Tutorial 1: Modeling

Finite dimensional Mass-spring system (MS): We consider the mass spring of Figure (1).

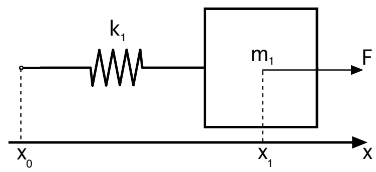

Figure 1: Mass spring system.

- What are the extensive variables characterizing this system ?
- Give the expression of the total energy as a function of the energy variables.
- Write the balance equations associated to this system.
- Give the port Hamiltonian representation of this system considering the external force at point 1 and the velocity at point 0 as inputs. What are the conjugated output ?

Chain of MS: We consider now the interconnexion of two MS systems as described in Figure 2.

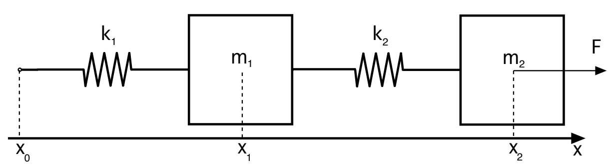

Figure 2: Interconnexion of MSD systems.

- Give the port Hamiltonian representation of each subsystem considering the same input/output than in the previous subsection.
- Write the interconnexion relations between the two subsystems.

- Give the port Hamiltonian representation of the overall system.
- Extend this representation to a n-elements chain.

Finite dimensional Mass-spring damper system (MSD) : We consider now that each element contains a viscous damping term.

- Give the port Hamiltonian representation of each subsystem considering the general form:

$$\begin{aligned} \dot{x} &= [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + [G(x) - P(x)]\, u \\ y &= [G(x) + P(x)]^\top \frac{\partial H}{\partial x}(x) + [M(x) + S(x)]\, u \end{aligned} \tag{1}$$

  where the matrices $J(x)$, $M(x)$, $R(x)$, $P(x)$, $S(x)$ satisfy the skew-symmetry conditions $J(x) = -J^\top(x)$, $M(x) = -M^\top(x)$, and the non negativity condition

$$\begin{pmatrix} R(x) & P(x) \\ P^\top(x) & S(x) \end{pmatrix} \geq 0, \quad x \in \mathcal{X} \tag{2}$$

- Write the corresponding energy balance equation.
- Give the port Hamiltonian representation of two interconnected sub systems.
- Extend this representation to a n-elements chain.

Longitudinal vibration of a beam: We consider now the longitudinal deformation of a beam of length L. $u(z,t)$ represents the deformation of the beam, with z the spatial variable, $z \in [0, L]$. The properties of the beam are defined by the distributed mass ρ and the Young modulus E (A is the cross section of the beam). The strain of the beam ε is defined by $\varepsilon = \frac{\partial u}{\partial z}(z,t)$. We consider the stress $\sigma(z,t)$ depends linearly to the strain $\sigma(z,t) = E\varepsilon(z,t)$.

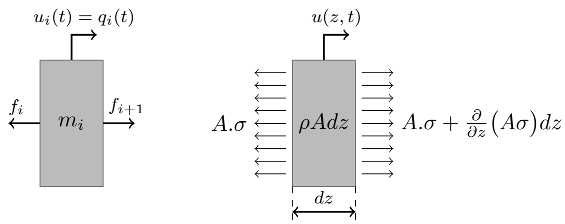

Figure 3: Local description of a beam.

- By analogy with the previous subsections, define the extensive variables associated with this system.
- Give the expression of the total energy of this system.
- Define the co-energy variables and deduce from the balance equations the port Hamiltonian representation of this system.
- Write the balance equation on the energy and the conditions associated with the skew symmetry of the differential operator.