# Tutorial 6: Stabilization - Control design

## Boundary control of a mass spring chain

**Modeling.** We consider a chain of mass spring systems attached at one side and controlled using force at the other side. In what follows for a sake of simplicity we consider the case $m_i = 1$ and $k_i = 1$, $i \in \{1, \cdots n\}$.

1. Recall the port Hamiltonian model for a two mass-spring chain.
2. Give the expression of the state matrices in the case of a n mass-spring chain.

**Control by interconnection.** We connect the system in a power preserving way to a PH controller.

1. Write the interconnection relations.
2. Give the closed loop equation of this system.
3. Choose the controller matrices such that there exist Casimir invariants of the form:
$$C(x, x_c) = x_c + \Gamma^T x$$
4. Write the resulting control law and do the projection in order to obtain a state feedback. Can it be reduced to a boundary feedback.
5. Simulate the closed loop response of the system.

## Boundary control of an elastic string

We consider the boundary control of an elastic string (that could be applied to any 1D wave equation) described by the following equation:

$$\frac{\partial^2 w}{\partial t^2}(\zeta, t) = \frac{1}{\rho(\zeta)} \frac{1}{\partial \zeta} \left[ T(\zeta) \frac{\partial w}{\partial \zeta}(\zeta, t) \right] \tag{1}$$

### Modeling - Control design (1)

1. Write the equivalent port Hamiltonian formulation of (1). Define the boundary port variables.
2. We consider we impose the velocity at the left side of the string and the force at the right side. Define the associated boundary control system.

3. We intend to stabilize the string by damping injection considering that the force at the right side is proportional to the velocity at the same point. Give the condition that such feedback has to fulfil in order to stabilize (asymptotically) the system.

**Simulations** In this section we use the files:

`physicParam.m`
`main.m`

1. Analyze briefly the structure of the file main.m.
2. By using the option "wave" simulate the open loop response of the system to a force step applied at the right side of the string.
3. Implement the previously defined control law. What is the main effect of tuning the control parameter ? What is the limitation of this approach.

**Control design (2)** We consider now a dynamic controller.

1. Proceed to the power preserving interconnection.
2. Write the closed loop equation of the system.
3. Choose the controller matrices such that there exist Casimir invariants.
4. Write the resulting control law and do the projection in order to obtain a state feedback. Can it be reduced to a boundary feedback.
5. Simulate the closed loop response of the system.