

Tutorial 4

1. Consider the following p.d.e.

$$\frac{\partial w}{\partial t}(\zeta, t) = -c \frac{\partial w}{\partial \zeta}(\zeta, t), \quad \zeta \in [0, \ell], \quad t \geq 0, \quad (1)$$

with $c > 0$ and boundary control

$$w(0, t) - w(\ell, t) = u(t) \quad t \geq 0, \quad (2)$$

and boundary measurement

$$w(\ell, t) = y(t) \quad t \geq 0, \quad (3)$$

- (a) Determine the transfer function.
 - (b) Is the transfer function positive real?
2. Consider a vibrating string of length ℓ meters whose position is fixed at the left end. The velocity is measured at the right end and which there is also a force applied. The model is given as

$$\begin{aligned} \rho \frac{\partial^2 w}{\partial t^2}(\zeta, t) &= T \frac{\partial^2 w}{\partial \zeta^2}(\zeta, t) \\ w(\zeta, 0) &= w_0(\zeta), \quad \frac{\partial w}{\partial t}(\zeta, 0) = w_1(\zeta) \\ \frac{\partial w}{\partial t}(0, t) &= 0, \quad \frac{\partial w}{\partial t}(\ell, t) = y(t), \quad \frac{\partial w}{\partial \zeta}(\ell, t) = u(t). \end{aligned} \quad (4)$$

$w(\zeta, t)$ represents the deviation from the rest position at spatial point $\zeta \in [0, \ell]$ and time $t \geq 0$, $w_0(\zeta)$ the initial profile and $w_1(\zeta)$ is the initial velocity. Furthermore, ρ is the positive constant mass density and T is the constant Young's modulus.

- (a) Determine the transfer function of (4).
- (b) Functions like exp, sin and cos can only work on unit-free variables. Check that this happens in the expression of your transfer function.
- (c) Determine the unit of the transfer function in two ways.