Tutorial 4

1. Consider the following p.d.e.

$$\frac{\partial w}{\partial t}(\zeta,t) = -c\frac{\partial w}{\partial \zeta}(\zeta,t), \qquad \zeta \in [0,\ell], \quad t \geq 0, \tag{1}$$

   with $c > 0$ and boundary control

$$w(0,t) - w(\ell,t) = u(t) \qquad t \geq 0, \tag{2}$$

   and boundary measurement

$$w(\ell,t) = y(t) \qquad t \geq 0, \tag{3}$$

   (a) Determine the transfer function.

   (b) Is the transfer function positive real?

2. Consider a vibrating string of length ℓ meters whose position is fixed at the left end. The velocity is measured at at the right end at which there is also a force applied. The model is given as

$$\begin{aligned} \rho\frac{\partial^2 w}{\partial t^2}(\zeta,t) =& \; T\frac{\partial^2 w}{\partial \zeta^2}(\zeta,t) \\ w(\zeta,0) =& \; w_0(\zeta), \quad \tfrac{\partial w}{\partial t}(\zeta,0) = w_1(\zeta) \\ \tfrac{\partial w}{\partial t}(0,t) =& \; 0, \quad \tfrac{\partial w}{\partial t}(\ell,t) = y(t), \quad \tfrac{\partial w}{\partial \zeta}(\ell,t) = u(t). \end{aligned} \tag{4}$$

   $w(\zeta,t)$ represents the deviation from the rest position at spatial point $\zeta \in [0,1]$ and time $t \geq 0$, $w_0(\zeta)$ the initial profile and $w_1(\zeta)$ is the initial velocity. Furthermore, ρ is the positive constant mass density and T is the constant Young's modulus.

   (a) Determine the transfer function of (4).

   (b) Functions like $\exp, \sin$ and $\cos$ can only work on unit-free variables. Check that this happens in the expression of your transfer function.

   (c) Determine the unit of the transfer function in two ways.