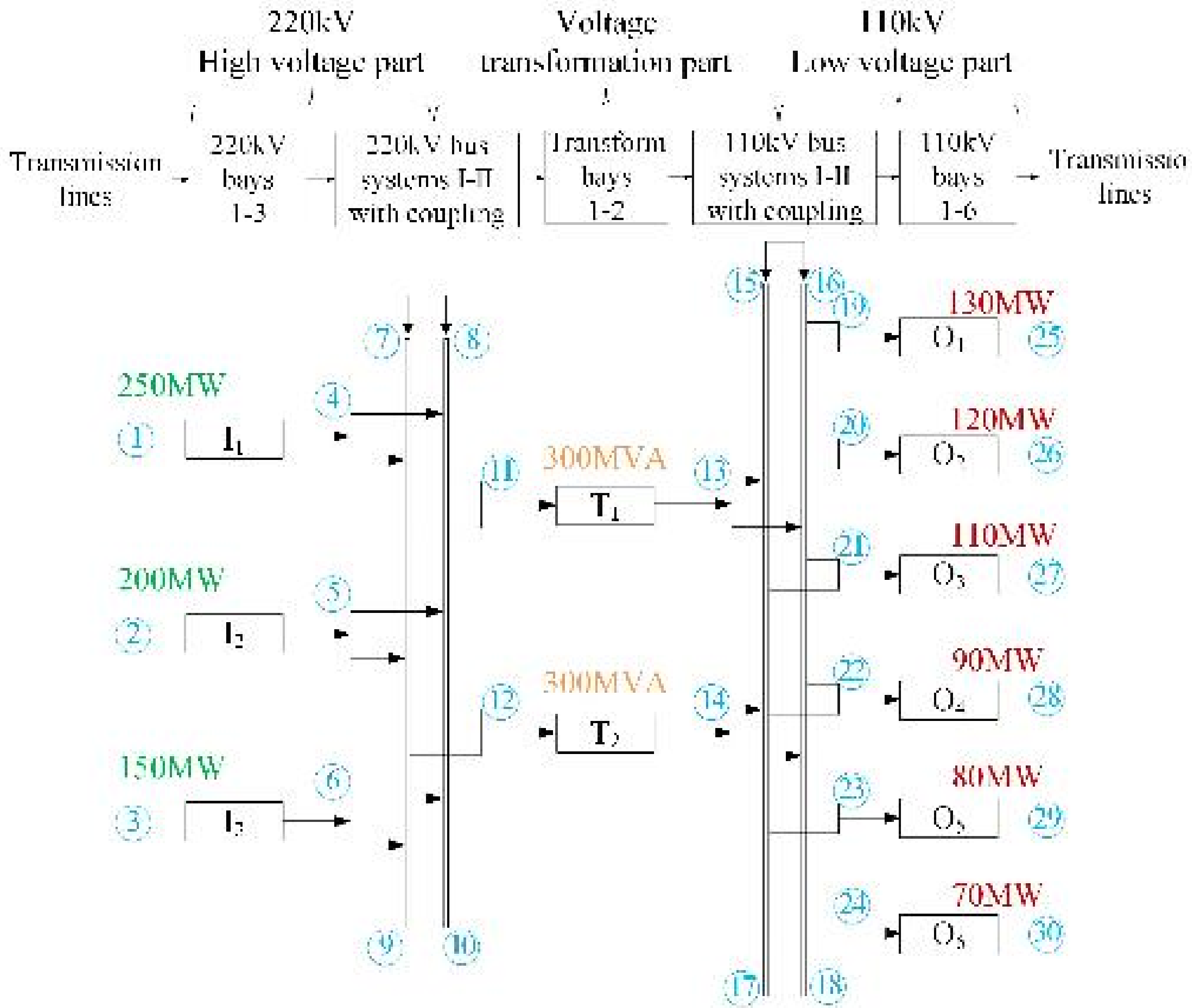




Application: Electric Substation Recovery

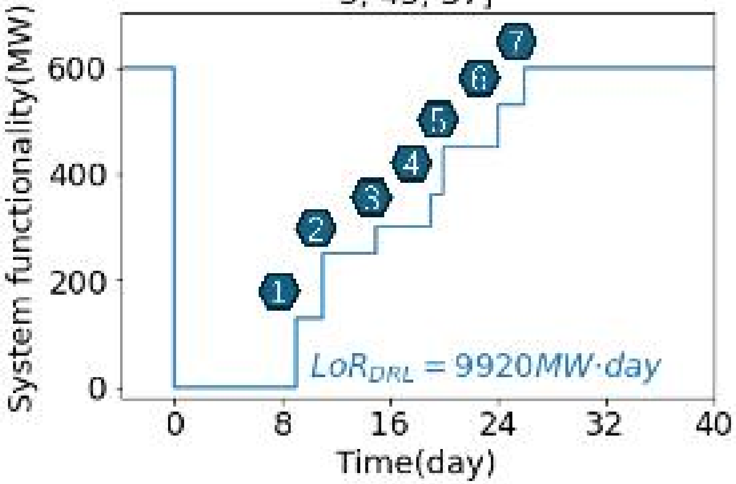
Aim

To optimize the post-disaster recovery of a 220 kV electrical substation system following an earthquake, aiming to restore functionality and enhance resilience efficiently.

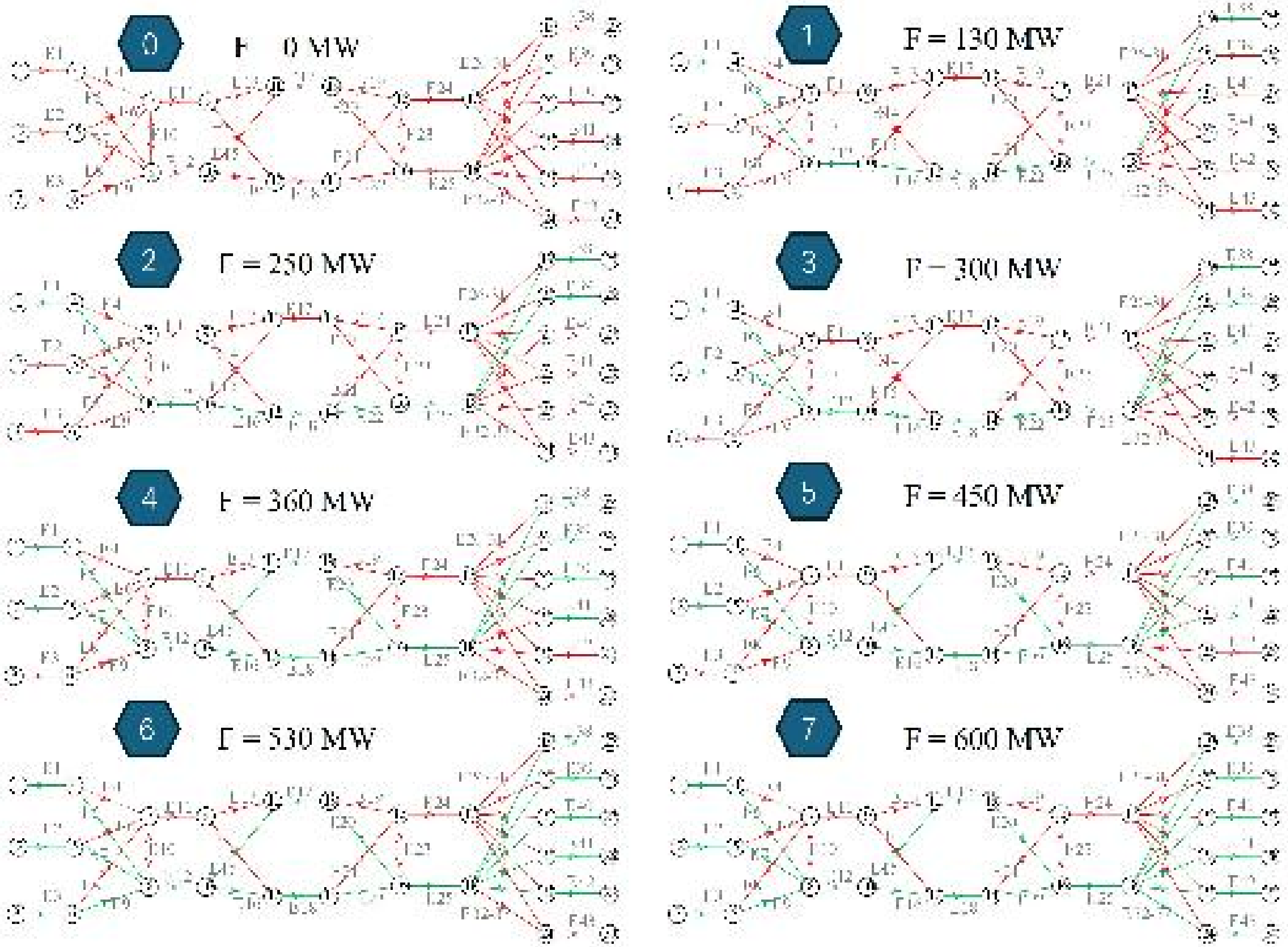


Application: Electric Substation Recovery

Resilience curve with DRL-based sequence: [12, 25, 1, 16, 5, 38, 32, 22, 18, 33, 39, 7, 34, 40, 2, 17, 20, 41, 15, 35, 9, 42, 36, 3, 43, 37]

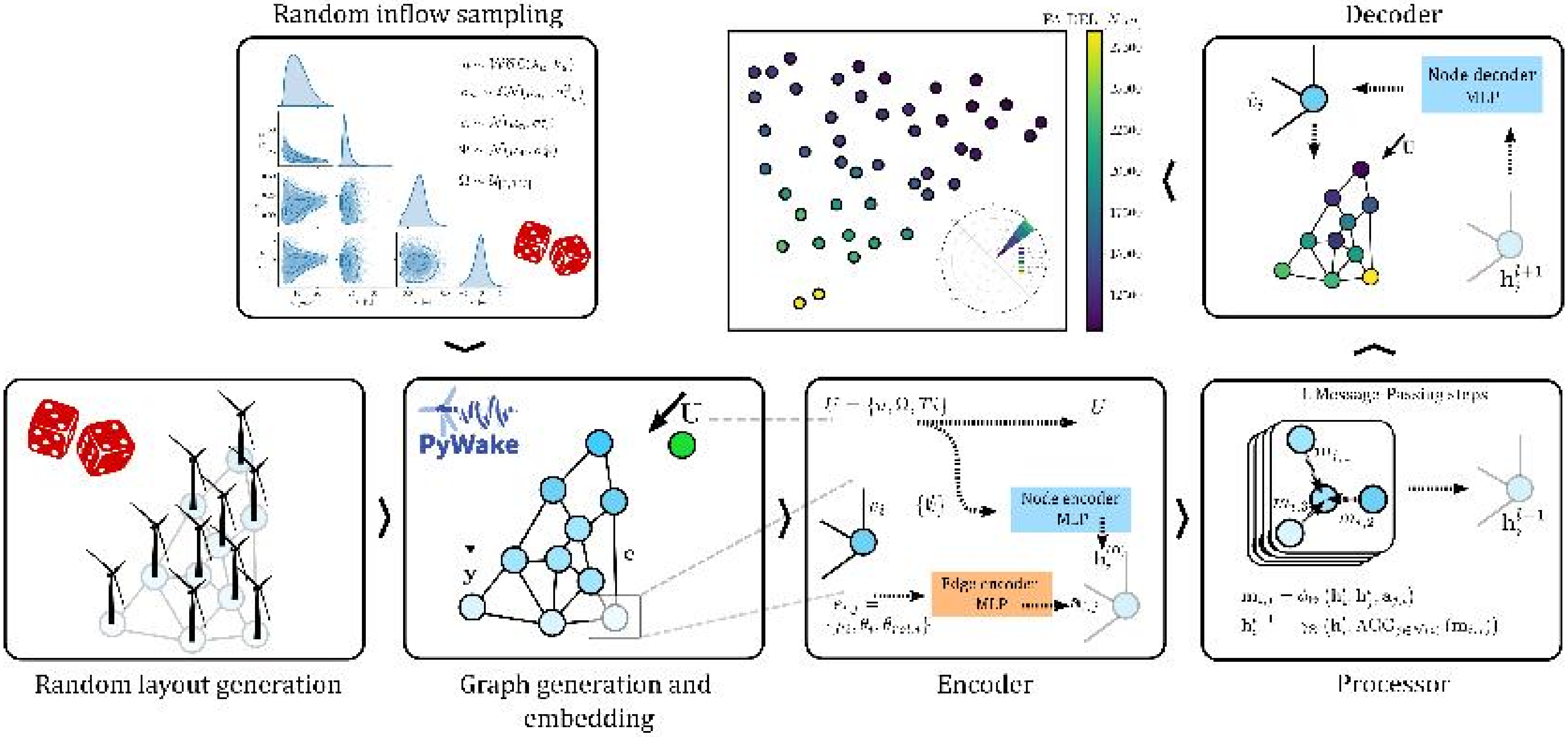


DS T: Damage 43 Repair 26



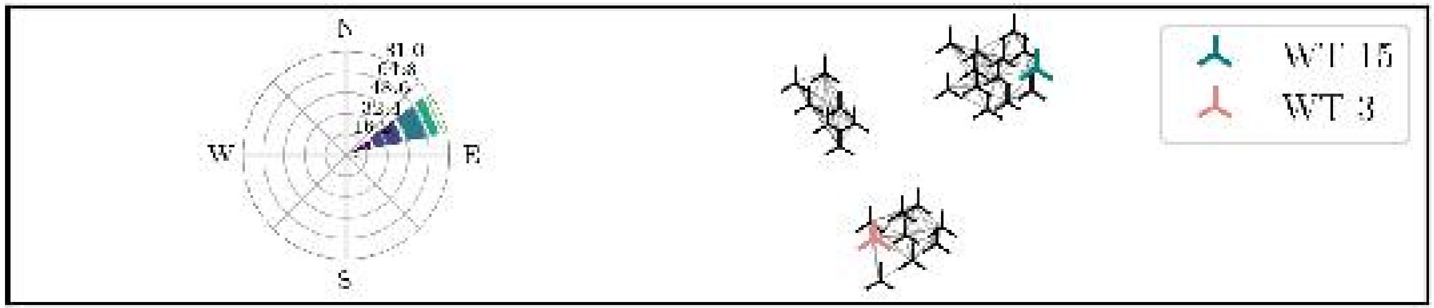
GNNs for Transfer across Ecosystems

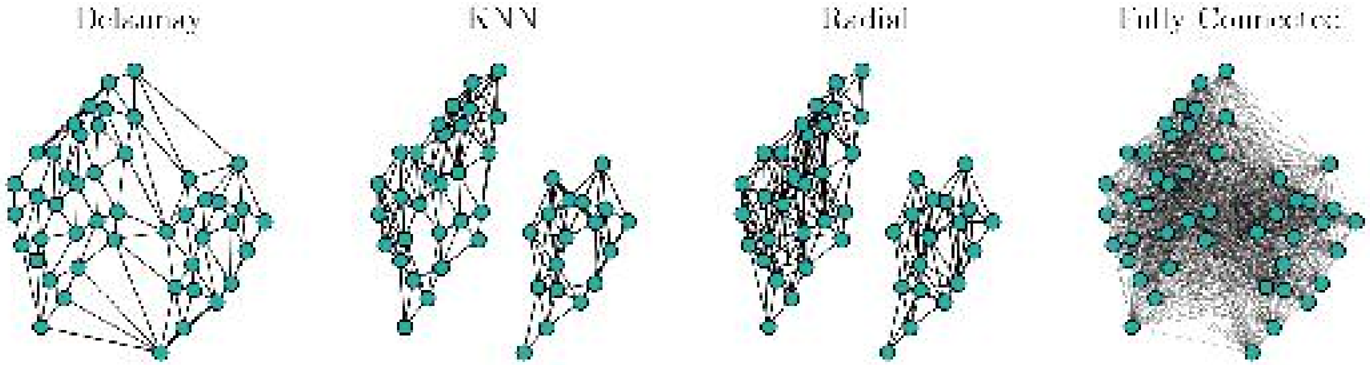
Harnessing the power of graph representations for transfer across populations/fleets

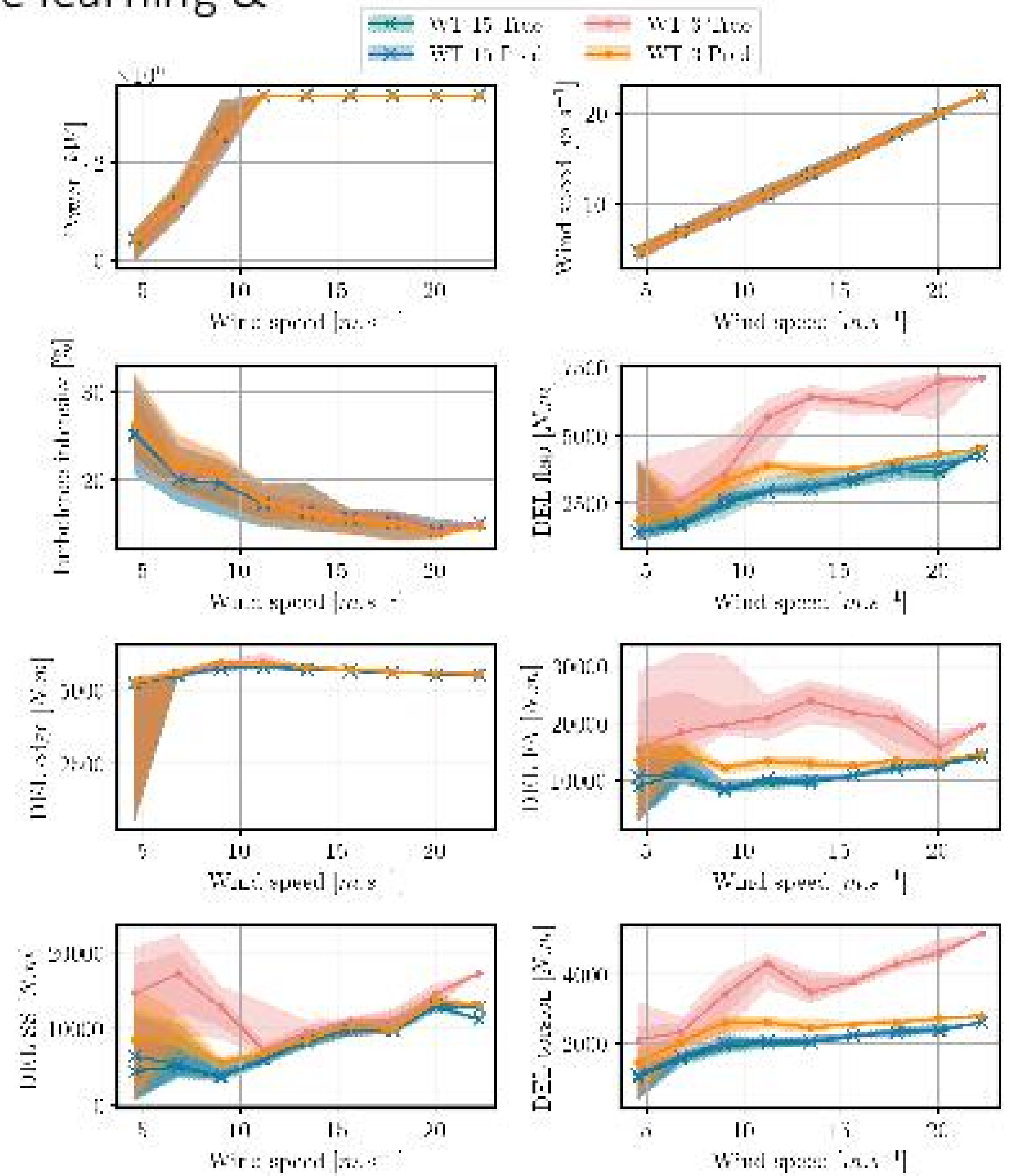


Application: Population-based Prediction for Wind Farms

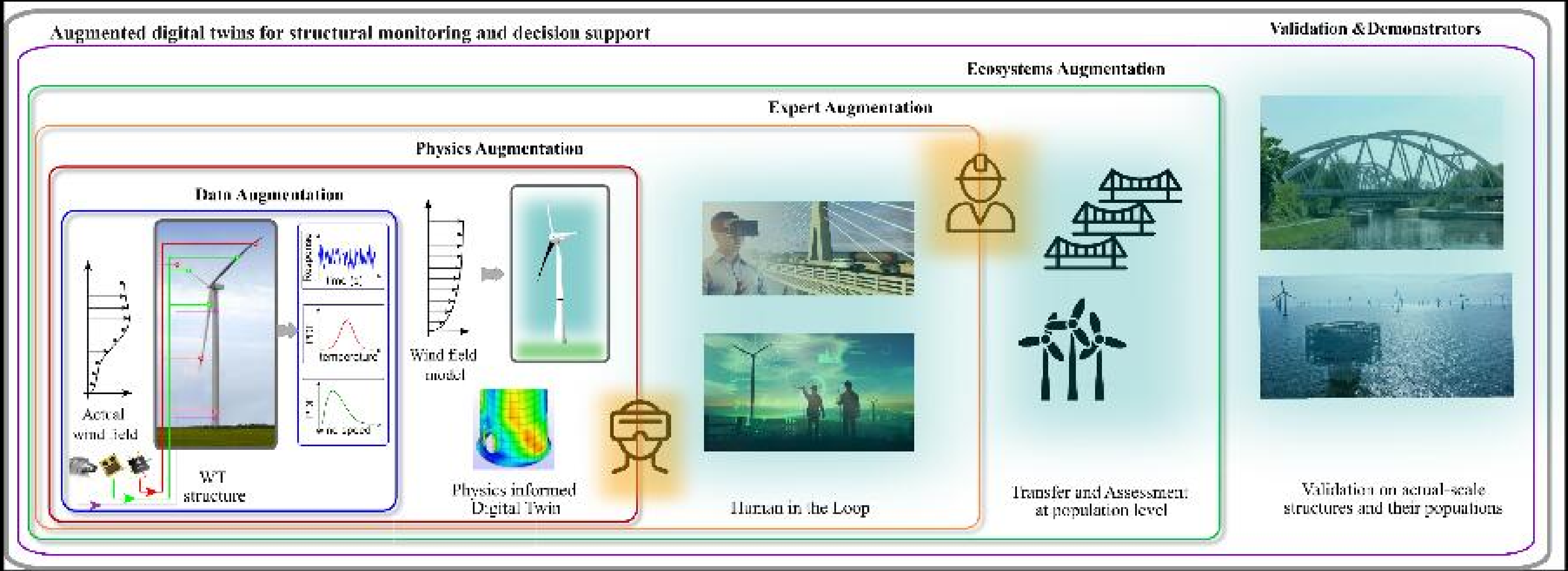
Harnessing the power of graph representations we can enhance learning & transfer across multiple digital/physical assets







Augmented digital twins for structural monitoring & decision support



Acknowledgments

- The European Research Council via the ERC Starting Grant WINDMIL (ERC-2015-StG #679843) on the topic of Smart Monitoring, Inspection and Life-Cycle Assessment of Wind Turbines.
- ITN Training Network, MSCA-ITN-2017, DyVirt: Dynamic Virtualisation: modelling performance of engineering structures
- H2020-MSCA-IF-2017 Grant, SiMAero, Simulation-Driven and On-line Condition Monitoring with Applications to Aerospace
- SANDIA National Labs
- TUM-IAS Hans Fischer Fellowship by TÜV SÜD

Sandia National Laboratories

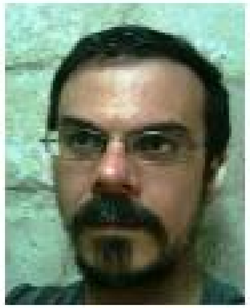
erc European Research Council

WINDMIL

SBB CFF FFS

NRF SINGAPORE

FNSNF



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