Tutorial 3

1. Consider a vibrating string of length one whose position is fixed at both ends. The model is given as

$$\begin{aligned} \frac{\partial^2 w}{\partial t^2}(\zeta,t) =& \; c^2\frac{\partial^2 w}{\partial \zeta^2}(\zeta,t) + \mathbb{1}_{[1/6,1/3]}(\zeta)u_1(t) + \sin(\pi\zeta)u_2(t), \\ w(\zeta,0) =& \; w_0(\zeta), \quad \tfrac{\partial w}{\partial t}(\zeta,0) = w_1(\zeta) \\ w(0,t) =& \; 0, \quad w(1,t) = 0. \end{aligned} \tag{1}$$

$w(\zeta,t)$ represents the deviation from the rest position at spatial point $\zeta \in [0,1]$ and time $t \geq 0$, $w_0(\zeta)$ the initial profile and $w_1(\zeta)$ is the initial velocity. Furthermore, c is a positive constant and u_1, u_2 denote the inputs.

   (a) Formulated (1) as the abstract differential equation on the (energy) state space $X = L^2((0,1);\mathbb{R}^2)$

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad x(0) = x_0$$

   (b) Show that for u identically zero, we have a unique weak solution not growing in the energy.

   (c) Prove that B is a bounded, linear operator from $\mathbb{R}^2$ to X.

   (d) Now we add the output

$$y(t) = \int_0^{\frac{1}{3}} w(\zeta,t)d\zeta$$

   Write this as $Cx(t)$ with C a bounded linear operator. (hard)

2. Consider the following p.d.e.

$$\frac{\partial w}{\partial t}(\zeta,t) = c\frac{\partial w}{\partial \zeta}(\zeta,t), \qquad \zeta \in [0,\ell], \quad t \geq 0, \tag{2}$$

with $c < 0$ and boundary control

$$w(0,t) - w(\ell,t) = u(t) \qquad t \geq 0, \tag{3}$$

and boundary measurement

$$w(\ell,t) = y(t) \qquad t \geq 0, \tag{4}$$

(a) Formulate (2)–(4) as a controlled port-Hamiltonian system and show that it is a well-defined input output system.

(b) Can the change of energy be expressed in the input and output?

3. Consider two vibrating strings which are fixed at the boundary, and there is a control force in the middle, see Figure 1. Furthermore, we

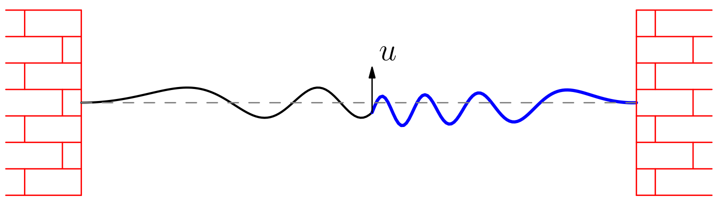

Figure 1: Vibrating string with control.

measure the velocity in the connection.

(a) Formulate the above as a controlled port-Hamiltonian system and show that it is a well-defined input output system.

(b) Can the change of energy be expressed in the input and output?